# Santilli's Isomathematical theory for changing modern mathematics 

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#### Abstract

We establish the Santilli's isomathematics based on the generalization of the modern mathematics. Isomultiplication $a \hat{\times} a=a b \hat{T}$, isodivision $a \hat{\div} b=\frac{a}{b} \hat{I}$, where $\hat{I} \neq 1$ is called an isounit, $\hat{T} \hat{I}=1, \hat{T}$ inverse of isounit. Keeping unchanged addition and subtraction, $(+,-, \hat{x}, \hat{\div})$ are four arithmetic operations in Santilli's isomathematics. Isoaddition $a \hat{+} b=a+b+\hat{0}$, isosubtraction $a \wedge=b=b-\hat{0}$ where $\hat{0} \neq 0$ is called isozero, $(\hat{+}, \hat{=}, \hat{x}, \hat{-})$ are four arithmetic operations in Santilli's new isomathematics. We give an example to illustrate the Santilli's isomathematics.


Santilli [1] suggests the isomathematics based on the generalization of the multiplication $\times$ division $\div$ and multiplicative unit 1 in modern mathematics. It
is epoch-making discovery. From modern mathematics we establish the foundations of Santilli's isomathematics and Santilli's new isomathematics. We establish Santilli's isoprime theory of both first and second kind and isoprime theory in Santilli's new isomathematics.

## (1) Division and multiplican in modern mathematics.

Suppose that

$$
\begin{equation*}
a \div a=\stackrel{0}{a} \neq \tag{1}
\end{equation*}
$$

where 1 is called multiplicative unit, 0 exponential zero.
From (1) we define division $\div$ and multiplication $\times$

$$
\begin{gather*}
a \div b=\frac{a}{b}, \quad b \neq 0, a \times b=  \tag{2}\\
a=a \times(a \div a)=a \times a^{0}=a \tag{3}
\end{gather*}
$$

We study multiplicative unit 1

$$
\begin{gather*}
a \times 1=a, a \div 1=a, 1 \div a=1  \tag{4}\\
\left(+1^{n}\right)=1,+\left({ }^{d} 1^{b}\right)=1,{ }^{n}(\neq)-{ }^{n}\left(1^{\prime}\right)^{a},{ }^{b}(=1) \tag{5}
\end{gather*}
$$

The addition + , subtraction - , multiplication $\times$ and division $\div$ are four arithmetic operations in modern mathematics which are foundations of modern mathematics. We generalize modern mathematics to establish the foundations of Santilli's isomathematics.
(2) Isodivision and isomultiplication in Santilli's isomathematics.

We define the isodivision $\hat{\doteqdot}$ and isomultiplication $\hat{x}$ [1-5] which are generalization of division $\div$ and multiplication $\times$ in modern mathematics.

$$
\begin{equation*}
a \hat{\div} a=\bar{a} \hat{a}=\{\nexists, \quad-0 \neq, \tag{6}
\end{equation*}
$$

where $\hat{I}$ is called isounit which is generalization of multiplicative unit $1, \overline{0}$ exponential isozero which is generalization of exponential zero.

We have

$$
\begin{equation*}
a \hat{\div} b=\hat{\wedge} \frac{a}{b}, \quad b \neq 0, \hat{a} a \times b=\hat{\wedge} \tag{7}
\end{equation*}
$$

Suppose that

$$
\begin{equation*}
a=a \hat{\times}(a \hat{\doteqdot} a)=a \hat{\times} a^{\overline{0}}=a \hat{T} \hat{I}=a \tag{8}
\end{equation*}
$$

From (8) we have

$$
\begin{equation*}
\hat{T} \hat{I}=1 \tag{9}
\end{equation*}
$$

where $\hat{T}$ is called inverse of isounit $\hat{I}$.
We conjectured [1-5] that (9) is true. Now we prove (9). We study isounit $\hat{I}$

$$
\begin{align*}
& a \hat{\times} \hat{I}=a \quad \hat{a} \hat{\div} I=, \hat{a} \hat{f} \div \hat{\epsilon^{\hat{I}}} \quad \hat{a}^{2}=1, \tag{10}
\end{align*}
$$

Keeping unchanged addition and subtraction, ( $+,-, \hat{x}, \hat{\doteqdot}$ ) are four arithmetic operations in Santilli's isomathematics, which are foundations of isomathematics. When $\hat{I}=1$, it is the operations of modern mathematics.

## (3) Addition and subtraction in modern mathematics.

We define addition and subtraction

$$
\begin{align*}
& x=a+b \quad y=a-  \tag{12}\\
& a+a-a=  \tag{13}\\
& a-a=0 \tag{14}
\end{align*}
$$

Using above results we establish isoaddition and isosubtraction
(4) Isoaddition and isosubtraction in Santilli's new isomathematics.

