# Signs of supersymmetry 

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#### Abstract

Particles resulting from the breaking of symmetries are arranged symmetrically about mass levels that descend in geometric progression from the Planck Mass within three sequences, of common ratio $1 / \pi, 2 / \pi$ and $1 / \mathrm{e}$. Particles arranged in this way include the weak gauge bosons $\mathrm{W}^{ \pm}$and $\mathrm{Z}^{0}$, the quark weak isospin doublets u-d, s-c and b-t, and all hadronic isospin doublets. Many pairs of hadrons with some common quark content also form symmetric partnerships. Often, the mass difference characterising a partnership is precisely equal to the mass of a level. The spin- 0 partners of the quarks and charged leptons are identified. The mass sequences may derive from the geometry of compact extra spaces of Planck scale.


## 1 Introduction

Particles occupy mass levels and sublevels within three sequences that descend in geometric progression from the Planck Mass $M_{\mathrm{Pl}}\left(1.220892 \times 10^{19} \mathrm{GeV}\right.$ [1]): Sequence 1 is of common ratio $1 / \pi$ [2], Sequence 2 is of common ratio $2 / \pi$ [3] and Sequence 3 is of common ratio $1 / \mathrm{e}$ [4]. Sublevels lie within sequences of common ratio $r^{1 / p}$, where $r$ is the common ratio of the principal sequence and $p$ is a power of 2 or 3 . The common ratios of the principal sequences will be related straightforwardly to the geometry of an orbifold (Sequences 1 and 2) and a sphere (Sequence 3).

Many particles form partnerships that are arranged symmetrically about mass levels or sublevels within one or more of the sequences. Most of the partnerships shown here are centred precisely upon levels within the principal sequences. Often, the mass difference of the partners is precisely equal to the mass of a level within one of the principal sequences.

Examples will be shown of partnerships between particles of equal spin and between particles of spin difference $1 / 2$. Groups of partnerships are displayed.

The values of particle mass used in this paper have been taken from the listings of the Particle Data Group [5].

## 2 Partnerships: signs of broken symmetry

## $\mathbf{W}^{ \pm}$and $\mathbf{Z}^{\mathbf{0}}$

The weak gauge bosons $\mathrm{W}^{ \pm}$and $\mathrm{Z}^{0}$, which result from symmetry breaking in the electroweak theory, are arranged symmetrically about first-order $(p=2)$ sublevels within Sequence 1 and Sequence 3, as shown in Figure 1. The mass difference characteristic of this partnership is equal to the mass of a low order $(p=16)$ sublevel within Sequence 1.

## Quark doublets

The up-down, strange-charm and bottom-top weak isospin doublets are all arranged symmetrically about principal $(p=1)$ levels or low-order sublevels within both Sequence 1 and Sequence 2. To substantiate this statement, the geometric mean of the two quark masses

[^0]of each doublet will be shown to lie upon a principal level or sublevel within Sequence 1 that coincides with a principal level or sublevel within Sequence 2. The geometric mean of the up and down quark masses is assigned a value of 5 MeV . For the strange-charm and bottom-top doublets, the central values of the Particle Data Group's quark mass evaluations are used to calculate the geometric mean representing the doublet. The values resulting from such calculations are, respectively, 365 MeV and 26.8 MeV . The three values of doublet mass are shown in Figure 2 to lie upon equally spaced coincident principal levels and sublevels within Sequences 1 and 2. The values of quark doublet mass suggested by the coincident levels of Sequences 1 and 2 are given in Table 1.


Figure 1: Locations of the weak gauge bosons $W^{ \pm}$and $Z^{0}$ within Sequences 1 and 3


Figure 2: Locations of the quark doublets within Sequences 1 and 2. The up-down doublet has been assigned a value of 5 MeV . The strange-charm and bottom-top doublets are each represented by the geometric mean of the two quark masses, using the central values of the Particle Data Group's evaluations.

| Doublet | Mass | Sequence 1 |  | Sequence 2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Level no. | Mass | Level no. | Mass |
| up-down | $\sim 5 \mathrm{MeV}$ | 43 | 5.12 MeV | 109 | 5.12 MeV |
| charm-strange | $365 \mathrm{MeV}^{*}$ | 39.25 | 375 MeV | 99.5 | 374 MeV |
| top-bottom | $26.8 \mathrm{GeV}^{*}$ | 35.5 | 27.4 GeV | 90 | 27.3 GeV |

* Geometric mean of the Particle Data Group's quark mass evaluations

Table 1: Quark doublet masses

## K meson isospin doublets

The K meson isospin doublets occupy a principal level within Sequence 1, as shown in Figure 3. The doublets are arranged symmetrically about a low-order sublevel within Sequence 2.


