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On an Azimuthally Symmetric Theory *of* Gravitation II

Outflows as a Gravitational Phenomena

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ABSTRACT

This is a second reading on the Azimuthally Symmetric Theory *of* Gravitation (ASTG) set-out in Nyambuya (2009). This theory is built on Poisson's well known equation and it has been shown there-in (Nyambuya 2009) that the ASTG is capable of explaining – from a purely classical physics standpoint; the precession of the perihelion of solar planets as being a consequence of the azimuthal symmetry emerging from the spin of the Sun - this symmetry has and must have an influence on the emergent gravitational field. We show herein that the emergent equations from the ASTG, do possess repulsive gravitational fields in the polar regions of the gravitating body in question, thus placing the ASTG on an interesting pedal to infer the origins of outflows as a gravitational phenomena. Outflows are an ubiquitous phenomena found in star forming systems and their true origins is a question yet to be settled. Given the current thinking on their origins, the direction that the present reading takes is nothing short of an asymptotic break from conventional wisdom; at the very least, it is a complete paradigm shift as gravitation is not at all associated; let alone considered to have anything to do with the out-pour of matter but is thought to be an all-attractive force that tries only to squash matter together into a single point. Additionally, we show that the emergent Azimuthally Symmetric Gravitational Field from the ASTG strongly suggests a solution to the supposed *Radiation Problem* faced by massive stars in their process of formation, *i.e.*, at $\sim 10\,\mathcal{M}_{\odot}$ radiation from the nascent star is expected to halt the accretion of matter onto the nascent star. We show that in-falling material will fall onto the equatorial disk and from there, this material will be channeled onto the forming star *via* the equatorial plane thus accretion of mass continues well past the curtain value of $\sim 10\,\mathcal{M}_{\odot}$ albeit *via* the disk. Along the equatorial plane, the net force (with the radiation force included) on any material there-on right up-till the surface of the star, is directed toward the forming star, hence accretion of mass by the nascent star is un-hampered.

Key words: azimuthal symmetry, core, spin, outflow, radiation problem, ring of maser.

1 INTRODUCTION

Champagne like bipolar molecular outflows are the unexpected and concurrently they are the most spectacular phenomena intimately associated with newly formed stars. Studies of bipolar outflows reveal that they [bipolar outflows] are ubiquitous toward High Mass Star (HMS) forming regions. These outflows in HMS forming regions are far more massive and energetic than those found associated with Low Mass Stars (LMS) forming regions (see *e.g.* Shepherd & Churchwell 1996a; Shepherd & Churchwell 1996b; Zhang *et al.* 2001; Zhang *et al.* 2005; Beuther 2002). Obviously this points to a correlation between the mass of the star and the outflow itself. Independent studies have established the existence of such a correlation; the mass outflow rate $\dot{\mathcal{M}}_{out}$ has been shown to be related to the bolometric luminosity L by the relationship $\dot{\mathcal{M}}_{out} \propto L^{0.6}$ for $0.3L_{\odot} \leqslant L \leqslant 10^{5}L_{\odot}$ (we shall use the term luminosity to mean bolometric luminosity). Another curious property of outflows is that the mass-flow rate, $\dot{\mathcal{M}}_{out}$, is related to the speed of the molecular outflow $\dot{\mathcal{M}}_{out} \propto v^{-\gamma}$ where $\gamma \sim 1.8$. How and why outflows come to exhibit these properties is an interesting field of research that is not part of the present. We simple want find out what powers these fountains and once we have a full-fledged numerical model, we would try to answer the above and other questions surrounding the nature of outflows.

Pertaining to their association with star formation activity, it is believed that molecular outflows are a necessary part of the star formation process because their existence may explain the apparent angu-

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lar momentum imbalance. It is well known that the amount of initial angular momentum in a typical star-forming cloud core is several orders of magnitude too large to account for the observed angular momentum found in formed or forming stars (see *e.g.* Larson 2003b). The sacrosanct Law *of* Conservation of angular momentum informs us that this angular momentum can not just disappear into the oblivion of interstellar spacetime; so the question is where does this angular momentum go to? It is here that outflows are thought to come to the rescue as they can act as a possible agent that carries away the excess angular momentum. This angular moment if it where to remain as part of the nascent star, it would, *via* the strong centrifugal forces, tear the star apart. This however does not explain, why they exist and how they come to exist but simple posits them as a vehicle needed to explain the mystery of "*The Missing Angular Momentum*" in star forming systems and the existence of stars in their intact and compact form as firery balls of gas. As we shall see – in accordance with the ASTG, this supposition may well have some basis.

In the existing literature *viz* the question why and how molecular outflows exist, there are about four proposed leading models that endeavor to explain the aforesaid: These four major proposals are:

Wind Driven Outflows: In this model, a wide-angle radial wind blows into the stratified surrounding ambient material, forming a thin swept-up shell that can be identified as the outflow shell (see Shu *et al.* 1991; Li & Shu 1996; Matzner & McKee 1999).

Jet Driven Bow Shocks: In this model, a highly collimated jet propagates into the surrounding ambient material producing a thin outflow shell around the jet (see Raga *et al.* 1993; Masson & Chernin 1993).

Jet Driven Turbulent Outflows: In this model, Kelvin-Helmholtz instabilities along the jet and or environmental boundary leading to the formation of a turbulent viscous mixing layer, through which the molecular cloud gas in entrained (see Cantó & Raga 1991; Raga *et al.* 1993; Stahler 1994; Lizano & Giovanardi 1996; Cantó, Raga & Riera 1993).

Circulation Flows: In this model, the molecular outflow is not entrained by an underlaying wind jet but is rather formed by in-falling matter that is deflected away from the protostar in the central torus of high magneto-hydrodynamic pressure through a quadrupolar circulation pattern around the protostar and is accelerated above escape speeds by local heating (see Fiege & Henriksen 1996a; Fiege & Henriksen 1996a).

All these models and some that are not mentioned here explain outflows as a feedback effect. The endeavor of this reading is to make an alternative suggestion albeit a complete, if not a radical departure from the already existing models briefly discussed above and thus we see no need to get into the details of these models. We say our model is a complete departure from the already existing models because – of all the agents that could lead to outflows, gravitation is not even considered to be a possible agent because it is thought of as, or assumed to be, an all-attractive force. Actually, the idea of a gravitating body such as a star producing a repulsive gravitational field, is at the very least unthinkable. Contrary to this, we show here that an azimuthally symmetric gravitational system does *in-principle* give rise to a bipolar repulsive gravitational field and this – in our view, clearly suggests that these regions of repulsive gravitation, possibly are the actual driving force of the bipolar molecular outflows. We also see that the ASTG provides a neat solution (possibly and very strongly so) to the so-called *Radiation Problem* (Larson & Starrfield 1971; Kahn 1974; Bonnell *et al.* 1998; Bonnell & Bate 2002; Palla & Stahler 1993) thought to bedevil and bewilder the formation of HMSs and as-well the observed *Ring of Masers* (Bartkiewicz *et al*. 2008; Bartkiewicz *et al*. 2009).

It is important that we mention here as part of the epilogue to this introductory section that this reading is fundamental in nature and because of this, we shall seek to begin whatever argument we seek to rise, from the soils of its very basic and fundamental level. This is done so that we are at the same level of understanding with the reader. With the aforesaid approach, if at any point we have errored, it would be easy to know and understand where and how we have errored.

2 THEORY

Newton's Law of universal gravitation can be written in a more general and condensed form as Poisson's Law, *i.e.*:

$$\vec{\nabla}^2\Phi = 4\pi G\rho, \tag{1}$$

where ρ is the density of matter and $G = 6.667\times10^{-11}kg^{-1}ms^{-2}$ is Newton's universal constant of gravitation and the operator $\vec{\nabla}^2$ written for a spherical coordinate system (see figure 2 for the coordinate setup) is given by:

$$\vec{\nabla}^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}, \tag{2}$$

where the symbols have their usual meanings. For a spherically symmetric setting, the solution to Poisson's equation outside the vacuum space (where $\rho = 0$) of a central gravitating body of mass $\mathcal{M}$ is the traditional Newtonian gravitation whose gravitational potential is given:

$$\Phi(r) = -\frac{G\mathcal{M}}{r}. \tag{3}$$

In the case where there is material surrounding this central mass $\mathcal{M} = \mathcal{M}(r)$, *i.e.*:

$$\mathcal{M}(r) = \int_0^r\int_0^{2\pi}\int_0^{2\pi} r^2\rho(r,\theta,\varphi)\sin\theta d\theta d\varphi dr. \tag{4}$$

we must make the replacement $\mathcal{M} \longmapsto \mathcal{M}(r)$ in (3). We shall solve (1) for both cases of empty and none-empty space and show from these solutions that Poisson's equation entails a repulsive bipolar gravitational field.

