Quantum indeterminacy is found sensitive to scaling and seen to vanish at the macroscopic

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Abstract. This article is one of a series explaining the nature of mathematical undecidability discovered present within quantum mechanics. In the measurement problem, the act of relating quantum effects to macroscopic reference systems is seen as the instrumental process. A brief outline is given telling how an axiomatic implementation of scalars in mathematical physics theoretically controls indeterminacy and cause. Wave mechanics of the free particle is outlined along these lines. General solutions for this system are mathematically undecidable, indeterminate formulae. The indeterminacy is seen to vanish for extremely large scales.

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1. Introduction

The revolutionary idea applied in this research is taken from well known phenomena in Mathematical Logic. While all scalars engage in the arithmetic of scalars; in terms of validity, different scalars are logically distinct [1]. The failure of mathematical physics to notice this distinction is the reason why quantum theory is logically at odds with quantum experiments.

Scalars are mathematical objects that satisfy the Field Axioms. The Field Axioms *prove* the existence of certain scalars while the wider set of all scalars merely *satisfies* them. Consider the examples:

$$\exists\alpha\,(\alpha \times \alpha = 4)\,; \tag{1}$$

$$\exists\alpha\,(\alpha \times \alpha = -1)\,. \tag{2}$$

Of these formulae, the Field Axioms prove only (1). (2) is neither provable nor disprovable. Even so, the square root of -1 is nevertheless an object that satisfies the Field Axioms and therefore, it does engage in their arithmetic. This number is intrinsic to quantum theory, but the standard formulation applies the arithmetic without recognising the said logical distinction. Misleadingly, standard theory introduces the square root of minus one axiomatically. This inadvertently adopted, additional axiom promotes (2) from a scalar that satisfies, to one which is provable, destroying the distinct logic.

The fundamental premise of this study is that there exist rules *in Nature*, isomorphic to the Field Axioms *in Theory*; and that *caused* effects in Nature correspond to *provable* formulae in Theory. This accords with our conventional concepts. Significantly however, formulae of the Theory involving the logically weaker scalars are neither provable nor disprovable: they are known as *mathematically undecidable.* These correlate to effects that are neither *caused* nor *not-caused;* but are *indeterminate.*

It is speculated that the act of measurement induces *indeterminate* quantum formulae to transform to formulae of *cause*, when related to macroscopic reference systems.

2. Background

Standard quantum mechanics derives the Canonical commutation relation from the homogeneity of space:

$$[\mathbf{p}, \mathbf{x}] = i\hbar\mathbf{1}. \tag{3}$$

But actually, homogeneity doesn't derive the i in this relation at all. Rigorous derivation shows that homogeneity, plus invariance of homogeneity under scaling, derives (4) instead, where η can be any scalar, that is: any number satisfying the Field Axioms [2]:

$$\forall\eta\exists\mathbf{x}\exists\mathbf{p}\left([\mathbf{p}, \mathbf{x}] = \eta\hbar\mathbf{1}\right). \tag{4}$$

3. Wave mechanics of the free particle under the Field Axioms

Here, in a logical version of the theory, I give an outline of the wave mechanics leading to general superposed solutions to wave equations for the free particle.

Assumption 1: Adopt the Field Axioms *a priori.*

The relation in (4) is taken as the fundamental formula of wave mechanics in place of (3). This new relation is satisfied by two representations which are equally valid: one in position space:

$$\forall\eta\left(\mathbf{p} = \eta\hbar\tfrac{\mathrm{d}}{\mathrm{d}x}\right); \qquad \mathbf{x} = x; \qquad \forall x\left(\mathbf{1}\left[\psi_{(x)}\right] = 1\psi\left(x\right)\right);$$

and the other in momentum space:

$$\forall\eta\left(\mathbf{x} = -\eta\hbar\tfrac{\mathrm{d}}{\mathrm{d}p}\right); \qquad \mathbf{p} = p; \qquad \forall p\left(\mathbf{1}\left[\phi_{(p)}\right] = 1\phi\left(p\right)\right).$$