Figure 3: Locations of the $K$ mesons within Sequences 1 and 2
$\eta^{\prime}-\Lambda-\Sigma^{0}-K^{*}$
These hadrons with strange quark content form a chain of partnerships. The spin- $0 \eta^{\prime}$ meson partners the $\Lambda$ baryon. The $\Lambda$ and $\Sigma^{0}$ baryons form a partnership. The $\Sigma^{0}$ baryon also partners the spin-1 K* isospin doublet. In partnerships, isospin doublets behave as single particles of mass equal to the geometric mean of the two hadron masses [6]. Each partnership in the chain is centred upon a principal level or low-order sublevel within Sequence 2, as shown in Figure 4. The mass difference of the two hadrons in each partnership is precisely equal to the mass of a principal level within either Sequence 1 or Sequence 2, as shown in Figure 5.


Figure 4: Locations within Sequence 2 of hadrons within the $\boldsymbol{\eta}^{\boldsymbol{\prime}} \boldsymbol{-} \boldsymbol{\Lambda}-\boldsymbol{\Sigma}^{\mathbf{0}}-\mathbf{K} *$ chain. The diamond marks the location corresponding to the geometric mean of the $\mathrm{K}^{*}$ and $\Sigma^{0}$ masses, and of the $\eta^{\prime}$ and $\Lambda$ masses.


Figure 5: Locations within Sequences 1 and 2 corresponding to the mass differences of the partnerships in the $\eta^{\prime}-\Lambda-\Sigma^{0}-K^{*}$ chain

The quark contents of the hadrons in each partnership within the chain are interrelated:

- $\eta^{\prime}(\bar{u} \bar{u}, d \bar{d}, s \bar{s})-\Lambda$ (uds)
- $\quad \Lambda$ (uds) $-\Sigma^{0}$ (uds)
- $\Sigma^{0}(\mathrm{uds})-\mathrm{K}^{*}(\mathrm{u} \overline{\mathrm{s}}, \mathrm{d} \overline{\mathrm{s}} ; \overline{\mathrm{u}} \mathrm{s}, \overline{\mathrm{d}} \mathrm{s})$


## Quark-vector meson partnerships

The quarks have been conjectured to occupy principal levels within Sequence 2 [3]. The hypothetical strange, charm and bottom quark masses are shown in Table 2. Using these values of quark mass, the strange, charm and bottom quarks have been found to form a set of beautiful and precise partnerships with, respectively, the $\phi(\bar{s})$, $\mathrm{Ds}^{*}(\mathrm{c} \overline{\mathrm{s}} ; \overline{\mathrm{c}} \mathrm{s})$ and $\mathrm{Bs}^{*}$ $(\mathrm{s} \overline{\mathrm{b}} ; \overline{\mathrm{s}} \mathrm{b})$ vector mesons upon the principal levels of Sequences 1 and 3, as shown in Figure 6.

| Quark | PDG mass evaluation <br> (central value) | Sequence 2 level <br> number | Level mass |
| :---: | :---: | :---: | :---: |
| strange | 105 MeV | 102 | 121 MeV |
| charm | 1.27 GeV | 97 | 1.16 GeV |
| bottom | 4.20 GeV | 94 | 4.48 GeV |

Table 2: The conjectured quark masses


Figure 6: Locations within Sequences 1 and 3 of the strange, charm and bottom quarks and their vector partners. For each partnership, a diamond marks the location corresponding to the geometric mean of the two masses.

## Quarks, the nucleon isospin doublet and their pseudoscalar partners

Partnerships of the light and charm quarks, and of the nucleon isospin doublet, with pseudoscalar mesons are centred upon the principal levels and low-order sublevels of Sequence 3, as shown in Figure 7.


Figure 7: Locations within Sequence 3 of the quarks and their pseudoscalar partners. For two of the partnerships, a diamond marks the location corresponding to the geometric mean of the two masses.

The up - down isospin doublet is partnered by the $\eta$ meson: the partnership is centred upon a low-order $(p=9)$ sublevel. The proton-neutron isospin doublet and the $\eta^{\prime}$ meson are arranged symmetrically about Level 44; the partnership is also centred upon low-order ( $p=16$ ) sublevels within Sequences 1 and 2. The strange quark is partnered by the $\pi^{0}$ meson: the partnership is centred upon Level 46 in Sequence 3. The charm quark is partnered by the $\eta_{\mathrm{c}}$ (c $\overline{\mathrm{c}}$ ) meson: the partnership is centred upon a low-order $(p=3)$ sublevel within Sequence 3. The neutral spin- 0 partners of the quarks also partner the antiparticles of the quarks.

The mass difference of the charm quark and $\eta_{c}$ is equal to the mass of a principal level within Sequence 2, as shown in Figure 8. The bottom quark is partnered by the $\eta_{\mathrm{b}}(\mathrm{b} \overline{\mathrm{b}})$ meson: the partnership is centred upon a low-order $(p=4)$ sublevel within Sequence 1. The mass difference of the bottom quark and $\eta_{\mathrm{b}}$ is precisely equal to the mass of a principal level within Sequence 1, as shown in Figure 8.