2.1 Empty Space Solutions

As already argued in Nyambuya (2009), for a scenario or setting that exhibits azimuthal symmetry such as a spinning gravitating body as the Sun and also the stars that populate the heavens (where the unexpected and spectacular champagne like bipolar molecular outflows

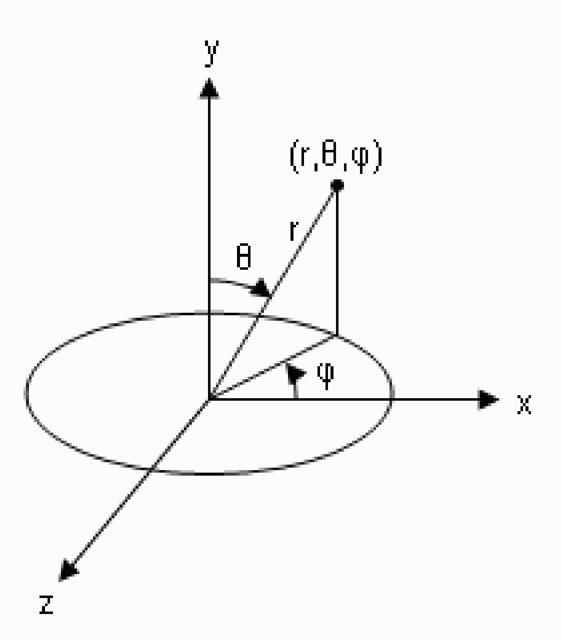

Figure 1. This figure shows a generic spherical coordinate system, with the radial coordinate denoted by r, the zenith (the angle from the North Pole; the colatitude) denoted by θ, and the azimuth (the angle in the equatorial plane; the longitude) by φ.

are the observed); we must have: $\Phi = \Phi(r, \theta)$. We have in Nyambuya (2009) "solved" the Poisson equation for empty space and this solution is:

$$\Phi(r, \theta) = -\sum_{\ell=0}^{\infty}\left[\lambda_\ell c^2\left(\frac{G\mathcal{M}}{rc^2}\right)^{\ell+1} P_l(\cos\theta)\right], \quad (5)$$

where λ_ℓ is an infinite set of dimensionless parameters with $\lambda_0 = 1$ and the rest of the parameters λ_ℓ for $\ell > 1$ will take values different from unity. We shall seek to determine these values from theory in the present reading and also propose them as a means to control outflows. We will show that there lays embedded in (5) a solution that is such that the polar regions of the gravitating central body will exhibit a repulsive gravitational field. It is this repulsive gravitational field that we shall propose as the driving force causing the emergence of outflows. But, we must bare in mind that outflows are seen in regions in which the central gravitating body is found in the immensement of ambient circumstellar material, thus we must solve the Poisson equation for the setting $\rho \neq 0$ for the azimuthally symmetric case (where the central gravitating body is spinning).

2.2 None-Empty Space Solutions

Clearly, in the event that $\rho \neq 0$ for the azimuthally symmetric case, we must have $\rho = \rho(r, \theta)$. This given, the question we wish to answer is; what form does $\Phi(r, \theta)$ take for a given mass distribution $\rho(r, \theta)$? or the reverse, what form does $\rho(r, \theta)$ take for a given $\Phi(r, \theta)$? It is reasonable to assume that gravitation is what influences the distribution of mass and not the other way round. Taking this as the case, then, we must have $\rho(r, \theta) = \rho(\Phi)$. We find that the form for $\rho(r, \theta)$ that meets the requirement $\rho(r, \theta) = \rho(\Phi)$ is:

$$\rho(r, \theta) = -\frac{1}{4\pi G}\left[\frac{2}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\right]\frac{\partial\Phi(r, \theta)}{\partial\theta}. \quad (6)$$

How did we arrive at this? We have but no choice but to answer this question. To make life very easy for us to arrive at the answer, we shall write Poisson's equation in rectangular coordinates, *i.e.*:

$$\left(\sum_{j=1}^{3}\frac{\partial^2}{\partial x_j^2}\right)\Phi(x, y, z) = 4\pi G\rho(x, y, z), \quad (7)$$

where $x_1 = x, x_2 = y, x_3 = z$. Now suppose we had a function $F(x, y, z)$ such that:

$$\left(\sum_{j=1}^{3}\frac{\partial}{\partial x_j}\right)^2 F(x, y, z) = 0. \quad (8)$$

This equation can be written as:

$$\left(\sum_{j=1}^{3}\frac{\partial^2}{\partial x_j^2}\right)F(x, y, z) = -\left(\sum_{j}^{3}\sum_{i\neq j}^{3}\frac{\partial^2}{\partial x_i \partial x_j}\right)F(x, y, z). \quad (9)$$

Now, if and only if the gravitational potential did satisfy (7), then, comparison of (7) with (9) requires $\Phi(x, y, z,) \equiv F(x, y, z)$ and:

$$\rho(x, y, z) = -\frac{1}{4\pi G}\left(\sum_{j}^{3}\sum_{i\neq j}^{3}\frac{\partial^2}{\partial x_i \partial x_j}\right)\Phi(x, y, z). \quad (10)$$

Now, if we transform to spherical coordinates, it is now understood as to why and how we came to the choice of ρ given in (6). At the end of the day, what this means is that we can choose whatever form for Φ, the density ρ will have to conform and prefigure to this setting of the gravitational field *via* (10). Only and only after accepting (10), do we have the mathematical legitimacy to choose to maintain the form (5) which we found for the case of empty space such that in the place of $\mathcal{M}$ we now can put $\mathcal{M}(r)$, hence thus in the case where a central gravitating condensation of mass is in the immensement of ambient circumstellar material, we must have:

$$\Phi(r, \theta) = -\sum_{\ell=0}^{\infty}\lambda_\ell c^2\left(\frac{G\mathcal{M}(r)}{rc^2}\right)^{\ell+1} P_l(\cos\theta), \quad (11)$$

where $\mathcal{M}(r)$ has been defined in the Appendix. We believe this answers the question "What form does $\rho(r, \theta)$ take for a given $\Phi(r, \theta)$?" and at the sametime we have justified (6) *viz* how we have come to it. Importantly, it should be noted that the observed radial density profile is maintained by the choice (10), *i.e.* $\rho(r) = \int_0^r \int_0^{2\pi} r^2 \rho(r, \theta) \sin\theta d\theta dr \propto r^{-\alpha_\rho}$. Also important to state clearly is that, all the above implies that the gravitational field is what influences the distribution of matter – this, in our view, resonates both with logic and intuition.

3 THE UNDETERMINED CONSTANTS λ_ℓ

As already stated in our earlier reading (Nyambuya 2009), one of the draw backs of the ASTG is that it is heavily dependent on observations *for* the values of λ_ℓ have to be determined from observations. Without knowledge of the $\lambda_\ell's$, one is unable to produce the hard numbers required to make any numerical quantifications – a theory incapable of making any numerical quantifications is useless. To avert this, we shall make use of the solar values of the $\lambda_\ell's$ determined in Nyambuya (2009), by making a *reasonable suggestion* and give a general form for these constants.

(1) First things first, *if* the constants λ_ℓ where all independent of each other, then, the theory would clearly be horribly complicated. If we take as guide the philosophy of Occam Razor of the simplicity of a theory, then, these constants must be dependent on each other somehow so as to reduce the labyrinth of complications. The simplest imaginable such dependence is $\lambda_\ell = F(\ell)\lambda_1$; in this way, the entire system of constants λ_ℓ is dependent on just the one constant λ_1.

(2) Second, we could like that on a practical level, only the second order approximation of the theory must suffice, this means the terms $\ell > 3$ must be practically negligible. We have already shown in Nyambuya (2009), that the second order approximation of the ASTG is able to explain a sizable amount of anomalous observations. With the ASTG written in its second order approximation and as we will show herein, one is able to explain from this second order approximation, the emergence of bipolar outflows as a gravitational phenomena without much difficulties. If the other terms beyond the second order approximation become practically significant, one will have difficulties to explain outflows. So in a way, we are not going to pretend but clearly state that, we want to fine tune the theory so that it is able to explain the emergence of bipolar. This is the strongest reason we want the terms for which $\ell > 3$ to be so small such that in practice one can neglect them entirely.

(3) Third and most important, the only data point we have of these constants is the determined values for the Sun, where $\lambda_\ell^\odot = 24.0\pm 7$ and $\lambda_2 = -0.2\pm 0.1$ (see Nyambuya 2009). If logic is to hold, then, our suggestion, $\lambda_1 = F(\ell)\lambda_1$; must be able to explain this.

We find that the following proposal:

$$\lambda_\ell = \left(\frac{(-1)^{\ell+1}}{(\ell^\ell)!\,(\ell^\ell)}\right)\lambda_1, \tag{12}$$

meets (1), (2) and (3). We shall assume this result until such a time evidence to the contrary is brought forth.

Checking on (3) we see that within the error margins $\lambda_2^\odot \simeq [(-1)^{2+1}/\left((2^2)!(2^2)\right]\lambda_1^\odot$. Further checking on (2); from (12) we will have $\lambda_4 = 3.40 \times 10^{-30}\lambda_1$ which is practically small and, the meaning of which is that all terms for which $\ell > 3$ can in practice be neglected entirely.