We define isoaddition $\hat{+}$ and isosubtraction $\hat{\sim}$.

$$
\begin{align*}
& a \hat{+} b=a+b+c_{1}, a \hat{\wedge} b=a-b-c_{2}  \tag{15}\\
& a=a \hat{+} a \hat{\wedge} a=a+c_{1}-c_{2}=a \tag{16}
\end{align*}
$$

From (16) we have

$$
\begin{equation*}
c_{1}=c_{2} \tag{17}
\end{equation*}
$$

Suppose that $c_{1}=c_{2}=\hat{0}$,
where $\hat{0}$ is called isozero which is generalization of addition and subtraction zero

We have

$$
\begin{equation*}
a \hat{+} b=a+b \hat{\theta}, \quad \hat{a}-b=a-\hat{\lambda} \tag{18}
\end{equation*}
$$

When $\hat{0}=0$, it is addition and subtraction in modern mathematics.
From above results we obtain foundations of santilli's new isomathematics

$$
\begin{align*}
& \hat{\times}=\times \hat{T} \times \hat{+}=+\hat{0}+; \hat{\div}=\times \hat{I} \div, \hat{\div}=-\hat{0}-; a \hat{\times} b=a b \hat{T}, a \hat{+} b=a+b+\hat{0} \\
& a \hat{\div} b=\frac{a}{b} \hat{I}, a \wedge b=a-b-\hat{0} ; a=a \hat{\times} a \hat{\div} a=a, a=a \hat{+} a \wedge a=a \\
& a \hat{\times} a=a^{2} T, a \hat{+} a=2 a+\hat{0} ; a \hat{\div} a=\hat{I} \neq 1, a \wedge a=-\hat{0} \neq 0 ; \hat{T I}=1 \tag{19}
\end{align*}
$$

$(\hat{+}, \hat{-}, \hat{x}, \hat{\doteqdot})$ are four arithmetic operations in Santilli's new isomathematics.
Remark, $a \hat{\times}(b \hat{+} c)=a \hat{\times}(b+c+\hat{0})$, From left side we have
$a \hat{\times}(b \hat{+} c)=a \hat{\times} b+a \hat{\times} \hat{+}+a \hat{\times} c)=a \hat{\times}(b+\hat{+}+c)=a \hat{\times}(b+\hat{0}+c)$, where $\hat{+}=\hat{0}$ also is a number.
$a \hat{\times}(b \wedge c)=a \hat{\times}(b-c-\hat{0})$. From left side we have $a \hat{\times}(b \wedge c)=a \hat{\times} b-a \hat{\times} \wedge-a \hat{\times} c)=a \hat{\times}(b-\hat{\wedge}-c)=a \hat{\times}(b-\hat{0}-c)$, where $\hat{\sim}=\hat{0}$ also is a number.
It is satisfies the distributive laws. Therefore $\hat{+}, \hat{=}, \hat{x}$ and $\hat{\dot{\dot{ }}}$ also are numbers.

It is the mathematical problems in the 21st century and a new mathematical tool for studying and understanding the law of world.
(5)An Example

We give an example to illustrate the Santilli's isomathematics.
Suppose that algebraic equation

$$
\begin{equation*}
y=q \times\left(b_{1}+q_{1}\right)+a_{2} \notin b_{2}- \tag{20}
\end{equation*}
$$

We consider that (20) may be represented the mathematical system, physical system, biological system, IT system and another system. (20) may be written as the isomathematical equation
$\hat{y}=a_{1} \hat{\times}\left(b_{1} \hat{+} c_{1}\right) \hat{+} a_{2} \hat{\doteqdot}\left(b_{2} \hat{\sim} c_{2}\right)=a_{1} \hat{T}\left(b_{1}+c_{1}+\hat{0}\right)+\hat{0}+a_{2} / \hat{T}\left(b_{2}-c_{2}-\hat{0}\right)$.
If $\hat{T}=1$ and $\hat{0}=0$, then $y=\hat{y}$.
Let $\hat{T}=2$ and $\hat{0}=3$. From (21) we have the isomathematical subequation

$$
\begin{equation*}
\hat{y}_{1}=2 a_{1}\left(b_{1}+c_{1}+3\right)+3+a_{2} / 2 \not k_{2}-c_{2} . \tag{22}
\end{equation*}
$$

Let $\hat{T}=5$ and $\hat{0}=6$. From (21) we have the isomathematical subequation

$$
\begin{equation*}
\hat{y}_{2}=5 a_{1}\left(b_{1}+c_{1}+6\right)+6+a_{2} / 5 / k_{2}-c_{2} . \tag{23}
\end{equation*}
$$

Let $\hat{T}=8$ and $\hat{0}=10$. From (21) we have the isomathematical subequation

$$
\begin{equation*}
\hat{y}_{3}=8 a_{1}\left(b_{1}+c_{1}+10\right)+1 \theta a_{2} \quad / \mathscr{B}_{2}\left(-c_{2}-.\right. \tag{24}
\end{equation*}
$$

From (21) we have infinitely many isomathematical subequations. Using (21)-(24), $\hat{T}$ and $\hat{0}$ we study stability and optimum structures of algebraic equation (20).

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