The two representations furnish their own wave mechanical theory: each with equal validity. The mathematician may not choose one in preference to the other; they should be regarded as facets of the same symmetry. The two wave mechanical theories afford their respective wave equations representing the free particle, thus:

$$\forall\eta\exists\psi\exists x\left(\eta\hbar^{\mathbb{Q}}\frac{\mathrm{d}}{\mathrm{d}x}\psi_{p^{\mathbb{Q}}}\left(x\right) = p^{\mathbb{Q}}\psi_{p^{\mathbb{Q}}}\left(x\right)\right);$$

$$\forall\eta\exists\phi\exists p\left(-\eta\hbar^{\mathbb{Q}}\frac{\mathrm{d}}{\mathrm{d}p}\phi_{x^{\mathbb{Q}}}\left(p\right) = x^{\mathbb{Q}}\phi_{x^{\mathbb{Q}}}\left(p\right)\right).$$

And these wave formulae are satisfied by general superposed solutions (5) and (6). The exponential in these is not a valid function under the Field Axioms but may be validly approximated to any degree of accuracy as a finite polynomial. I adopt the $\mathrel{\hat{=}}$ equality as notation for this:

$$\forall\eta\forall a\exists\Psi\left(\Psi\left(x^{\mathbb{Q}}\right) \mathrel{\hat{=}} \int_{\mathbb{Q}} a\left(p^{\mathbb{Q}}\right)\exp\left(\frac{p^{\mathbb{Q}}x^{\mathbb{Q}}}{\eta\hbar^{\mathbb{Q}}}\right)\mathrm{d}p^{\mathbb{Q}}\right); \tag{5}$$

$$\forall\eta\forall b\exists\Phi\left(\Phi\left(p^{\mathbb{Q}}\right) \mathrel{\hat{=}} \int_{\mathbb{Q}} b\left(x^{\mathbb{Q}}\right)\exp\left(\frac{p^{\mathbb{Q}}x^{\mathbb{Q}}}{-\eta\hbar^{\mathbb{Q}}}\right)\mathrm{d}x^{\mathbb{Q}}\right). \tag{6}$$

But both are *invalid* formulae under the Field Axioms because the quantifiers $\forall a$ and $\forall b$ are not entirely satisfied: not all functions $a\left(p^{\mathbb{Q}}\right)$ and $b\left(x^{\mathbb{Q}}\right)$ yield integrals that exist. Weaker propositions can be constructed where $\exists a$ and $\exists b$ replace $\forall a$ and $\forall b$:

$$\forall\eta\exists a\exists\Psi\left(\Psi\left(x^{\mathbb{Q}}\right) \mathrel{\hat{=}} \int_{\mathbb{Q}} a\left(p^{\mathbb{Q}}\right)\exp\left(\frac{p^{\mathbb{Q}}x^{\mathbb{Q}}}{\eta\hbar^{\mathbb{Q}}}\right)\mathrm{d}p^{\mathbb{Q}}\right); \tag{7}$$

$$\forall\eta\exists b\exists\Phi\left(\Phi\left(p^{\mathbb{Q}}\right) \mathrel{\hat{=}} \int_{\mathbb{Q}} b\left(x^{\mathbb{Q}}\right)\exp\left(\frac{p^{\mathbb{Q}}x^{\mathbb{Q}}}{-\eta\hbar^{\mathbb{Q}}}\right)\mathrm{d}x^{\mathbb{Q}}\right). \tag{8}$$

This narrows the scope to weaker propositions that do satisfy the Field Axioms. We now hypothesise the existence of coincident pairs of functions that might share information back and forth between (7) and (8), in the following:

Assumption 2:

$$\forall a\exists\Phi\left(a\left(p^{\mathbb{Q}}\right) = \Phi\left(p^{\mathbb{Q}}\right)\right);$$
$$\forall b\exists\Psi\left(b\left(p^{\mathbb{Q}}\right) = \Psi\left(p^{\mathbb{Q}}\right)\right).$$

And conditional on the validity of Assumption 2 we may write:

$$\forall\eta\exists\Phi\exists\Psi\left(\Psi\left(x^{\mathbb{Q}}\right) \mathrel{\hat{=}} \int_{\mathbb{Q}} \Phi\left(p^{\mathbb{Q}}\right)\exp\left(\frac{p^{\mathbb{Q}}x^{\mathbb{Q}}}{\eta\hbar^{\mathbb{Q}}}\right)\mathrm{d}p^{\mathbb{Q}}\right); \tag{9}$$

$$\forall\eta\exists\Phi\exists\Psi\left(\Phi\left(p^{\mathbb{Q}}\right) \mathrel{\hat{=}} \int_{\mathbb{Q}} \Psi\left(x^{\mathbb{Q}}\right)\exp\left(\frac{p^{\mathbb{Q}}x^{\mathbb{Q}}}{-\eta\hbar^{\mathbb{Q}}}\right)\mathrm{d}x^{\mathbb{Q}}\right). \tag{10}$$