4 OUTFLOWS AS A GRAVITATIONAL PHENOMENA

None-Empty Space Solutions

Now, if one accepts what has been presented thus far – as will be shown in this section; it follows that outflows may-well be a gravitational phenomena. First, from the previous section, it follows that we must take the ASTG only up to second order, *i.e.*:

$$\Phi = -\frac{G\mathcal{M}}{r}\left[1 + \frac{\lambda_1 G\mathcal{M}\cos\theta}{rc^2} + \lambda_2\left(\frac{G\mathcal{M}}{rc^2}\right)^2 \frac{3\cos^2\theta - 1}{2}\right], \tag{13}$$

where it will be understood that $\mathcal{M} = \mathcal{M}(r)$ since we are dealing with none-empty space case of Poisson's equation. We know that the gravitational field intensity $\vec{\mathbf{g}}(r,\theta) = -\boldsymbol{\nabla}\Phi(r,\theta) = g_r(r,\theta)\hat{\mathbf{r}} + g_\theta(r,\theta)\hat{\boldsymbol{\theta}}$, this means:

$$g_r = g_N\left[\overbrace{1 + \frac{2\lambda_1 G\mathcal{M}\cos\theta}{rc^2}}^{\textbf{Term I}} + \overbrace{3\lambda_2\left(\frac{G\mathcal{M}}{rc^2}\right)^2\left(\frac{3\cos^2\theta - 1}{2}\right)}^{\textbf{Term II}}\right], \tag{14}$$

where $g_N = -G\mathcal{M}(r)/r^2$ is the Newtonian gravitational field intensity and:

$$g_\theta = g_N r^2 \sin\theta\left[\frac{\lambda_1 G\mathcal{M}}{rc^2} + 9\lambda_2\left(\frac{G\mathcal{M}}{rc^2}\right)^2\cos\theta\right]. \tag{15}$$

For gravitation to be attractive as in the normal case $[g_r(r,\theta) > 0]$ and $[g_\theta(r,\theta) > 0]$. From (14) and (15), it is clear that regions of repulsive gravitation will exist and these will occur where $[g_r(r,\theta) < 0]$ and or $[g_\theta(r,\theta) < 0]$. Let us start by treating the case $[g_r(r,\theta) < 0]$. From (14), if $[g_r(r,\theta) < 0]$, then (**Term I** < 0) and (**Term II** < 0) as well. The condition (**Term I** < 0) implies:

$$r < -\lambda_1\left(\frac{2G\mathcal{M}(r)}{c^2}\right)\cos\theta, \tag{16}$$

and this can be written in the equivalent form:

$$r < \left|\lambda_1\left(\frac{2G\mathcal{M}(r)}{c^2}\right)\cos\theta\right|. \tag{17}$$

where the brackets $|[]|$ represents the absolute value – we have to explain this. From (16), it is seen that this inequality includes negative values of r; we need to explain this to avoid any confusion as to what these negative values of r really mean.

Let O, A and P be distinct and separate points on a plane with O and A being fixed and P is a variable point. In polar coordinates, as in the present case, a point P is characterized by two numbers: the distance $r \geqslant 0$ to the fixed pole or origin O, and the angle θ the line OP makes with the fixed reference line OA. The angle θ is only defined up to a multiple of 360° (or 2π; in radians). This is the conventional definition. Sometimes it is convenient as in the present case to relax the condition $r \geqslant 0$ and allow r to be assigned a negative value such that the point (r,θ) and $(-r,\theta+180^\circ)$ represent the same-point, hence thus when ever we have $(-r,\theta)$ this must be replaced by $(r,\theta-180^\circ)$. This can be found in any good mathematics textbook on polar coordinates – hence thus, we have justified (17). Hereafter, whenever a similar scenario arises where negative values of r emerge, we will automatically and without notification assume that $(-r, \theta)$ is $(r, \theta - 180^\circ)$ and this will come with the introduction of the absolute value sign as has been done in (17).

Proceeding ... as has already been explained at the beginning of this section, we have to substitute (A11) in the place of $\mathcal{M}(r)$ in (17) and having done so we would have to make r the subject. This would lead to a horribly complicated inequality that would require the use of the Newton-Ralphson approach to solve – ours in the present is but a qualitative analysis. We can make some very realistic simplifying assumptions that can make our life much easier. If the radial spatial extent of the star is small compared to that of the core, *i.e.* $\mathcal{R}_{star} \ll \mathcal{R}_{core} \Rightarrow \mathcal{R}_{core}^{3-\alpha_\rho} - \mathcal{R}_{star}^{3-\alpha_\rho} \simeq \mathcal{R}_{core}^{3-\alpha_\rho}$ (see Appendix) and the mass of the star is small compared to the mass of the core *i.e.* $\mathcal{M}_{star} \ll \mathcal{M}_{core} \Rightarrow \mathcal{M}_{csl} \simeq \mathcal{M}_{core}$, then the Mass Distribution Function (MDF) is given:

$$\mathcal{M}(r) \simeq \mathcal{M}_{core}\left(\frac{|r|}{\mathcal{R}_{core}(t)}\right)^{3-\alpha_\rho}. \quad (18)$$

Inserting this into (17) and thereafter performing some basic algebraic computations that see r as the subject of the formula, one is lead to:

$$r < \left(\left|\lambda_1\left(\frac{2G\mathcal{M}_{core}}{c^2\mathcal{R}_{core}}\right)\cos\theta\right|^{\frac{1}{2-\alpha_\rho}}\right)\mathcal{R}_{core}. \quad (19)$$

Now, if we set:

$$\epsilon_1 = \left(\left[\lambda_1\left(\frac{2G\mathcal{M}_{core}}{c^2\mathcal{R}_{core}}\right)\right]^{\frac{1}{2-\alpha_\rho}}\right)\left(\frac{\mathcal{R}_{core}}{\mathcal{R}_{star}}\right), \quad (20)$$

then (19) reduces to:

$$r < \epsilon_1\mathcal{R}_{star}\left|\cos\theta\right|^{\frac{1}{2-\alpha_\rho}} = l_{max}\left|\cos\theta\right|^{\frac{1}{2-\alpha_\rho}}, \quad (21)$$

where $l_{max} = \epsilon_1\mathcal{R}_{star}$. On the *xy*-plane as shown in figure (2), the equation $r = l_{max}\left|\cos\theta\right|^{\frac{1}{2-\alpha_\rho}}$ describes two lobs. For the purposes of this reading, let the volume of revolution of the lob be called a lobiod. The loboid above the *x*-axis shall be called the upper loboid and likewise the lobiod below the *x*-axis shall be called the lower lobiod.

Now, proceeding ... the condition (**Term II** < 0) implies $\theta < \cos^{-1}(\pm 1/\sqrt{3})$, which means: $-54.7 < \theta < 54.7$.

Now, for $g_\theta(r, \theta) < 0$, we will have from (15), that:

$$r < -\left(\frac{9\lambda_2}{2\lambda_1^2}\right)\left(\frac{2\lambda_1 G\mathcal{M}(r)}{c^2}\right)\cos\theta, \quad (22)$$

Now going through the same procedure as above, (22) can be written as:

$$r < l_{min}\left|\cos\theta\right|^{\frac{1}{2-\alpha_\rho}}, \quad (23)$$

where:

$$l_{min} = \left(\left|\frac{9\lambda_2}{2\lambda_1^2}\right|^{\frac{1}{2-\alpha_\rho}}\right)l_{max}. \quad (24)$$

Thus radially, the region of repulsive gravitation is:

$$[l_{min} < r < l_{max}]\ \&\ \left[\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) < \theta < \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right], \quad (25)$$

and azimuthally, the region of repulsive gravitation is that described by:

$$\left[r < l_{min}\left|\cos\theta\right|^{\frac{1}{2-\alpha_\rho}}\right]\ \&\ \left[\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) > \theta > \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]. \quad (26)$$

In the region:

$$\left[r < l_{min}\left|\cos\theta\right|^{\frac{1}{2-\alpha_\rho}}\right]\ \&\ \left[\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) < \theta < \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right], \quad (27)$$

the gravitational field is both radially and azimuthally repulsive. Pictorially, a summary of the emergent picture of the repulsive gravitational field in shown in figure (2). This picture – in our view, fits the description of outflows, the limiting factors are the sizes of l_{max} and l_{min}, these values all depend on the one parameter λ_1, hence thus, this parameter is the crucial parameter which determines the properties of outflows. Shortly, we will discuss this picture but before this, it is necessary that we go through the empty space solutions first.

Empty Space Solutions

As will be demonstrated in this section, the picture imaging from the empty space solution is not different from that of the none-empty space solution. However, there is an important difference between these two pictures and this difference need to be stated. If our spinning gravitating body is not giving off material like the Sun, then the region of repulsive gravitation will occur inside the this body. We shall consider the star to be a point mass, *i.e.*, all of its mass is concentrated at the star's center of mass.

