# A METHOD OF SOLVING CERTAIN NONLINEAR DIOPHANTINE EQUATIONS 

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#### Abstract

. In this paper we propose a method of solving a Nonlinear Diophantine Equation by converting it into a System of Diophantine Linear Equations.


## Introduction.

Let's consider a polynomial with integer coefficients, of degree $m$

$$
P\left(X_{1}, \ldots, X_{n}\right)=\sum_{\substack{0 \leq i_{1}+\ldots+i_{n} \leq m \\ 0 \leq i_{j} \leq m, j=1, n}} a_{i_{1} \ldots i_{n}} X_{1}^{i_{1}} \ldots X_{n}^{i_{n}}
$$

which can be factored out in linear factors (that can eventually be established through the undetermined coefficients method):

$$
P\left(X_{1}, \ldots, X_{n}\right)=\left(A_{1}^{(1)} X_{1}+\ldots+A_{n}^{(1)} X_{n}+A_{n+1}^{(1)}\right) \cdots\left(A_{1}^{(m)} X_{1}+\ldots+A_{n}^{(m)} X_{n}+A_{n+1}^{(m)}\right)+B
$$

with all $A_{j}^{(k)}, B$ in $\mathbb{Q}$, but which by bringing to the same common denominator and by eliminating it from the equation $P\left(X_{1}, \ldots, X_{n}\right)=0$ they can be considered integers.. Thus the equation transforms in the following system:

$$
\left\{\begin{array}{l}
A_{1}^{(1)} X_{1}+\ldots+A_{n}^{(1)} X_{n}+A_{n+1}^{(1)}=D_{1} \\
\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\
A_{1}^{(m)} X_{1}+\ldots+A_{n}^{(m)} X_{n}+A_{n+1}^{(m)}=D_{m}
\end{array}\right.
$$

where $D_{1}, \ldots, D_{m}$ are the divisors for $B$ and $D_{1} \cdots D_{m}=B$.
We separately solve each linear Diophantine equation and then we intersect the equations' solutions.

## Example 1.

Solve in integer numbers the equation:

$$
-2 x^{3}+5 x^{2} y+4 x y^{2}-3 y^{3}-3=0 .
$$

We'll write the equation in another format

$$
(x+y)(2 x-y)(-x+3 y)=3 .
$$

Let $m, n$ and $p$ be the divisors of $3, m \cdot n \cdot p=3$. Thus

$$
\left\{\begin{array}{r}
x+y=m \\
2 x-y=n \\
-x+3 y=p
\end{array}\right.
$$

For this system to be compatible it is necessary that

$$
\left(\begin{array}{rrr}
1 & 1 & m \\
2 & -1 & n \\
-1 & 3 & p
\end{array}\right)=0,
$$

or

$$
\begin{equation*}
5 m-4 n-3 p=0 \tag{1}
\end{equation*}
$$

In this case

$$
\begin{equation*}
x=\frac{m+n}{3} \text { and } y=\frac{2 m-n}{3} \tag{2}
\end{equation*}
$$

Because $m, n, p \in \mathbb{Z}$, from (1) it results - by solving in integer numbers - that:

$$
\left\{\begin{array}{l}
m=3 k_{1}-k_{2} \\
n=\quad k_{2} \\
p=5 k_{1}-3 k_{2}
\end{array} \quad k_{1}, k_{2} \in \mathbb{Z}\right.
$$

which substituted in (2) will give us $x=k_{1}$ and $y=2 k_{1}-k_{2}$. But $k_{2} \in D(3)=\{ \pm 1, \pm 3\}$; thus the only solution is obtained for $k_{2}=1, k_{1}=0$ from where $x=0$ and $y=-1$.

## Example 2.

Analogously, it can be shown that, for example the equation:

$$
-2 x^{3}+5 x^{2} y+4 x y^{2}-3 y^{3}=6
$$

does not have solutions in integer numbers.

## REFERENCES

[1] Marius Giurgiu, Cornel Moroti, Florică Puican, Stefan Smărăndoiu Teme şi teste de Matematică pentru clasele IV-VIII - Ed. Matex, Rm. Vîlcea, Nr. 3/1991
[2] Ion Nanu, Lucian Tuțescu - "Ecuații Nestandard", Ed. Apollo şi Ed. Oltenia, Craiova, 1994.

