# CONSIDERATIONS ON NEW FUNCTIONS IN NUMBER THEORY 

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#### Abstract

: New functions are introduced in number theory, and for each one a general description, examples, connections, and references are given.

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## Introduction.

In this paper a small survey is presented on eighteen new functions and four new sequences, such as: Inferior/Superior f-Part, Fractional f-Part, Complementary function with respect with another function, S-Multiplicative, Primitive Function, Double Factorial Function, S-Prime and S-Coprime Functions, Smallest Power Function.

1) Let $f: Z \quad--\gg Z$ be a strictly increasing function and $x$ an element in R. Then:
a) Inferior f-Part of $\mathbf{x}$,

ISf(x) is the smallest $k$ such that $f(k)<=x<f(k+1)$.
b) Superior f-Part of $\mathbf{x}$,
$\operatorname{SSf}(\mathrm{x})$ is the smallest k such that $\mathrm{f}(\mathrm{k})<\mathrm{x}<=\mathrm{f}(\mathrm{k}+1)$.

Particular cases:
a) Inferior Prime Part:

For any positive real number $n$ one defines ISp(n) as the largest prime number less than or equal to $n$. The first values of this function are (Smarandache[6] and Sloane[5]): $2,3,3,5,5,7,7,7,7,11,11,13,13,13,13,17,17,19,19,19,19,23,23$.
b) Superior Prime Part: For any positive real number $n$ one defines $\operatorname{SSp}(\mathrm{n})$ as the smallest prime number greater than or equal to $n$.
The first values of this function are (Smarandache[6] and Sloane[5]): $2,2,2,3,5,5,7,7,11,11,11,11,13,13,17,17,17,17,19,19,23,23,23$.
C) Inferior Square Part:

For any positive real number $n$ one defines ISs(n) as the largest square less than or equal to $n$.

The first values of this function are (Smarandache[6] and Sloane[5]): $0,1,1,1,4,4,4,4,4,9,9,9,9,9,9,9,16,16,16,16,16,16,16,16,16,25,25$.
b) Superior Square Part:

For any positive real number $n$ one defines SSs(n) as the smallest square greater than or equal to $n$.
The first values of this function are (Smarandache[6] and Sloane[5]): $0,1,4,4,4,9,9,9,9,9,16,16,16,16,16,16,16,25,25,25,25,25,25,25,25,25,36$.
d) Inferior Cubic Part:

For any positive real number $n$ one defines ISc(n) as the largest cube less than or equal to $n$.
The first values of this function are (Smarandache[6] and Sloane[5]): $0,1,1,1,1,1,1,1,8,8,8,8,8,8,8,8,8,8,8,8,8,8,8,8,8,8,8,27,27,27,27$.
e) Superior Cube Part:

For any positive real number $n$ one defines $S S s(n)$ as the smallest cube greater than or equal to $n$.
The first values of this function are (Smarandache[6] and Sloane[5]): $0,1,8,8,8,8,8,8,8,27,27,27,27,27,27,27,27,27,27,27,27,27,27,27,27,27$.
f) Inferior Factorial Part:

For any positive real number $n$ one defines ISf(n) as the largest factorial less than or equal to $n$.
The first values of this function are (Smarandache[6] and Sloane[5]): $1,2,2,2,2,6,6,6,6,6,6,6,6,6,6,6,6,6,6,6,6,6,6,24,24,24,24,24,24,24$.
g) Superior Factorial Part:

For any positive real number $n$ one defines $\operatorname{SSf(n)}$ as the smallest
factorial greater than or equal to $n$.
The first values of this function are (Smarandache[6] and Sloane[5]):
$1,2,6,6,6,6,24,24,24,24,24,24,24,24,24,24,24,24,24,24,24,24,24,24,120$.
Remark 1: This is a generalization of the inferior/superior integer part of a number (floor function).
2) Let $f: Z \quad--->Z$ be a strictly increasing function and $x$ an element in R. Then:

## Fractional f-Part of $x$,

 FSf(x) $=x-\operatorname{ISf}(x)$,where ISf(x) is the Inferior f-Part of $x$ defined above.

Particular cases:
a) Fractional Prime Part:

FSp (x) $=x$ - ISp(x), where ISp(x) is the Inferior Prime Part defined above. Example: $\operatorname{FSp}(12.501)=12.501-11=1.501$.
b) Fractional Square Part:

FSS (x) $=x-I S s(x)$, where ISs(x) is the Inferior Square Part defined above. Example: $\operatorname{FSs}(12.501)=12.501-9=3.501$.