Let us take stock of the validity of (9) and (10): individually, they satisfy (4); they also satisfy the mechanics of the free particle and they are logically valid under the Field Axioms [1]. Even so (9) and (10) are mutually inconsistent. Assumed validity of either denies validity of the other. This is proved by any attempt to *invert* them in the manner of the Fourier transform [2]. But inversion does occur in cases where the exponential functions form orthogonal spaces. Existence of such spaces relies on the existence of particular instances of the scalings η. Exchanging the $\forall\eta$ quantifiers for $\exists\eta$, in (9) and (10), narrows the scope further to propositions that *are* consistent:

$$\exists\eta\exists\Phi\exists\Psi\left(\Psi\left(x^{\mathbb{Q}}\right) \mathrel{\hat{=}} \int_{\mathbb{Q}} \Phi\left(p^{\mathbb{Q}}\right)\exp\left(\frac{p^{\mathbb{Q}}x^{\mathbb{Q}}}{\eta\hbar^{\mathbb{Q}}}\right)\mathrm{d}p^{\mathbb{Q}}\right); \tag{11}$$

$$\exists\eta\exists\Phi\exists\Psi\left(\Phi\left(p^{\mathbb{Q}}\right) \mathrel{\hat{=}} \int_{\mathbb{Q}} \Psi\left(x^{\mathbb{Q}}\right)\exp\left(\frac{p^{\mathbb{Q}}x^{\mathbb{Q}}}{-\eta\hbar^{\mathbb{Q}}}\right)\mathrm{d}x^{\mathbb{Q}}\right). \tag{12}$$

At this point, values of η validly satisfying (9) and (10), are yet to be explicitly stated.

Assumption 3: Assume existence of the following value for η:

$$\exists\eta\left(\eta^2 = -\left(s^{\mathbb{Q}}\right)^2\right). \tag{13}$$

$s^{\mathbb{Q}}$ is chosen to be rational, indicated by the superscript, and so is logically valid [1], It is also entirely variable. This renders η unavoidably imaginary: $\eta = \pm is^{\mathbb{Q}}$. Existence of $i = \sqrt{-1}$ is logically independent of the Field Axioms and so is mathematically undecidable and logically indeterminate [1]. Substitution of (13) into (11) and (12) gives:

$$\exists\Phi\exists\Psi\left(\Psi\left(x^{\mathbb{Q}}\right) \mathrel{\mathring{=}} \int_{\mathbb{Q}} \Phi\left(p^{\mathbb{Q}}\right) \exp\left(\frac{p^{\mathbb{Q}}x^{\mathbb{Q}}}{\pm is^{\mathbb{Q}}\hbar^{\mathbb{Q}}}\right) \mathrm{d}p^{\mathbb{Q}}\right); \tag{14}$$

$$\exists\Phi\exists\Psi\left(\Phi\left(p^{\mathbb{Q}}\right) \mathrel{\mathring{=}} \int_{\mathbb{Q}} \Psi\left(x^{\mathbb{Q}}\right) \exp\left(\frac{p^{\mathbb{Q}}x^{\mathbb{Q}}}{\mp is^{\mathbb{Q}}\hbar^{\mathbb{Q}}}\right) \mathrm{d}x^{\mathbb{Q}}\right). \tag{15}$$

The undecidability is entrained in (14) and (15). Taking stock once more, (14) and (15) are mutually consist. Together they satisfy (4); they satisfy the mechanics of the free particle but now logical validity under the Field Axioms is lost and replaced by logical indeterminacy.

4. Logical transition under scaling

Although in general (14) and (15) are logically indeterminate, there are instances where their indeterminacy vanishes in favour of validity. This behaviour is governed by detail in the ratio:

$$\frac{p^{\mathbb{Q}}x^{\mathbb{Q}}}{s^{\mathbb{Q}}\hbar^{\mathbb{Q}}}. \tag{16}$$

In circumstances where this ratio vanishes to zero, the imaginary unit is no longer a factor in the exponent of the exponential, and furthermore, the exponential tends to unity. Under this condition, (14) and (15) tend propositions looking like this:

$$\exists\Phi\exists\Psi\left(\Psi\left(x^{\mathbb{Q}}\right) = \int_{\mathbb{Q}} \Phi\left(p^{\mathbb{Q}}\right) \mathrm{d}p^{\mathbb{Q}}\right); \tag{17}$$

$$\exists\Phi\exists\Psi\left(\Phi\left(p^{\mathbb{Q}}\right) = \int_{\mathbb{Q}} \Psi\left(x^{\mathbb{Q}}\right) \mathrm{d}x^{\mathbb{Q}}\right). \tag{18}$$

In such instances, the undecidable information carried by the imaginary unit is expunged and the previously undecidable propositions transform to ones that are logically valid: as are (17) and (18).