Figure 8: Locations within Sequences 1 and 2 corresponding to the mass differences of the heavy, charm and bottom, quarks and their pseudoscalar partners

## The charged leptons and their pseudoscalar partners

The electron and the $\mathrm{K}^{ \pm}$mesons occupy principal levels within Sequence 1, and are arranged symmetrically about Level 42, as shown in Figure 9. The muon and the $\pi^{ \pm}$mesons, and the tau lepton and the $\mathrm{D}^{ \pm}$mesons, are arranged symmetrically about Levels 102 and 96, respectively, within Sequence 2, as shown in Figure 9. The presumed e- $\mathrm{K}^{-}, \mu-\pi^{-}$and $\tau$ - $\mathrm{D}^{-}$ partnerships are all centred upon principal levels for which the level number is a multiple of 3 .

The locations within Sequences 1 and 3 corresponding to the mass differences characterising the charged lepton-pseudoscalar meson partnerships are shown in Figure 10. Curiously, the $\mu$ $\pi^{ \pm}$and $\tau-D^{ \pm}$mass differences are equal to the $\pi^{0}-\pi^{ \pm}$mass difference multiplied by powers of e.


Figure 9: Locations of the charged leptons and their pseudoscalar partners within Sequences 1 and 2


A Mass difference of the electron and the $\mathrm{K}^{ \pm}$mesons

B Mass difference of the tau lepton and the $\mathrm{D}^{ \pm}$mesons

C Mass difference of the muon and the $\pi^{ \pm}$mesons

D Mass difference of $\pi^{0}$ and $\pi^{ \pm}$

Figure 10: Locations within Sequences 1 and 3 corresponding to the mass differences of the charged leptons and their pseudoscalar partners

## Principal partnerships

The following partnerships are all centred upon principal mass levels:

- The up-down quark doublet (Sequence 1 and Sequence 2)
- The bottom-top quark doublet (Sequence 2)
- The uds baryon partnership $\Lambda-\Sigma^{0}$ (Sequence 2)
- The strange quark- $\phi$ vector meson partnership (Sequence 3)
- The charm quark- $\mathrm{D}_{\mathrm{s}} *$ vector meson partnership (Sequence 1)
- The bottom quark- $\mathrm{B}_{\mathrm{s}}{ }^{*}$ vector meson partnership (Sequence 1)
- The strange quark $-\pi^{0}$ pseudoscalar meson partnership (Sequence 3)
- The proton-neutron isospin doublet- $\eta^{\prime}$ pseudoscalar meson partnership (Sequence 3)
- The electron- $\mathrm{K}^{-}$meson partnership (Sequence 1)
- The muon- $\pi^{-}$meson partnership (Sequence 2)
- The tau lepton-D meson partnership (Sequence 2)


## Spin-0 partners of the quarks and charged leptons

The pseudoscalar partners of the quarks and charged leptons have been identified, and are as shown in Table 3. Also included is the nucleon doublet and its pseudoscalar partner.

| Particle | Spin-0 partner |
| :---: | :---: |
| (up, down) isospin doublet | $\eta^{\prime}$ meson |
| (proton, neutron) isospin doublet | $\eta^{\prime}$ meson |
| strange quark | $\pi^{0}$ meson |
| charm quark | $\eta_{\mathrm{c}}$ meson |
| bottom quark | $\eta_{\mathrm{b}}$ meson |
|  | $\mathrm{K}^{-}$meson |
| electron | $\pi^{-}$meson |
| muon | $\mathrm{D}^{-}$meson |

Table 3: Spin-0 partners of the quarks, nucleon doublet and charged leptons

## 3 Discussion

Particles resulting from the breaking of symmetries are arranged symmetrically about mass levels or sublevels within one or more of the three principal sequences identified here. Partnerships are also characterised by mass differences that are equal to the masses of levels or sublevels. Partnerships between quarks and pseudoscalar mesons, and between charged leptons and pseudoscalar mesons, provide evidence for broken supersymmetry.

The common ratios $1 / \pi$ and $2 / \pi$ of Sequence 1 and Sequence 2 recall the lengths $\pi r_{c}$ and $(\pi / 2) r_{\mathrm{c}}$ of the $S^{1} / Z_{2}$ and $S^{1} / Z_{2} \times Z_{2}$ orbifolds, $r_{\mathrm{c}}$ being the compactification radius of the extra dimensions. These orbifolds are often employed in extra-dimensional models to break the symmetries of the higher-dimensional theory. The quotient of the Planck length scale and the compactification length scale of a Planck-scale $S^{1} / Z_{2}$ orbifold is equal to $1 / \pi$, the common ratio of Sequence 1. The quotient of the Planck length scale and the compactification length scale of a Planck-scale $S^{1} / Z_{2} \times Z_{2}$ orbifold is equal to $2 / \pi$, the common ratio of Sequence 2 . Sequence 1 and Sequence 2 may derive from the geometry of an $S^{1} / Z_{2} \times S^{1} / Z_{2} \times Z_{2}$ toroidal orbifold. The common ratio of Sequence $3,1 / e$, recalls the exponential warp factor $\exp (-k y)$, where $k \sim$ Planck scale and $y$ is the coordinate of an extra dimension, of the Randall and Sundrum braneworld models [7, 8], and may derive from the geometry of a Planck-scale sphere, the radius of which is warped with Planck curvature. The warp factor on the sphere is 1/e.

## 4 References

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