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c) Fractional Cubic Part:
    FSc(x) = x - ISc(x),
    where ISc(x) is the Inferior Cubic Part defined above.
    Example: FSc(12.501) = 12.501 - 8 = 4.501.
d) Fractional Factorial Part:
    FSf(x) = x - ISf(x),
    where ISf(x) is the Inferior Factorial Part defined above.
    Example: FSf(12.501) = 12.501 - 6 = 6.501.
Remark 2.1: This is a generalization of the fractional part of a number.
Remark 2.2: In a similar way one defines:
- the Inferior Fractional f-Part:
    IFSf(x) = x - ISf(x) = FSf(x);
- and the Superior Fractional f-Part:
    SFSf(x) = SSf(x) - x;
    for example: Superior Fractional Cubic Part of 12.501
    = 27 - 12.501 = 14.499.
```

3) Let $g: A$ - $-->A$ be a strictly increasing function, and let "~" be a
given internal law on $A$. Then we say that
f: A ---> A is complementary with respect to the
function $g$ and the internal law "~" if:
$f(x)$ is the smallest $k$ such that there exists a $z$ in $A$ so that
$x \sim k=g(z)$.

Particular cases:
a) Square Complementary Function:
$\mathrm{f}: \mathrm{N}--->\mathrm{N}, \mathrm{f}(\mathrm{x})=$ the smallest k such that xk is a perfect square.
The first values of this function are (Smarandache[6] and Sloane[5]): $1,2,3,1,5,6,7,2,1,10,11,3,14,15,1,17,2,19,5,21,22,23,6,1,26,3,7$.
b) Cubic Complementary Function:
$\mathrm{f}: \mathrm{N}$---> $\mathrm{N}, \mathrm{f}(\mathrm{x})=$ the smallest $k$ such that xk is a perfect cube.
The first values of this function are (Smarandache[6] and Sloane[5]):
$1,4,9,2,25,36,49,1,3,100,121,18,169,196,225,4,289,12,361,50$.
More generally:
c) m-power Complementary Function:
$\mathrm{f}: \mathrm{N}$---> $\mathrm{N}, \mathrm{f}(\mathrm{x})=$ the smallest k such that xk is a perfect m-power.
d) Prime Complementary Function:
$f: N--->N, f(x)=$ the smallest $k$ such that $x+k$ is a prime. The first values of this function are (Smarandache[6] and Sloane[5]): $1,0,0,1,0,1,0,3,2,1,0,1,0,3,2,1,0,1,0,3,2,1,0,5,4,3,2,1,0,1,0,5$.

## 4) S-Multiplicative Function:

```
\(\star \quad \star\)
A function \(f\) : \(N\)--> N which, for any (a, b) = 1, verifies \(f(a b)=\max \{f(a), f(b)\}\); (i.e. it reflects the main property of the Smarandache function[8]).
```

References:
[1] Castillo, Jose, "Other Smarandache Type Functions", http://www.gallup.unm.edu/~smarandache/funct2.txt
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[6] Smarandache, Florentin, "Only Problems, not Solutions!", Xiquan Publishing House, Phoenix-Chicago, 1990, 1991, 1993; ISBN: 1-879585-00-6. (reviewed in <Zentralblatt fur Mathematik> by P. Kiss: 11002, 744, 1992; and in <The American Mathematical Monthly>, Aug.-Sept. 1991);
[7] "The Florentin Smarandache papers" Special Collection, Arizona State University, Hayden Library, Tempe, Box 871006, AZ 85287-1006, USA; (Carol Moore \& Marilyn Wurzburger: librarians).
[8] Tabirca, Sabin, "About S-Multiplicative Functions", <Octogon>, Brasov, Vol. 7, No. 1, 169-170, 1999.

## 5) Smarandache-Kurepa Function:

For p prime, $S K(p)$ is the smallest integer such that !SK(p) is divisible by $p$, where ! $S K(p)=0!+1!+2!+\ldots+(p-1)!$

For example:

| $p$ | 2 | 3 | 7 | 11 | 17 | 19 | 23 | 31 | 37 | 41 | 61 | 71 | 73 | 89 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

References:
[1] Ashbacher, C., "Some Properties of the Smarandache-Kurepa and Smarandache-Wagstaff Functions", in <Mathematics and Informatics Quarterly>, Vol. 7, No. 3, pp. 114-116, September 1997.
[2] Weisstein, Eric W., "Concise Encyclopedia of Mathematics", CRC Press, Boca Raton, Florida, 1998.
6) Smarandache-Wagstaff Function:

For $p$ prime, $S W(p)$ is the smallest integer such that $W(S W(p))$ is divisible by $p$, where $W(p)=1!+2!+\ldots+(p)!$

For example:

| p | 3 | 11 | 17 | 23 | 29 | 37 | 41 | 43 | 53 | 67 | 73 | 79 | 97 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\mathrm{SW}(\mathrm{p})$ | 2 | 4 | 5 | 12 | 19 | 24 | 32 | 19 | 20 | 20 | 7 | 57 | 6 |

Reference:
[1] Ashbacher, C., "Some Properties of the Smarandache-Kurepa and Smarandache-Wagstaff Functions", in <Mathematics and Informatics Quarterly>, Vol. 7, No. 3, pp. 114-116, September 1997.
[2] Weisstein, Eric W., "Concise Encyclopedia of Mathematics", CRC Press, Boca Raton, Florida, 1998.
7) Smarandache Ceil Functions of $n$-th Order:

Sk(n) is the smallest integer for which $n$ divides $S k(n)^{\wedge} k$.
For example, for $k=2$, we have:
$\begin{array}{lllllllllllllllll}\mathrm{n} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16\end{array}$
$\begin{array}{lllllllllllllllll}\mathrm{S} 2(\mathrm{n}) & 2 & 4 & 3 & 6 & 10 & 12 & 5 & 9 & 14 & 8 & 6 & 20 & 22 & 15 & 12 & 7\end{array}$
References:
[1] Ibstedt, H., "Surfing on the Ocean of Numbers -- A Few Smarandache Notions and Similar Topics", Erhus University Press, Vail, USA, 1997; pp. 27-30.
[2] Begay, A., "Smarandache Ceil Functions", in <Bulletin of Pure and Applied Sciences>, India, Vol. 16E, No. 2, 1997, pp. 227-229.
[3] Weisstein, Eric W., "Concise Encyclopedia of Mathematics", CRC Press, Boca Raton, Florida, 1998.
8) Pseudo-Smarandache Function:
$Z(n)$ is the smallest integer such that $1+2+\ldots+Z(n)$ is divisible by $n$.

For example:

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Z}(\mathrm{n})$ | 1 | 3 | 2 | 3 | 4 | 3 | 6 |

Reference:
[1] Kashihara, K., "Comments and Topics on Smarandache Notions and Problems", Erhus University Press, Vail, USA, 1996.
[2] Weisstein, Eric W., "Concise Encyclopedia of Mathematics", CRC Press, Boca Raton, Florida, 1998.
9) Smarandache Near-To-Primordial Function:
$\operatorname{SNTP}(\mathrm{n})$ is the smalest prime such that either $\mathrm{p}-1, \mathrm{p}$, or $\mathrm{p}+1$ is divisible by $n$,
*
where $p$, of a prime number $p$, is the product of all primes less than or equal to $p$.

For example:

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | $\ldots$ | 59 | $\ldots$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\operatorname{SNTP}(\mathrm{n})$ | 2 | 2 | 2 | 5 | 3 | 3 | 3 | 5 | $?$ | 5 | 11 | $\ldots$ | 13 | $\ldots$ |

References:
[1] Mudge, Mike, "The Smarandache Near-To-Primordial (S.N.T.P.) Function", <Smarandache Notions Journal>, Vol. 7, No. 1-2-3, August 1996, p. 45.
[2] Ashbacher, Charles, "A Note on the Smarandache Near-To-Primordial Function", <Smarandache Notions Journal>, Vol. 7, No. 1-2-3, August 1996, pp. 46-49.
[3] Weisstein, Eric W., "Concise Encyclopedia of Mathematics", CRC Press, Boca Raton, Florida, 1998.

## 10) Double-Factorial Function:

$\operatorname{SDF}(\mathrm{n})$ is the smallest number such that $\operatorname{SDF}(\mathrm{n})$ !! is divisible by n , where the double factorial
$m!!=1 x 3 x 5 x . . . x m$, if $m$ is odd;
and $m!!=2 x 4 x 6 x . . . x m$, if $m$ is even.
For example:

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{SDF}(\mathrm{n})$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 4 | 9 | 10 | 11 | 6 | 13 | 14 | 5 | 6 |

Reference:
[1[] Dumitrescu, C., Seleacu, V., "Some notions and questions in number theory", Erhus Univ. Press, Glendale, 1994, Section \#54 ("Smarandache Double Factorial Numbers").

## 11) Primitive Functions:

Let p be a positive prime.
$S_{p}$ : N ---> N, having the property that $\left(S_{p}(n)\right)$ ! is divisible by $p$,
and it is the smallest integer with this property.
For example:
S (4) = 9, because 9 ! is divisible by $3^{\wedge} 4$, and it is the smallest one 3
with this property.

These functions help computing the Smarandache Function.

## Reference:

[1] Smarandache, Florentin, "A function in number theory", <Analele Universitatii Timisoara>, Seria St. Mat., Vol. XVIII, fasc. 1, 1980, pp. 79-88.

## 12) Smarandache Function:

```
        S : N ---> N, S(n) is the smallest integer such that S(n)! is
```

        divisible by \(n\).
    
## Reference:

[1] Smarandache, Florentin, "A function in number theory", <Analele Universitatii Timisoara>, Seria St. Mat., Vol. XVIII, fasc. 1, 1980, pp. 79-88.
13) Smarandache Functions of the First Kind:

```
        S : N* --> N
        n
```

            \(r\)
        i) If \(n=u\) (with \(u=1\), or \(u=p\) prime number), then
            \(S\) (a) \(=k\), where \(k\) is the smallest positive integer such that
            n
            \(k\) ! is a multiple of \(u^{r a}\);
    ii) If $n=p 1^{r 1} \cdot p 2^{r 2} \ldots t^{r t}, \operatorname{then} S(a)=\underset{n}{\max }\left\{S_{j<=t} \underset{j}{ } \quad\right.$ (a) $\}$.
pj
14) Smarandache Functions of the Second Kind:

```
    k * * k *
S : N --> N , S (n) = S (k) for k in N,
    n
where S are the Smarandache functions of the first kind.
            n
```

15) Smarandache Function of the Third Kind:
b
$S_{a}(n)=S_{a}^{s}\left(b_{n}\right)$, where $S$ is the Smarandache function of the
```
first kind, and the sequences (a ) and (b ) are different from
```

the following situations:
i) $\mathrm{a}_{\mathrm{n}}=1$ and $\underset{\mathrm{n}}{\mathrm{b}}=\mathrm{n}$, for n in N ;
ii) $\mathrm{a}_{\mathrm{n}}=\mathrm{n}$ and $\mathrm{b}_{\mathrm{n}}=1$, for n in $\mathrm{N}^{\text {* }}$.
Reference:
[1] Balacenoiu, Ion, "Smarandache Numerical Functions", <Bulletin of Pure and Applied Sciences>, Vol. 14E, No. 2, 1995, pp. 95-100.
16) S. Prime Functions are defined as follows:

```
P : N --> {0, 1}, with
    P(n)={}{\begin{array}{ll}{0,}&{\mathrm{ if n is prime;}}\\{1,}&{\mathrm{ otherwise. }}
For example P(2)=P(3)=P(5)=P(7)=P(11)=0, whereas
P(0)=P(1)=P(4)=P(6)=... = 1.
```

More general:
$\mathrm{P}_{\mathrm{k}}: \mathrm{N}^{\mathrm{k}}-->\{0,1\}$, where k is an integer $>=2$, and

17) S. Coprime Functions are similarly defined:
$\mathrm{C}: \mathrm{N}_{\mathrm{k}}^{\mathrm{k}}-->\{0,1\}$, where k is an integer $>=2$, and


```
Reference:
    [1] F. Smarandache, "Collected Papers", Vol. II, 200 p., <Functii
        Prime and Coprime>, p. 137, Kishinev University Press,
        Kishinev, 1997.
```

    18) The Smallest Power Function:
        \(S P(n)\) is the smallest number \(m\) such that \(m \wedge k\) is
        divisible by n , where \(\mathrm{k}>=2\) is given.
        The following sequence \(S P(n)\) is generated:
        \(1,2,3,2,5,6,7,4,3,10,11,6,13,14,15,4,17,6,19,10\),
        \(21,22,23,6,5,26,3,14,29,30,31,4,33,34,35,6,37,38\),
        39, 20, 41, 42, ...
        Remarks:
        If \(p\) is prime, then \(S P(p)=p\).
        If \(r\) is square free, then \(S P(r)=r\).
    
If $n=p^{\wedge} s$, where $p$ is prime, then:
p, if $1<=s<=p$;
SP (n) =
$\mathrm{p}^{\wedge} 2$, if $\mathrm{p}+1<=\mathrm{s}<=2 \mathrm{p}^{\wedge} 2$;
$p^{\wedge} 3$, if $2 p^{\wedge} 2+1<=s<=3 p^{\wedge} 3$;
. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
$p^{\wedge} t$, if $(t-1) p^{\wedge}(t-1)+1<=s<=t p^{\wedge} t$.
Generally, if $n=\left(\underset{1}{p} \mathrm{~s}_{1}\right) \mathrm{x} . . \mathrm{x}\left(\underset{\mathrm{k}}{\mathrm{p}} \mathrm{s}_{\mathrm{k}}^{\mathrm{s}}\right)$, with all p prime, then:
$\operatorname{SP}(\mathrm{n})=\left(\mathrm{p}_{1} \wedge \mathrm{t}_{1}\right) \mathrm{x} \ldots \mathrm{x}(\mathrm{p} \wedge \mathrm{t})$, where

for 1 <= i <= k.
a) A second function ( $k=2$ ):

