# Programming gravitational orbitals (gravitons) as units of $\hbar c$ in a Planck unit simulation hypothesis 

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#### Abstract

The Simulation Hypothesis proposes that all of reality is an artificial simulation, analogous to a computer simulation. Here is described a method for programming gravity between macro objects in a Planck level simulation (where all events occur at unit Planck time). A continuous gravitational force between objects is replaced with discrete units of $\hbar c$ (defined as gravitational orbitals or gravitons) that directly link the individual object particles with each other and measure in terms of orbital momentum and velocity. The orbital angular momentum of the planetary orbits derives from the sum of the planet-sun particle-particle orbital angular momentum irrespective of the angular momentum of the sun itself and particle-particle rotational angular momentum contributes to the planets rotational angular momentum. Particles oscillate between a wave-state to a Plancktime Planck-mass point-state (a Planck micro black-hole), gravitational interactions as interactions between these micro black-holes. Each graviton is a single unit of $\hbar c$ albeit with a variable orbital momentum and velocity component defined in terms of a gravitational equivalent to the principal quantum number $n$. As orbits have different momentum densities, movement between orbits occurs via a change in the graviton momentum:velocity ratio, an orbital buoyancy, such that moving the earth to a different galaxy will change this ratio, the number of graviton units of $\hbar c$ however remaining the same. As the simulation uses digital instead of analog time all particle point-states will share a common time frame as measured in units of Planck time.


## 1 Introduction

The Simulation Hypothesis proposes that all of reality, including the earth and the universe, is in fact an artificial simulation, analogous to a computer simulation.

Planck units are suitable for use in deep universe simulations as they are by definition discrete units, however Planck unit model simulations, even with the availability of massive computer resources, are difficult to implement when all events occur at unit Planck time.

A method for programming Planck units for mass, length, time and charge from a mathematical electron has been proposed [1]. This approach uses frequencies (the frequency of occurrence of an event at unit Planck time) instead of probabilities.

Here is introduced a method by which gravity can likewise be simulated by replacing a (continuous) gravitational force between objects with discrete units of $\hbar c$ (defined here as gravitational orbitals or gravitons) that link all particles in the objects respectively at (digital) unit Planck time (all particles share this common time frame).

Simplifying wave-particle duality at the Planck level to an oscillation between an electric wave-state to a (discrete) unit of Planck-mass (for 1 unit of Planck-time) point-state (a Planck micro black-hole), and by assigning graviton links between all particles that are simultaneously in the point-state (for any chosen unit of Planck time), we can sum their respective orbital angular momentum to obtain the observed gravitational orbit.

Gravitational interactions between massive objects is
thereby reduced to individual interactions between micro black-holes at the Planck level.

Although each graviton is a single discrete unit of $\hbar c$, its orbital momentum and velocity components adjust for distance. As orbits have different momentum densities, movement between orbits occurs via a change in the graviton length:velocity ratio, an orbital buoyancy.

While the momentum of the orbital keeps satellites following their orbits, it is the momentum:velocity ratio that keeps the satellite from 'floating' off into space or 'falling' to the earth.

## 2 Gravitons

2.1. The gravitational coupling constant $\alpha_{G}$ characterizes the gravitational attraction between a given pair of elementary particles in terms of the electron mass to Planck mass ratio;

$$
\begin{equation*}
\alpha_{G}=\frac{G m_{e}^{2}}{\hbar c}=\frac{m_{e}^{2}}{m_{P}^{2}}=1.75 \ldots x 10^{-45} \tag{1}
\end{equation*}
$$

If we replace wave-particle duality with an electric wavestate to Planck-mass (for 1 unit of Planck-time) pointstate oscillation then at any unit of Planck time $t$ a certain number of particles will simultaneously be in the Planck mass point-state. For example a 1 kg satellite orbits the earth, for any $t$, satellite (A) will have $1 \mathrm{~kg} / \mathrm{m}_{P}$ $=45.9 \times 10^{6}$ particles in the point-state. The earth (B) will have $5.97 \times 10^{24} \mathrm{~kg} / m_{P}=0.274 \times 10^{33}$ particles in the point-state. If we assign a graviton to link each respective point-state then for any given unit of Planck time
the number of gravitons;

$$
\begin{equation*}
N_{\text {gravitons }}=\frac{m_{A} m_{B}}{m_{P}^{2}}=0.126 \times 10^{41} \tag{2}
\end{equation*}
$$

The observed satellite orbit around the earth derives from the sum of these $0.126 x 10^{41}$ gravitons. If A and B are respectively Planck mass particles then $N_{\text {gravitons }}=$ 1. If A and B are respectively electrons then

$$
\begin{equation*}
N_{\text {gravitons }}=\alpha_{G}=\frac{m_{e}^{2}}{m_{P}^{2}}=1.75 \ldots x 10^{-45} \tag{3}
\end{equation*}
$$

The frequency of an electron oscillation $=\left(m_{P} / m_{e}\right) t_{p}$ and so the probability that any 2 electrons are simultaneously in the mass point-state for any chosen $t=$ $\left(m_{P} / m_{e}\right)^{2}=1 / \alpha_{G}$. $N_{\text {gravitons }}$ is simply the sum of all the respective particle $\alpha_{G}$ 's between both objects at any $t$, as a consequence for objects whose mass is less than Planck mass there will be units of time $t$ when there are no graviton links and electric wave-state interactions will predominate. Gravitational interactions becomes the sum of discrete interactions between individual units of Planck mass.
2.2. Although atomic orbitals have an unknown geometry, gravitational orbits are an average of all the underlying gravitational orbitals (gravitons) and so more closely approximate a classical geometry, it is therefore not necessary to know the individual graviton (orbital) structure. Consequently we can adapt the Bohr model to gravitational orbits albeit $n$, being an average of all the individual graviton $n$ 's, is not an integer.