In the examination of the ratio's detail, $\hbar^{\mathbb{Q}}$ is regarded as an absolute valued, rational constant characterising the quantum scale; and scaling factor $s^{\mathbb{Q}}$ is a variable whose value specifies the scale of any reference system against which the quantum free particle is to be related. At microscopic scales, where $\hbar^{\mathbb{Q}}$ and $s^{\mathbb{Q}}$ are equitable and at scales where $\hbar^{\mathbb{Q}}$ dominates $s^{\mathbb{Q}}$, the indeterminate logic we see in quantum systems prevails. At macroscopic scales where $s^{\mathbb{Q}}$ overwhelmingly dominates, the indeterminacy vanishes and validity prevails.

Under this scaling, it is of great interest to learn that indeterminacy does not vanish homogeneously. Consider (14); in this, the integral sweeps the entire range of $p^{\mathbb{Q}}$, so zeros in $p^{\mathbb{Q}}$ are ruled out. But $x^{\mathbb{Q}}$ is free to be mapped to $\Psi\left(x^{\mathbb{Q}}\right)$. For extremely large $s^{\mathbb{Q}}$ specifying a macroscopic frame of reference, where $x^{\mathbb{Q}}$ is close to zero, the ratio (16) easily vanishes. In contrast to this where $x^{\mathbb{Q}}$ is extremely large also, $x^{\mathbb{Q}}$ may still dominate $s^{\mathbb{Q}}$, preventing the vanishing ratio at extremities of $x^{\mathbb{Q}}$. In these places, the indeterminacy is not overcome as it is centrally.

References

[1] Faulkner S 2011 *Indeterminate scalars under the Field Axioms: foundation for Reichenbach's quantum logic* http://www.vixra.org/pdf/1101.0045v1.pdf

[2] Faulkner S 2011 *Gödelian features found at quantum indeterminacy inconsistency, undecidability and self reference* http://www.vixra.org/pdf/1101.0075v1.pdf

Appendix A. The Field Axioms

	The Field Axioms	
	Additive Group	
FA0	$\forall\alpha\forall\beta\exists\gamma\,(\gamma = \alpha + \beta)$	Closure
FA1	$\exists 0\forall\alpha\,(0 + \alpha = \alpha)$	Identity 0
FA2	$\forall\alpha\exists\beta\,(\alpha + \beta = 0)$	Inverses
FA3	$\forall\alpha\forall\beta\forall\gamma\,((\alpha + \beta) + \gamma = \alpha + (\beta + \gamma))$	Associativity
FA4	$\forall\alpha\forall\beta\,(\alpha + \beta = \beta + \alpha)$	Commutativity
	Multiplicative Group	
FM0	$\forall\alpha\forall\beta\exists\gamma\,(\gamma = \alpha \times \beta)$	Closure
FM1	$\exists 1\forall\alpha\,(1\alpha = \alpha 1 = \alpha \wedge 0 \neq 1)$	Identity 1
FM2	$\forall\alpha\exists\beta\,(\alpha \times \beta = 1 \wedge \alpha \neq 0)$	Inverses
FM3	$\forall\alpha\forall\beta\forall\gamma\,((\alpha \times \beta) \times \gamma = \alpha \times (\beta \times \gamma))$	Associativity
FM4	$\forall\alpha\forall\beta\,(\alpha \times \beta = \beta \times \alpha)$	Commutativity
FAM	$\forall\alpha\forall\beta\forall\gamma\,(\alpha \times (\beta + \gamma) = (\alpha \times \beta) + (\alpha \times \gamma))$	Distributivity

Table A1. The Field Axioms written as sentences in *first-order logic*. The variables: $\alpha, \beta, \gamma, 0, 1$ represent mathematical objects complying with these axioms. The semantic interpretations of these objects are known as *scalars*.