```
1, 2, 3, 2, 5, 6, 7, 4. 3, 10, 11, 6, 13, 14, 15, 4, 17, 6, 19, 10,
21, 22, 23, 12, 5, 26, 9, 14, 29, 30, 31, 8, 33, ...
2
( S2(n) is the smallest integer m such that m is divisible by n )
```

b) A third function $(k=3)$ :
$1,2,3,2,5,6,7,8,3,10,11,6,13,14,15,4,17,6,19,10$,
$21,22,23,6,5,26,3,14,29,30,31,4,33, \ldots$
( $\mathrm{S} 3(\mathrm{n})$ is the smallest integer m such that m is divisible by n )

## 19) A 3n-digital subsequence:

13, 26, 39, 412, 515, 618, 721, 824, 927, 1030, 1133, 1236, ... (numbers that can be partitioned into two groups such that the second is three times biger than the first)

## 20) A 4n-digital subsequence:

14, 28, 312, 416, 520, 624, 728, 832, 936, 1040, 1144, 1248, ... (numbers that can be partitioned into two groups such that the second is four times biger than the first)

## 21) A 5n-digital subsequence:

15, 210, 315, 420, 525, 630, 735, 840, 945, 1050, 1155, 1260, ... (numbers that can be partitioned into two groups such that the second is five times bigger than the first)

## 22) Sequences of Sub-sequences

```
For all of the following sequences:
```

a) Crescendo Sub-sequences:
$1,1,2,1,2,3,1,2,3,4,1,2,3,4,5,1,2,3,4,5,6$, $1,2,3,4,5,6,7,1,2,3,4,5,6,7,8, \quad . \quad . \quad$.
b) Decrescendo Sub-sequences:

```
1, 2, 1, 3, 2, 1, 4, 3, 2, 1, 5, 4, 3, 2, 1, 6, 5, 4, 3, 2, 1,
7, 6, 5, 4, 3, 2, 1, 8, 7, 6, 5, 4, 3, 2, 1, . . .
```

C) Crescendo Pyramidal Sub-sequences:

```
1, 1, 2, 1, 1, 2, 3, 2, 1, 1, 2, 3, 4, 3, 2, 1,
1, 2, 3, 4, 5, 4, 3, 2, 1, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, . . .
```

d) Decrescendo Pyramidal Sub-sequences:

