A METHOD OF SOLVING CERTAIN NONLINEAR DIOPHANTINE EQUATIONS

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Abstract.

In this paper we propose a method of solving a Nonlinear Diophantine Equation by converting it into a System of Diophantine Linear Equations.

Introduction.

Let's consider a polynomial with integer coefficients, of degree m

$$P(X_1,...,X_n) = \sum_{\substack{0 \le i_1 + ... + i_n \le m \\ 0 \le i_i \le m, j = 1, n}} a_{i_1...i_n} X_1^{i_1} ... X_n^{i_n}$$

which can be factored out in linear factors (that can eventually be established through the undetermined coefficients method):

 $P(X_1, ..., X_n) = \left(A_1^{(1)}X_1 + ... + A_n^{(1)}X_n + A_{n+1}^{(1)}\right) \cdots \left(A_1^{(m)}X_1 + ... + A_n^{(m)}X_n + A_{n+1}^{(m)}\right) + B$

with all $A_j^{(k)}$, *B* in \mathbb{Q} , but which by bringing to the same common denominator and by eliminating it from the equation $P(X_1, ..., X_n) = 0$ they can be considered integers. Thus the equation transforms in the following system:

$$\begin{cases} A_1^{(1)}X_1 + \dots + A_n^{(1)}X_n + A_{n+1}^{(1)} = D_1 \\ \dots \\ A_1^{(m)}X_1 + \dots + A_n^{(m)}X_n + A_{n+1}^{(m)} = D_m \end{cases}$$

where $D_1, ..., D_m$ are the divisors for B and $D_1 \cdots D_m = B$.

We separately solve each linear Diophantine equation and then we intersect the equations' solutions.

Example 1.

Solve in integer numbers the equation:

 $-2x^3 + 5x^2y + 4xy^2 - 3y^3 - 3 = 0.$

We'll write the equation in another format

(x+y)(2x-y)(-x+3y) = 3.

Let *m*, *n* and *p* be the divisors of $3, m \cdot n \cdot p = 3$. Thus

$$\begin{cases} x+y = m\\ 2x-y = n\\ -x+3y = p \end{cases}$$

For this system to be compatible it is necessary that

$$\begin{pmatrix} 1 & 1 & m \\ 2 & -1 & n \\ -1 & 3 & p \end{pmatrix} = 0 ,$$

or

$$5m - 4n - 3p = 0$$
 (1)

In this case

$$x = \frac{m+n}{3}$$
 and $y = \frac{2m-n}{3}$ (2)

Because $m, n, p \in \mathbb{Z}$, from (1) it results – by solving in integer numbers – that:

$$\begin{cases} m = 3k_1 - k_2 \\ n = k_2 \\ p = 5k_1 - 3k_2 \end{cases} \quad k_1, k_2 \in \mathbb{Z}$$

which substituted in (2) will give us $x = k_1$ and $y = 2k_1 - k_2$. But $k_2 \in D(3) = \{\pm 1, \pm 3\}$; thus the only solution is obtained for $k_2 = 1$, $k_1 = 0$ from where x = 0 and y = -1.

Example 2.

Analogously, it can be shown that, for example the equation:

$$2x^3 + 5x^2y + 4xy^2 - 3y^3 = 6$$

does not have solutions in integer numbers.

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