As before; from (14) and (15), it is clear that regions of repulsive gravitation will exist and these will occur where $[g_r(r,\theta) < 0]$ and or $[g_\theta(r,\theta) < 0]$. We shall as before start by treating the case $[g_r(r,\theta) < 0]$. From (14), if $[g_r(r,\theta) < 0]$, then (**Term I** < 0) and (**Term II** < 0) as well. The condition (**Term I** < 0) implies:

$$r < -\lambda_1\left(\frac{2G\mathcal{M}}{c^2}\right)\cos\theta, \quad (28)$$

where in the present case $\mathcal{M} = \mathcal{M}_{star}$ and this can be written in the equivalent form:

$$r < \left|\lambda_1\left(\frac{2G\mathcal{M}_{star}}{c^2}\right)\cos\theta\right|. \quad (29)$$

Now, if we set:

$$\epsilon_1 = \lambda_1\left(\frac{\mathcal{R}_{star}^s}{\mathcal{R}_{star}}\right), \quad (30)$$

where $\mathcal{R}_{star}^s = 2G\mathcal{M}_{star}/c^2$ is the Schwarzchild radius of the star, then (29) reduces to:

$$r < \epsilon_1\mathcal{R}_{star}|\cos\theta| = l_{max}|\cos\theta|. \quad (31)$$

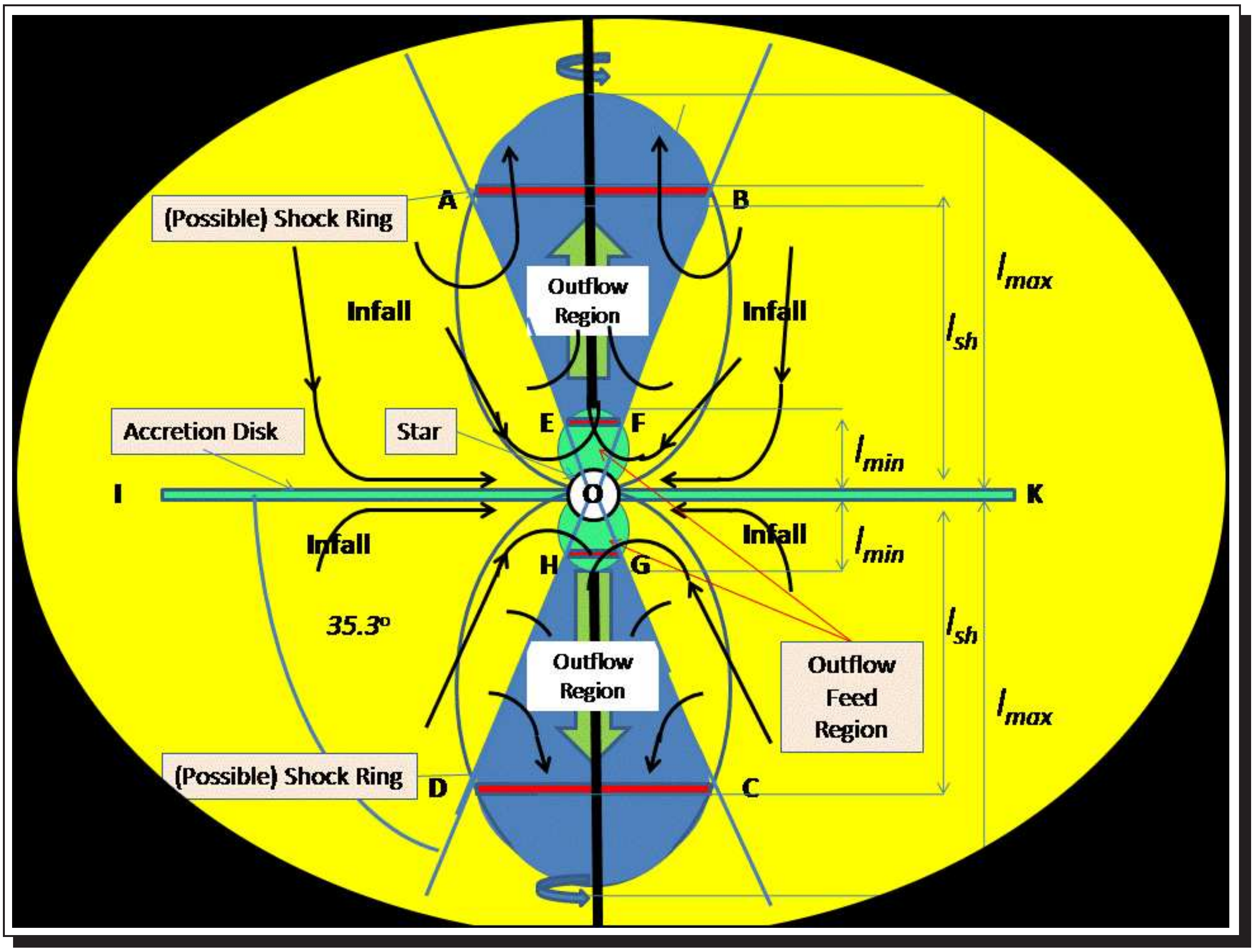

Figure 2. This figure illustrates the emergent picture from the azimuthally symmetric considerations of the Poisson equation. While fanning out matter in the region of repulsive gravitation, the rotating star is surrounded by an equatorial disk; once the outflow switches-on, this disk is the only channel *via* which the mass of the star feeds. The disk is not affected by radiation in the sense that some of its material close to the nascent star will be swept away by the radiation field, no! The force of gravity along this disk is purely radial and is directed toward the nascent.

Proceeding ... the condition (**Term II** <0), as before, implies $\theta < \cos^{-1}(\pm 1/\sqrt{3})$, which means: $-54.7 < \theta < 54.7$.

Again as before, for $g_\theta(r,\theta) < 0$, we will have from (15), that:

$$r < \left|\left(\frac{9\lambda_2}{2\lambda_1^2}\right)\left(\frac{2\lambda_1 G\mathcal{M}_{star}}{c^2}\right)\cos\theta\right|, \tag{32}$$

and we need not explain anymore why the above can be written as:

$$r < l_{min}|\cos\theta|, \tag{33}$$

where this time:

$$l_{min} = \left|\frac{9\lambda_2}{2\lambda_1^2}\right| l_{max}. \tag{34}$$

Thus radially, the region of repulsive gravitation is:

$$[l_{min} < r < l_{max}] \,\&\, \left[\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) < \theta < \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right], \tag{35}$$

and azimuthally, the region of repulsive gravitation is that described by:

$$\left[r < l_{min}\,|\cos\theta|^{\frac{1}{2-\alpha_\rho}}\right] \,\&\, \left[\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) > \theta > \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]. \tag{36}$$

In the region:

$$\left[r < l_{min}\,|\cos\theta|^{\frac{1}{2-\alpha_\rho}}\right] \,\&\, \left[\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) < \theta < \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right], \tag{37}$$

the gravitational field is both radially and azimuthally repulsive. The emergent picture is no different from that of the case of none-empty space. The important difference is that the outflow is confined inside the interior of the star, it is not visible outside unless $\epsilon_1 > 1$. If $\epsilon_1 < 1$, then we would expect there to exist a repulsive bipolar gravitational field. In the interior of the star, the solutions obtained for the case of none-empty space is what must apply.

5 OUTFLOW ANATOMY

Briefly, we shall look into the anatomy of the outflow – we say *briefly* because each of the issues we shall look into requires a separate reading to fully address them. First before we do that, it is important to find-out when does the outflow switch-on and also when does it switch-off. That at some point in time in the evolution of star, outflows switch-on and off is not debatable. So, before we even look into them, it makes perfect sense to investigate this. From figure (2), we see that the anatomy of the outflow has been identified with four regions, *i.e.*, the *Outflow Feed Region*, the *Outflow Region* and the *Shock Ring*. After investigating the switching-on and off of the outflow, we will look into the nature of these regions. Our analysis is qualitative rather than quantitative. We believe a quantitative analysis will require a fully-fledged numerical code. Work on this numerical code is underway.

5.1 Switching-on *of* Outflows

Let us call the lobiod described by (21) the outflow lobiod and likewise the lobiod described by (23) the outflow feed loboid. From the preceding section, it is abundantly clear that we are going to have repulsive bipolar regions whose surface is described by a cone and a outflow lobiod section. From this, we know that the maximum spacial extend of the repulsive gravitational field region will be given by the maximum spatial length of the lobes which occurs when $\cos\theta = 1$, *i.e.* $l_{max} = \epsilon_1 \mathcal{R}_{star}$. Now, to ask the question when does the outflow switch-on amounts to asking when is l_{max} equal to the radius of the star? because the repulsive gravitational field will only manifest beyond the surface of the star *if and only if* the maximum spatial extent of the region of repulsive gravitation is at least equal to the radius of the star, *i.e.*: $l_{max} \geqslant \mathcal{R}_{star}$, this means, $l_{max} = \epsilon_1 \mathcal{R}_{star}$; clearly, this will occur when $\epsilon_1 = 1$. Therefore, outflows will switch-on when the condition $\epsilon_1 = 1$ is reached, otherwise when $\epsilon_1 < 1$, the repulsive gravitational field is confined inside the star.