```
1, 2, 1, 2, 3, 2, 1, 2, 3, 4, 3, 2, 1, 2, 3, 4,
5, 4, 3, 2, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, 2, 3, 4, 5, 6, . . .
```

e) Crescendo Symmetric Sub-sequences:
$1,1,1,2,2,1,1,2,3,3,2,1,1,2,3,4,4,3,2,1$, $1,2,3,4,5,5,4,3,2,1,4,2,3,4,5,6,6,5,4,3,2,1, . .$.
f) Decrescendo Symmetric Sub-sequences:
$1,1,2,1,1,2,3,2,1,1,2,3,4,3,2,1,1,2,3,4$, $5,4,3,2,1,1,2,3,4,5,3,5,4,3,2,1,1,2,3,4,5,6, . .$.
g) Permutation Sub-sequences:
$1,2,1,3,4,2,1,3,5,6,4,2,1,3,5,7,8,6,4,2$,
$1,3,5,7,9,10,8,6,4,2,1,3,5,7,9,10,8,6,4,2, . .$.
Find a formula for the general term of the sequence.

Solutions:

For purposes of notation in all problems, let
a (n)
denote the $n$-th term in the complete sequence and
b (n)
the $n$-th subsequence. Therefore, $a(n)$ will be $a$ number and $b(n) a$ sub-sequence.
a) Clearly, b(n) contains $n$ terms. Using a well-known summation formula, at the end of $b(n)$ there would be a total of

$$
\mathrm{n}(\mathrm{n}+1)
$$

$\qquad$
2
terms.
Therefore, since the last number of $b(n)$ is $n$,
$a((n(n+1)) / 2)=n$.
Finally, since this would be the terminal number in the sub-sequence $\mathrm{b}(\mathrm{n})=1,2,3, . . \quad, \mathrm{n}$
the general formula is

$$
a(((n(n+1) / 2)-i)=n-i
$$

for $\mathrm{n}>=1$ and $0<=\mathrm{i}<=\mathrm{n}-\mathrm{i}$.
b) With modifications for decreasing rather than increasing, the proof is essentially the same. The final formula is

$$
a(((n(n+1)) / 2)-i)=1+i
$$

for $\mathrm{n}>=1$ and $0<=\mathrm{i}<=\mathrm{n}-1$.
c) Clearly, $b(n)$ has $2 n-1$ terms. Using the well-known formula of summation

$$
1+3+5+. . .+(2 n-1)=n
$$

## $2 \quad 2$

the last term of $b(n)$ is in position $n$ and $a(n)=1$. The largest number in $b(n)$ is $n$, so counting back $n-1$ positions, they increase in value by one each step until $n$ is reached.

## 2

$a(n-i)=1+i, \quad$ for $0<=i<=n-1$.
2
After the maximum value at $n-1$ positions back from $n$, the values decrease by one. So at the nth position back, the value is $n-1$, at the ( $n-1$ ) st position back the value is $n-2$ and so forth.

2
$a(n-n-i)=n-i-1$
for $0<=i<=n-2$.
d) Using similar reasoning

2
$a(\mathrm{n})=\mathrm{n}$ for $\mathrm{n}>=1$
and

$$
2
$$

a(n - i) $=n-i$, for $0<=i<=n-1$ 2

$$
a(n-n-i)=2+i, \text { for } 0<=i<=n-2 \text {. }
$$

e) Clearly, $b(n)$ contains $2 n$ terms. Applying another well-known summation formula

$$
2+4+6+. . .+2 n=n(n+1), \text { for } n>=1
$$

Therefore, $a(n(n+1))=1$. Counting backwards $n-1$ positions, each term
decreases by 1 up to a maximum of $n$.

$$
a((n(n+1))-i)=1+i, \text { for } 0<=i<=n-1
$$

The value $n$ positions down is also $n$ and then the terms decrease by one back down to one.

$$
a((n(n+1))-n-i)=n-i, \text { for } 0<=i<=n-1
$$

f) The number of terms in $b(n)$ is the same as that for (e). The only difference is that now the direction of increase/decrease is reversed.

$$
\begin{aligned}
& a((n(n+1))-i)=n-i, \text { for } 0<=i<=n-1 . \\
& a((n(n+1))-n-i)=1+i, \text { for } 0<=i<=n-1 .
\end{aligned}
$$

g) Given the following circular permutation on the first n integers.

$$
\underset{\mathrm{n}}{\text { phi }} \quad=\left|\begin{array}{cccccccccc}
1 & 2 & 3 & 4 & \cdot & \cdot & \cdot & \mathrm{n}-2 & \mathrm{n}-1 & \mathrm{n} \\
1 & 3 & 5 & 7 & \cdot & \cdot & . & 6 & 4 & 2
\end{array}\right|
$$

Once again, $b(n)$ has $2 n$ terms. Therefore,

$$
a(n(n+1))=2
$$

Counting backwards $n-1$ positions, each term is two larger than the successor

$$
a((n(n+1))-i)=2+2 i, \text { for } 0<=i<=n-1
$$

The next position down is one less than the previous and after that, each term is again two less the successor.

$$
a((n(n+1))-n-i)=2 n-1-2 i, \text { for } 0<=i<=n-1
$$

As a single formula using the permutation
$a((n(n+1)-i)=\operatorname{phi}(2 n-i)$, for $0<=i<=2 n-1$. n

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The First International Conference on Smarandache Type Notions in Number Theory, August 21-24, Department of Mathematics, University of Craiova, Romania; This Conference has been organized by Dr. C. Dumitrescu \& Dr. V. Seleacu, under the auspices of UNESCO.