This strongly suggests that if we are to use the ASGT to model outflows, then we must think of ϵ_1 (hence λ_1) as an evolutionary parameter of the star *i.e.*, this value starts of from a given absolute minimum value (say $\epsilon_1 = 0$) and as the star evolves, this value gets larger and larger until such a time that the repulsive gravitational field is switched on when $\epsilon_1 = 1$, and thereafter it continues to grow and as it grows so does the spatial extend of the outflow (since this parameter controls the spatial size of the region of the repulsive gravitational field).

If the outflow switches on – as it must, the dire question is; "Why does it switch on at that moment when it switches on and not at any other moment? What is so special about that moment when it switches on that triggers it [outflows] to switch on?" As already pointed out at the beginning of this reading; that, it is believed that molecular outflows are a necessary part of the star formation process because their existence does explain – *in principle* – the apparent angular momentum imbalance – *i.e.* to say; the amount of initial angular momentum in a typical star-forming cloud core is several orders of magnitude too large to account for the observed angular momentum found in formed or forming stars (see *e.g.* Larson 2003b) and according to the Law *of* Conservation of angular momentum, this angular momentum can not just disappear into oblivion; so the question is where does this angular momentum go to? This angular momentum if it where to remain as part of the nascent star, it would tear it [star] apart. *Perhaps*, outflows are there to save the nascent star from this catastrophe and fate?!

If the above has any correspondence with reality, then, it makes sense to imagine that at the moment the centrifugal forces are about to tear apart the star, outflows will switch-on. The centrifugal forces have their maximum toll on the equatorial surface of the star. The centrifugal force on the surface of the star acting on a particle of mass m is $F_{ctr} = m\omega^2\mathcal{R}_{star}$ (ω is the spin angular frequency of the star) and the gravitational force on the same particle is $F_g = -G\mathcal{M}m/\mathcal{R}^2_{star}$. If the particle where to stay put on the surface of the star, then we will have $F_{ctr} + F_g = 0$; and if the particle where to fly *off* the surface, we will have $F_{ctr} + F_g > 0$. This means the critical condition before the star begins to be torn apart is that its spin angular frequency must not exceed the critical spin angular frequency $\omega^2_* = G\mathcal{M}/\mathcal{R}^3_{star}$; this means outflows will occur when $\omega^2 > \omega^2_* \Longrightarrow \omega^2/\omega^2_* > 1$. From this it makes sense to set:

$$\epsilon_1 = \left(\frac{\omega}{\omega_*}\right)^a, \tag{38}$$

where a is some constant – in the none-empty space solution, ϵ_1 is given by (40) and in the empty space solution it is given by (30). If and only if ϵ_1 is the same for both the empty and none-empty space solutions, then we can use the solar value $\lambda^\odot_1$ to estimate a. We have $\lambda^\odot_1 \simeq 24$, $\omega^\odot_* = 6.27 \times 10^{-4} Hz$, $\omega^\odot = 2.73 \times 10^{-7} Hz$, $\mathcal{R}_\odot = 6.96 \times 10^8 m$ and $\mathcal{R}^s_\odot = 2.96 \times 10^3 m$, and $a = \log(\lambda^\odot_1 \mathcal{R}^s_\odot/\mathcal{R}_\odot)/\log(\omega^\odot/\omega^\odot_*)$; from all this, it follows that: $a = 1.19$, therefore:

$$\epsilon_1 = \left(\frac{\omega}{\omega_*}\right)^{1.19}. \tag{39}$$

It is important to note that in the empty space case, ϵ_1 is a property of the star while in the none-empty space case, this is a property of the core (*i.e.* Star + Circumstellar material).

In the above, we have looked at the switching-on of outflows and rather tacitally, we have made a suggestion as to what may trigger them, now we have to move further. We know that outflows are not always present, at somepoint in the evolution of the star, they switch-off. What could cause them to do so? Given the reality that within the outflow lobiod, there is the outflow feed lobiod; this too, grows in size as the outflow lobiod grows; at somepoint the outflow and the outflow feed lobiod will become equal – leaving the outflow with no feed point. At this point when the outflow and outflow feed lobiods become equal, clearly, the outflow must switch-off. This occurs when $l_{max} = l_{min}$ and from (24) this means the condition for this to occur is $|\lambda_2| = 2\lambda^2_1/9$ and given that $\lambda_2 = -\lambda_1/96$, this means $\lambda^{max}_1 = 0.046875$. The minimum value of λ_1 needed for outflows to switch-on, occurs when $\epsilon_1 = 1$ and from (40), this means:

$$\lambda^{min}_1 = \left(\frac{\mathcal{R}_{core}}{\mathcal{R}^s_{core}}\right)\left(\frac{\mathcal{R}_{star}}{\mathcal{R}_{core}}\right)^{2-\alpha_\rho}, \tag{40}$$

hence thus outflow activity will take place during which time when ($\lambda_1 : \lambda_1^{min} < \lambda_1 < \lambda_1^{max}$). Specifically, the outflow will switch-on when $\lambda_1 = \lambda_1^{min}$ and switch-off when $\lambda_1 = \lambda_1^{max}$.

5.2 Outflow Feed Region

In the Outflow Feed Region – *i.e.* the region described by (27), both the azimuthal and the radial components of the gravitational force are repulsive, *i.e.* ($g_\theta > 0$) and ($g_r > 0$) respectively. Any material that enters this region is going to be channeled into the Outflow Region, the radial component of the gravitational field is going to channel this matter radially outward while the azimuthal component is going to going to channel this outward radially moving material toward the spin axis, hence it is expected that most of the matter will enter the Outflow Region along the the spin axis of the star. It is important to state that no matter the radiation from the star, there will be no reversal of in-falling matter outside the region of repulsive gravitation due to the radiation field of the nascent star – we shall discuss this in §(7).

5.3 Outflow Region

The Outflow Region is comprised of a section of a cone (OAB & OCD), the outflow lobiod minus the Outflow Feed Region. In this Outflow Region, the gravitational force is only radially repulsive *i.e.*, $g_\theta > 0$ and $g_r < 0$. This means, once the repulsive gravitational force is switched-on and it is in a fully fledged phase, all material found in this region is going to be channeled out of this region both radially while azimuthally being channel to the edge of the fanning cone. The repulsive radial component pushes the material out radially, while the azimuthal component of the gravitational force draws this material close to the edges of the cone. This means the bulk of the outflow material must be found along t the edge of the cone.

Where the cone meets the outflow lobiod, *i.e.*, along AB and CD, there is going to be rings. Considering the ring AB, it is clear that this ring (as CD) must be a shock front since on this ring, along the radial line OA, the in-coming material will meet the outgoing material with equation but opposite radial forces. This equal and but opposite forces must create (radially) a stationery shock. This shock is going to have a ring structure – let us call this the *Shock Ring*. As the rings AB & CD, EF & GH will be rings too, but not shock things. These rings EF & GH are the month of the outflow and matter enters in to the outflow region *via* this opening.

5.4 Shock Rings and Methanol Masers

Given that; (1) AB & CD are shock rings; (2) that methanol masers (amongst other pumping mechanisms) are thought to arise in shock regions and (3) the observations of Bartkiewicz *et al.* (2005) where these authors discovered a ring distribution of $6.7\,GHz$ methanol masers; it is logical to assume that this shock ring may well be a hub of methanol masers arising from the shock present on this ring. Recent and further work by these authors strongly suggests that a Ring *of* Masers is a natural occurrence in star forming regions as (Bartkiewicz *et al*. 2009).

This ring distribution of masers components, they believe strongly suggests the existence of a central source – this is the case here, the central source must exists and it is the forming star. They found an infrared object coinciding with the center of the ring of masers within $78mas$ and this source is cataloged in the 2MASS survey as $2MASS$*183451.56-08182114*. They believe this is an evolving evolving protostar driving this masers *via* circular shocks – this is in line with the the present. Very strongly, the Bartkiewicz Ring *of* Masers suggests – in our opinion that; our outflow model may very well contain an element of truth, that our model contains the possible seeds of resolution of this puzzling occurrence of Ring Masers.

About this shock ring; when viewed from the projection as shown in figure (2), the distance of the shock ring from the star will be:

$$l_{sh} = l_{max}(3)^{-\frac{3-\alpha_\rho}{4-2\alpha_\rho}}, \tag{41}$$

and the radius of this shock ring will be:

$$\mathcal{R}_{ring} = l_{max}(1.5)^{-\frac{3-\alpha_\rho}{4-2\alpha_\rho}}. \tag{42}$$

Clearly, for an isolated system, depending on the orientation relative to the observer, this ring can appear as a linear structure, a circular or an elliptical ring.

At present more than 500 $6.7\,GHz$ methanol masers sources are known to exist (Malyshev & Sobolev 2002; Pestalozzi *et al.* 2005; Xu *et al.* 2003) and are associated with a very early evolutionary phase of high mass star formation. The methanol maser emitting at the $6.7\,GHz$ frequency first discovered by Menten (1991) is the second strongest centimeter masing transition of any molecule (after the $22\,GHz$ water transition) and is commonly found toward star formation regions. It is typically stronger than $12.2\,GHz$ methanol masers (discovered by Batrla *et al.* 1987) observed toward the same region. Methanol masers have become well established tracers or sign spots of high mass star formation regions. It is thought that methanol masers occur in the very early stages of massive star formation.

While methanol masers are found in regions of massive star formation, some have been found with no associated high mass star formation actively (see *e.g.* Ellingsen *et al.* 1996, Szymczak *et al.* 2002. Besides this non-association, some methanol masers are and have been observed to exist in close spatial proximity of massive stars. This has lead to the classification of methanol masers into Class I and Class II. Class I masers emit at the frequencies 25.0, 44.0, $36.0\,GHz$ etc while class II methanol masers emit at 6.7, 12.2, $157.0\,GHz$ *etc* methanol masers is classified as Class II. Class I methanol masers are often observed to exist apart from the continuum sources , while Class II are observed to exist very close, albeit, both classes often co-exist in the same star forming region inside an HII regions (*e.g.* Sobolev *et al.* 2004). Clearly, $l_{sh} = l_{sh}(t)$ and $\mathcal{R}_{ring} = \mathcal{R}_{ring}(t)$ and as the star evolves, l_{sh} and $\mathcal{R}_{ring}$ get larger. This means in the case of young stars, if this ring is a hub of methanol masers, it is expected that methanol masers will be found closer to the star for young HMS and likewise, for more evolved massive stars, methanol masers will be found further from the nascent star. If this is correct, then it may explain the aforesaid; why Class II methanol masers are mostly found close to the nascent star and why Class I methanol masers are found existing further from the nascent star.

High resolution imaging of the 6.7 and 12.2 GHz methanol masers has found that many exhibit a simple elongated linear or curved spatial morphologies (Norris *et al.* 1988; Norris *et al.* 1993; Minier *et al.* 2000) and as already stated, depending on the orientation of the observer relative to the star forming system, the ring may appear as a linear structure. These linear structures have lengths of 50 to 1300 AU. Because of this, one of the possible interpretations that has been entertained for sometime is that the masers originate in the circumstellar accretion disc surrounding the newly formed star (Edris *et al.* 2005) and besides this; because of their strong association with outflows (see *e.g.* Plambeck & Menten 1990; Kalenskii *et al.* 1992; Bachiller *et al.* 1995; Johnston *et al.* 1992), other than originating from the circumstellar disk, also, it has been entertained that methanol maser may originate from outflows (see *e.g.* Pratap & Menten 1992; de Buizer *et al.* 2000). Clearly, the outflow origin of methanol masers resonates with the present ideas. If the ideas herein are correct, then, this reading would of value to researchers seeking an outflow origin of methanol masers.

Further, if viewed from the same view as in figure (2), and if as argued above that masers are found on the ring, one will expect to observe a linear alignment of masers above and below the the nascent star. This would explain the observed linear alignment of methanol masers and also the observed linear alignment of masers above an below the IRAS source found in molecular cloud *G69.489-0.785* (see Fish 2007). Given Fish's observations of blue and red-shifted masers in ON1 (Fish 2007), the suggested model of this ring of masers is interesting as it may offer an explanation of this unexplained and puzzle of red and blue-shifted masers at opposite sides of the IRAS source associated with ON1.

6 COLLIMATION FACTOR

We can calculate the collimation factor of the outflow since we know the extent (l_{max}) and the breath of the outflow which is the size of the shock rings *i.e.*, the collimation factor could be $q_{col} = \mathcal{R}_{ring}/l_{max}$, which can also be written as:

$$q_{col} = (1.5)^{\frac{3-\alpha_\rho}{4-2\alpha_\rho}}, \qquad (43)$$

(this has been deduced from equation 42). From this, we see that as $\alpha_\rho \longmapsto 2$ from $\alpha_\rho = 0$, *i.e.* $\alpha_\rho : 0 \longmapsto 2$, then we will have $q_{col} \longmapsto \infty$. For this setting, generally $q_{col} > 1$. We also realize that now as $q_{col} \longmapsto \infty$ when $\alpha_\rho : 3 \longmapsto 2$ (see Appendix for this notation), then $q_{col} > 1$ and if $\alpha_\rho : 0 \longmapsto 2$. For this setting, generally $q_{col} \geqslant 1.36$. Since we believe the latter is what we must have in nature, that is $\alpha_\rho : 0 \longmapsto 2$ and not $\alpha_\rho : 3 \longmapsto 2$, we should not have outflows with a collimation factors $1 < q_{col} < 1.36$. Because of projection effects, it is difficult if not impossible to verify this result.

Also, because of projection effects, the collimation factor that we measure in real life is not the actual collimation factor but the projected collimation factor. If we know the actual collimation factor, we will be able to know the density index since from (43) we can deduce that:

$$\alpha_\rho = 2 - \left(\frac{\log q_{col}^2}{\log 1.5} - 1\right)^{-1} = \frac{\log(q_{col}^4/8)}{\log(1.5)}. \qquad (44)$$

LMSs are known to have relatively low outflow collimation factors ($q_{col} \sim 20$) while HMSs have significantly high outflow collimation factors, sometimes reach $q_{col} \sim 20$. From (44) the aforesaid implies, assuming these collimation factors are a good representation of the real collimation factor, that LMSs cores have density index $\alpha_\rho = 1.56$ and HMS cores have density index $\alpha_\rho = 1.98$. This is not unreasonable but very much expected. The fact that for HMS forming cores, we have $\alpha_\rho = 1.98$ and for LMS forming cores we have $\alpha_\rho = 1.56$, means HMS cores are much more dense compared to LMS forming cores.

7 RADIATION PROBLEM

While the main thrust of this reading is not to focus on the *Radiation Problem* associated with massive stars but outflows, we find that the ASTG affords us a window of opportunity to visit this problem. *Vis* the radiation problem, it is taken as bona-fide scientific knowledge that our understanding of the formation of massive stars is lacking both theoretically and observationally. On the theoretical front-lines, in the gestation period of a star's life, its mass will grow *via* the infalling envelope and also through the forming accretion disk laying along it's equator – this, as far as our theoretical understanding is concerned, works well for stars less than $\sim 10\mathcal{M}_\odot$. In the literature, it is said that the problem of massive stars ($\mathcal{M}_{star} \geqslant 10\mathcal{M}_\odot$) arises because as the central protostar's mass grows, so does the radiation pressure from it, and at $\sim 10\,\mathcal{M}_\odot$, the star's radiation pressure becomes powerful enough to *halt* any further in-fall (hence accretion) of matter onto the protostar and the disk (Larson & Starrfield 1971; Kahn 1974; Bonnell & Bate 2002; Palla & Stahler 1993). So the problem is - how does the star continue to accumulate more mass beyond the $\sim 10\,\mathcal{M}_\odot$ limit?

If the radiation field really did reverse any further accretion of matter and protostars exclusively accumulated mass *via* direct in-fall and the accretion disk, it could set a mass upper limit of $\sim 10\,\mathcal{M}_\odot$ for any star in the Universe. Unfortunately or maybe fortunately this is not what we observe. It therefore means that some process responsible for the formation of stars beyond the $\sim 10\,\mathcal{M}_\odot$ limit definitely must be a work hence a solution to the problem must be sought. As will be seen very shortly, the reason we have arrived at this conclusion is because we have assumed a spherically symmetric gravitational field and additionally; with this spherically symmetric gravitational field we have not taken into to account the ambient circumstellar material surrounding the nascent star.

If this is the case *i.e.* the radiation problem really did exist as stated above and assuming that we have used the correct gravitational field in our analysis, the solution to the conundrum would be to seek a star formation model that overcomes the radiation pressure problem and at the sametime allowing for the star to form (accumulate all of its mass) before it exhausts its nuclear fuel. Two such models have been put forward, that is (1) the Accelerated Accretion Model (Yorke 2002; Yorke 2003) and (2) the Coalescence Model (Bonnell *et al.* 1998; Bonnell & Bate 2002; Bonnell *et al.* 2006; Bonnell *et al.* 2007).

The second scenario, *i.e.*, the coalescence model (Bonnell *et al.* 1998) is born out of the observational fact that massive stars are generally found in the centers of dense clusters (Hillenbrand 1997; Clarke *et al.* 2000). In these dense environments, the probability of collision of proto-stellar objects is significant, hence the coalescence model. This model easily by-passes the radiation-pressure problem and despite the fact that not a single observation to date has confirmed it (directly or indirectly), it [the coalescence model] appears to be the most natural mechanism by which massive stars form given the said observational fact about massive stars and their preferential environment.

The other alternative, which is less pursued, would be to seek a physical mechanism that overcomes the radiation pressure problem as has been conducted by the authors Krumholz *et al.* (2005). These authors (Krumholz *et al.* 2005) believe that the radiation problem does not exist because radiation-driven bubbles that block accreting gas are subject to Rayleigh-Taylor instability which occurs anytime a dense, heavy fluid is being accelerated by light fluid for example when a cloud receives a shock, or when a fluid of a certain density floats above a fluid of lesser density, such as dense oil floating on water. The Rayleigh-Taylor instabilities allows fingers of dense gas to break into the evacuated bubbles and reach the stellar surface while in addition, outflows from massive stars create optically thin cavities in the accreting envelope. These channel radiation away from the bulk of the gas and reduce the radiation pressure it experiences. In this case, the radiation pressure feedback is not the dominant factor in setting the final size of massive stars and accretion will proceed albeit at much higher rates.

Now, let us define the radiation problem as it is understood in the current literature and for this, we shall follow Yorke (2002); for direct radial accretion and accretion *via* the disk to occur onto the nascent star, explicitly, it is required that the Newtonian gravitational force, $G\mathcal{M}_{star}/r^2$, at a point distance r from the star of mass $\mathcal{M}_{star}$ and luminosity L_{star} at time any given point in time, must exceeds the radiation force $\kappa_{eff}L_{star}/4\pi cr^2$ *i.e.*:

$$\frac{G\mathcal{M}_{star}}{r^2} > \frac{\kappa_{eff}L_{star}}{4\pi cr^2}, \qquad (45)$$

where κ_{eff} is the effective opacity which is the measure of the gas's state of being opaque, a measure of the gas imperviousness to the rays of light and is measured in m^2kg^{-1}. This analysis by Yorke (2002) which is also reproduced in Zinnecker & Yorke (2007), is a standard and well accepted analysis that assumes spherical symmetry and at the sametime it does not take into account the material outside the nascent star. On the other hand, star formation is not a truly spherically phenomena and already argue here since stars are known to have spin angular momentum (also see *e.g.* reviews by Zinnecker & Yorke 2007; McKee & Ostriker 2007) but this simple calculation suffices in as far probing the conditions when radiation pressure becomes a significant player on the star formation podium. In Nyambuya (2008), this same calculation albeit with the circumstellar material taken into account has been performed and a conclusion is arrived at (therein Nyambuya 2008) to the effect that, yes the radiation problem will exist but with the important difference that it will not push all the circumstellar material away but it will create a cavity that grows with the increasing radiation pressure from the nascent star.

This calculation by Yorke (2002) proceeds as follows: the inequality (45), sets a maximum condition for accretion of material, namely $\kappa_{eff} < 4\pi cG\mathcal{M}/L$, and evaluating this, one obtains:

$$\kappa_{eff} < 1.3\times10^4\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)\left(\frac{L}{L_\odot}\right)^{-1}, \qquad (46)$$

where $\mathcal{M}$ and L are in solar units. Given that, $L_{star} = L_\odot\left(\mathcal{M}/\mathcal{M}_\odot\right)^3$, implies that:

$$\kappa_{eff} < 1.3\times10^4\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{-2}. \qquad (47)$$

Now, given that the interstellar medium's (ISM) opacity is measured to be $\sim 20.0\ m^2kg^{-1}$, this sets an upper mass limit for stars of $\sim 10\mathcal{M}_\odot$ for gravitation to dominate the scene before radiation does, thus halting any further in-fall. It is clear here that the ISM's opacity and or the opacity of the molecular cloud material is what sets the $\sim 10\mathcal{M}_\odot$ mass limit thus if there is a way to lower the opacity inside the gas cloud in which the star is forming, the radiation problem would be solved.

The AAM finds some of its grounds around the alteration of the opacity. For example, if the opacity inside the gas cloud is significantly lower then the ISM value, then accretion can proceed via the AAM Model. To reduce the opacity inside the gas cloud, the AAM posits as one of the its options that optical and UV radiation inside the accreting material is shifted from the optical/UV into the far IR and also the that the opacity may be lower than the ISM value because the opacity will be reduced by the accretion of optically thick material in the blobs of the accretion disk. Thus reducing the opacity or finding a physical mechanism that reduces the opacity to values lower than the ISM is a viable solution to the radiation problem. The above mechanism to reduce the opacity are rather mechanical and dependent on the environment. Is there any physical mechanism that exists naturally that can alter the opacity to values lower than the ISM inside the cloud? The ASTG offers a solution and in the subsequent paragraphs, we present this.

In the face of the radiation field, the effective radial component of the gravitational field in the ASTG is given by equation (48) (see overleaf). The radial component is unaffected by the radiation field. Inspection of (48) reveals that along the equation, *i.e.* on the plane where $\theta = 90^\circ$, we will have $g_r(r,\theta) < 0$, the meaning of which is that the effective radial force is attractive right up to the surface of the star! Since $\lambda_2 < 0$, when $\theta = 90^\circ$, $(3\cos^2 90^\circ - 1)/2 = -1/2 < 0$ hence (**Term II** > 0). For **Term I**, we see that when $\theta = 90^\circ$ Term I $= 1 - \kappa L/4\pi G\mathcal{M}c > 0$. We should take note of the fact in writing **Term I** $= 1 - \kappa L/4\pi G\mathcal{M}c > 0$, $\mathcal{M} = \mathcal{M}_{star}$ but $\mathcal{M} = \mathcal{M}(r)$. If $\mathcal{M} = \mathcal{M}_{star}$, then $1 - \kappa L/4\pi G\mathcal{M}c < 0$, thus making (**Term I** < 0), but this not the case. Because $\mathcal{M}(r) < \mathcal{M}_{star}$, it follows that $1 - \kappa L/4\pi G\mathcal{M}c > 0$ thus making (**Term I** > 0). Combing all this, we arrive at the conclusion that $g_r(r,\theta) < 0$, thus on the equator, the effective force is attractive right up to the surface of the star, hence thus, when outflows switch-on, the accretion disk becomes the only official channel *via* which the star's ever-growing appetite for material is serviced.

Clearly, if $\lambda_2 > 0$, then $g_r(r,\theta) > 0$, the meaning of which is that the accretion disk would not exist as it would be dispersed by the force due to **Term II**. This obviously is at odds with experience

$$g_r(r,\theta) = -\frac{G\mathcal{M}}{r^2}\left[\overbrace{1-\frac{\kappa L}{4\pi G\mathcal{M}c}+\frac{2\lambda_1 G\mathcal{M}\cos\theta}{rc^2}}^{\textbf{Term I}}+\overbrace{3\lambda_2\left(\frac{G\mathcal{M}}{rc^2}\right)^2\frac{3\cos^2\theta-1}{2}}^{\textbf{Term II}}\right]. \quad (48)$$

hence thus we have the strongest reason for setting $\lambda_2 > 0$, otherwise the ASTG would be seriously at odds with physical and natural reality as we know it. Though no detailed study of accretion disk has been made (see *e.g.* Brogen *et al.* 2007; Araya *et al.* 2008), it has long been thought that the accretion disk is a means by which accretion of matter on the nascent stars continues soon after radiation has (significantly) sounded her presence on the star formation podium – if our investigation proof correct, as we believe they will, then, we have been right.

That said (the above); to really know whether or not disk accretion is indeed the mechanism *via* which HMSs feed their mass, what needs to be done is to study the necessary accretion rate needed for a massive star of a given mass to form before it can exhaust its nuclear fuel and check this with the predicted disk accretion according to the ASTG. Whether or not the accretion is or is not the mechanism *via* which HMSs feed their mass; from what we have discussed earlier, one thing is clear, accretion *via* the disk can not be halted by radiation at any-point in the evolution of the nascent star.

Before leaving this section, we must ask the question: How does the introduction of the radiation field affect the repulsive force field? Since the radiation field is by nature repulsive, it must only act so as to enhance it. To find out, let us investigate the condition $g_r(r,\theta) > 0$, for this repulsive force field. What constraint does this bring to (r,θ). If $g_r(r,\theta) > 0$, then we must have (**Term I** < 0) and (**Term II** < 0). The condition (**Term II** < 0) leads us back to the original θ-constraint, *i.e.* $125.3 < \theta < 54.7$ and the condition (**Term I** < 0), leads to:

$$1-\frac{\kappa L}{4\pi G\mathcal{M}c}+\frac{2\lambda_1 G\mathcal{M}\cos\theta}{rc^2} < 0. \quad (49)$$

Significantly further from the star $\kappa L/4\pi G\mathcal{M}(r)c \sim 0$, hence thus

$$1+\frac{2\lambda_1 G\mathcal{M}\cos\theta}{rc^2} < 0, \quad (50)$$

and this leads to the same result (21) as before, therefore the region of repulsive gravitation is not affected in as far as its geometry is concerned. also, this mean material in the Outflow Feed Region does not face any in-fall reversal due to the intense radiation pressure from the nascent star. What we would expect is that the radiation will enhance the speed of the outflow and also the channeling of material into the outflow region *via* the Outflow Feed Zone.

8 DISCUSSION AND CONCLUSION

Rather than a final and complete solution, this reading should be taken more as a genesis that lays down the mathematical foundations that seek to lead to the resolution of the problem of outflows, *vis*, what their origin is. Also, we should say that, if this reading is anything go by, *i.e.*, if it proves itself to have a real direct correspondence with the experience of physical reality, then not only have we laid down the mathematical foundations that may lead to the understanding of outflows; but we have laid a three fold foundation that could lead to the resolution of three problems, and these problems are:

- The *Origins and Nature of Outflows*
- The *Radiation Problem* thought to exist for HMS.
- The *Origin of Linear & Ring Structures of Methanol Masers*.

All this we have arrived at after the consideration of the azimuthal symmetry arising from the spin of a gravitating body. This symmetry has been applied to the gravitational field and where upon we have come up with the ASTG. In Nyambuya (2009), we did show that the ASTG can explain the perihelion shift of planets in the solar system. In Nyambuya (2009), the ASTG as it lays therein suffers the setback that the "constants" λ_ℓ are unknown. We have gone so far in the present as to suggest a way to solve this problem but this suggestion is subject to revision pending any new data.

It should be said that, to the best of what I can remember ever-since I learnt that the force of gravity is what causes an apple to fall to the ground and that the very same force causes the moon and the planets to stay in their orbs; I have never really convinced of gravitation as being a repulsive force, let alone that it possibly can have anything to do with the power behind outflows. Just as anyone would find these ideas in violation of their intuition, I find myself in the same bracket. But one thing is clear, the picture emerging from the mathematics thereof, is hard to dismiss. It calls one to make a closer look at the what the Poisson equation is "saying to us".

In closing, allow me to say that as things stand in the present – while we firmly believe we have discovered something worthwhile; it is difficult to make any bold conclusions. Perhaps I should only mention that work has began on a numerical model of outflows based on what we have discovered herein. Only then – I believe; it will be possible to make any bold conclusions.

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APPENDIX A: RADIAL MASS DISTRIBUTION FUNCTION

As already said in the text of the main body of this reading, stellar systems such as cores and molecular clouds are found to exhibit a radial density profiles given by:

$$\rho(r) = \rho_0 \left(\frac{r_0}{|r|}\right)^{\alpha_\rho} \tag{A1}$$

where ρ_0 and r_0 are dynamic normalization constants. We have insert the absolute value of r in (A1) because is demsostrated in the main body that r can take negative values, so we need to take care of this. We shall understood that $r := |r|$.

In order to make sense of this density profile (A1) we shall have to calculate these normalization constants. In its bare form, the power law equation (A1) as it stands implies an infinite density at $r = 0$. Power laws have this property. Obvious one has to deal with this. The usual or typical way is to impose a minimum value for r say $r_{min} = r_0$ and assign a density there. The questions to be answered in this Appendix are:

- (1) What are the physically permisable values of α_ρ?
- (2) What is the emergent radial mass distribution expected from the density profile given in (A1)?

We know that for a radially dependant density profile, the mass distribution is calculated from the integral:

$$\mathcal{M}(r) = \int_{r_0}^{r} 4\pi r^2 \rho(r) dr. \tag{A2}$$

Inserting the density function (A1) into the above integral and then evaluating the resultant integral, we are lead to:

$$\mathcal{M}(r) = \frac{4\pi\rho_0 r_0^{\alpha_\rho}}{3-\alpha}\left(r^{3-\alpha_\rho} - r_0^{3-\alpha_\rho}\right), \tag{A3}$$

The case $\alpha_\rho = 3$ leads to the special form of the MDF:

$$\mathcal{M}(r) = 4\pi\rho_0 r_0^3 \ln\left(\frac{r}{r_0}\right). \tag{A4}$$

We shall not consider this case. Now, what we shall do here is to constrain the α and show that $0 < \alpha < 3$.

Let $r_1 > r_2$. For this setting, we expect that $\mathcal{M}(r_1) > \mathcal{M}(r_2)$ and this is obvious thing because as we radially zooms out of the cloud, one would expect in the sphere of radius r_2 that there will be at least more matter than the engulfed sphere of radius r_1. The condition $\mathcal{M}(r_1) > \mathcal{M}(r_2) \implies \mathcal{M}(r_1) - \mathcal{M}(r_2) > 0$. Using equation (A11), we have:

$$\mathcal{M}(r_1) - \mathcal{M}(r_2) = \frac{4\pi\rho_0 r_0^{\alpha_\rho}}{3-\alpha}\left(r_1^{3-\alpha_\rho} - r_2^{3-\alpha_\rho}\right) > 0, \tag{A5}$$

and for $\alpha > 3$ we have $3 - \alpha < 0$ so when we divide by the term $(4\pi\rho_0 r_0^{\alpha_\rho})/(3-\alpha_\rho)$ on both sides of the inequality, we must change the sign of the inequality from $>$ to $<$ because $(4\pi\rho_0 r_0^{\alpha_\rho})/(3-\alpha_\rho)$ is a negative number. So doing, we will have from this:

$$r_1^{3-\alpha_\rho} - r_2^{3-\alpha_\rho} < 0, \tag{A6}$$

and this implies $r_1^{\alpha-3} < r_2^{\alpha-3}$ and from this, the relationship:

$$r_1 < r_2, \tag{A7}$$

follows directly. This leads us to **contradiction** because it violates our initial condition $r_1 > r_2 \Longrightarrow \mathcal{M}(r_1) > \mathcal{M}(r_2)$. We therefore conclude that: $\alpha < 3$.

Going further, if $3-\alpha_\rho > 3$, it means as one zooms out of the cloud from the center, the cloud's average material density increases. This scenario is unphysical because gravity is an attractive inverse distance law and thus will always pack more and more material in the center than in the outer regions as one zooms out of the clouds from its center and hence the only material configuration that can emerge from this setting is one in which the average density of material decreases as one zooms out of the cloud. This implies $3 - \alpha_\rho < 3$ which leads to $\alpha_\rho > 0$, hence combining the two results we have:

$$0 < \alpha_\rho < 3, \tag{A8}$$

which is our main result and what follows is just a completion of this exersice *i.e.* to compute the emergent radial Mass Distribution Function (MDF).

Now we have to normalize the MDF by imposing some boundary conditions. The usual or traditional boundary condition is to set $\mathcal{M}(r_0) = 0$ and this in actual fact means there will be a cavity of radius r_0 in the cloud. What we shall do is different from this normal or traditional normalization. We shall set $\mathcal{M}(r_0) = \mathcal{M}_{star}$ where $\mathcal{M}_{star}$ is the mass of the central star. Thus what we have done is to place the nascent star in the cavity. This means we must write our MDF as:

$$\mathcal{M}(r) = \frac{4\pi\rho_0 r_0^{\alpha_\rho}}{3-\alpha_\rho}\left(r^{3-\alpha_\rho} - \mathcal{R}_{star}^{3-\alpha_\rho}\right) + \mathcal{M}_{star}, \tag{A9}$$

and this applies for $\mathcal{R}_{star} \leqslant r \leqslant \mathcal{R}_{core}$.

Now, if the mass enclosed inside the core remains constant throughout, then we must have at $r = \mathcal{R}_{core}$ the boundary condition $\mathcal{M}(\mathcal{R}_{core}) = \mathcal{M}_{core}$, thus the circumstellar material $\mathcal{M}_{csl} = \mathcal{M}_{core} - \mathcal{M}_{star}$, and hence:

$$\frac{4\pi\rho_0 r_0^{\alpha_\rho}}{3-\alpha} = \frac{\mathcal{M}_{csl}}{\left(\mathcal{R}_{core}^{3-\alpha_\rho} - \mathcal{R}_{star}^{3-\alpha_\rho}\right)}, \tag{A10}$$

and this means the MDF can now be written as:

$$\mathcal{M}(r) = \overbrace{\mathcal{M}_{csl}\left(\frac{|r|^{3-\alpha_\rho} - \mathcal{R}_{star}^{3-\alpha_\rho}}{\mathcal{R}_{core}^{3-\alpha_\rho} - \mathcal{R}_{star}^{3-\alpha_\rho}}\right)}^{\textbf{Circumstellar Material in Region Radius r}} + \overbrace{\mathcal{M}_{star}}^{\textbf{Mass of the nascent star}}, \tag{A11}$$

where $\alpha_\rho \neq 1$. We shall take this as the final form of our MDF.

The Case $\alpha_\rho = 2$**:** If we have $\alpha_\rho = 2$, then it follows from (A11) that we must have $\mathcal{M}(r) = constant = \mathcal{M}_0$ for $r \geqslant 0$. What does this mean? It means that even at $r = 0$ we must have $\mathcal{M}(0) = \mathcal{M}_0$. From this it follows that this mass is concentrated at a single point. In reality it can not be that all the mass is concentrated onto a single point; what this means is that the density profile with density index $\alpha_\rho = 2$ is unattainable and at the sametime it is the sort for density index by matter when it collapses to form stars. What this effectively means is that if a mass distribution where to start with $\alpha_\rho : 2 < \alpha_\rho \leqslant 3$, then α_ρ must decrease as $\alpha_\rho \longmapsto 2$, *i.e.* as this distribution of mass tries to muzzle itself into a single point. Also if $\alpha_\rho : 0 \leqslant \alpha_\rho < 2$ then α_ρ must increase as $\alpha_\rho \longmapsto 2$ when this distribution of mass tries to muzzle itself into a single point. Observations of number of star forming regions indicate that $\alpha_\rho : 0 \leqslant \alpha_\rho < 2$, thus it is safe to assume that $\alpha_\rho : 0 \longmapsto 2$.