$$
\begin{equation*}
N_{o b j e c t}=\frac{M_{o b j e c t}}{m_{P}} \tag{4}
\end{equation*}
$$

As we are calculating gravity only between point-states (units of Planck mass per unit Planck time), then for any 2 points ( $\mathrm{i}, \mathrm{j}$ )

$$
\begin{equation*}
r=\alpha n_{i, j}^{2} 2 l_{p} \tag{5}
\end{equation*}
$$

If object B with mass $=m_{P}$ orbits (planet) $\mathrm{A}\left(m_{A} \gg\right.$ $m_{B}$ ) then we may calculate the average distance between point B and each individual point in A giving

$$
\begin{equation*}
r_{\text {average }}=\alpha n_{a, b}^{2} 2 l_{p} \tag{6}
\end{equation*}
$$

However this average $n_{a, b}$ would apply to the distance between the mass center point of A to point B (i.e.: $N_{\text {object }}=1$ ), in order to include the mass of A we assign;

$$
\begin{equation*}
n^{2}=\frac{n_{a, b}^{2}}{N_{A}} \tag{7}
\end{equation*}
$$

This divides $r_{\text {average }}$ into discrete segments of length $\alpha n^{2} 2 l_{p}$. The fine structure constant $=137.03599 \ldots$

$$
\begin{equation*}
r_{g}=N_{\text {object }}\left(\alpha n^{2} 2 l_{p}\right) \tag{8}
\end{equation*}
$$

$$
\begin{gather*}
v_{g}=\frac{c}{\sqrt{2 \alpha} n}  \tag{9}\\
a_{g}=\frac{1}{N_{o b j e c t}} \frac{c^{2}}{2 \alpha^{2} n^{4} 2 l_{p}}  \tag{10}\\
T_{g}=\frac{r_{g}}{v_{g}}=N_{\text {object }}\left(\frac{2 \pi \sqrt{2 \alpha} \alpha n^{3} 2 l_{p}}{c}\right) \tag{11}
\end{gather*}
$$

> 2.2.1. Example - Earth radius $=6371 \mathrm{~km}$
> $\mu_{\text {earth }}=3.986004418(9) x 10^{14}($ std grav. parameter [5] $)$
> $N_{\text {earth }}=M_{\text {earth }} / m_{P}=.2744385886 \ldots x 10^{33}$
> $\quad r_{g}=6371.0 \mathrm{~km}(\mathrm{n}=2289.408 \ldots)$
> $a_{g}=9.820 \mathrm{~m} / \mathrm{s}^{2}$
> $\quad T_{g}=5060.837 \mathrm{~s}$
> $\quad v_{g}=7909.792 \mathrm{~m} / \mathrm{s}$

Geosynchronous orbit
$r_{g}=42164.0 \mathrm{~km}(\mathrm{n}=5889.66 \ldots)$
$a_{g}=0.2242 \mathrm{~m} / \mathrm{s}^{2}$
$T_{g}=86163.6 \mathrm{~s}$
$v_{g}=3074.666 \mathrm{~m} / \mathrm{s}$
Moon orbit ( $\mathrm{d}=84600 \mathrm{~s}$ )
$r_{g}=384400 \mathrm{~km}(\mathrm{n}=17783.25 \ldots)$
$a_{g}=.0026976 \mathrm{~m} / \mathrm{s}^{2}$
$T_{g}=27.4519 \mathrm{~d}$
$v_{g}=1.0183 \mathrm{~km} / \mathrm{s}$
2.2.2. The energy that was required to lift that 1 kg satellite into a geosynchronous orbit is the difference between the energy of each of the 2 orbits (geosynchronous and earth).

$$
\begin{gather*}
E_{\text {graviton }}=\frac{h c}{2 \pi r_{6371}}-\frac{h c}{2 \pi r_{42164}}  \tag{12}\\
N_{\text {gravitons }}=M_{\text {earth }} m_{\text {satellite }} / m_{P}^{2}=0.126 x 10^{41} \\
E_{\text {total }}=E_{\text {graviton }} . N_{\text {gravitons }}=53 M J / \mathrm{kg}
\end{gather*}
$$

2.2.3. Planetary orbits
$m_{\text {sun }}=1.32712440018 e 20 / G=.19889 \times 10^{31} \mathrm{~kg}$

$$
\begin{gather*}
N_{\text {sun }}=M_{\text {sun }} / m_{P}=.9137 x 10^{38} \\
n=\sqrt{\frac{r_{(\text {sun-planet })}}{N_{\text {sun }} \alpha 2 l_{p}}}  \tag{13}\\
T=N_{\text {sun }}\left(\frac{2 \pi \sqrt{2 \alpha} \alpha n^{3} 2 l_{p}}{c}\right) \tag{14}
\end{gather*}
$$

mercury $r=57909000 \mathrm{~km}, T=87.969 d, v=47.87 \mathrm{~km} / \mathrm{s}$ venus $r=108208000 \mathrm{~km}, T=224.698 d, v=35.02 \mathrm{~km} / \mathrm{s}$ earth $r=149600000 \mathrm{~km}, T=365.26 d, v=29.78 \mathrm{~km} / \mathrm{s}$ mars $r=227939200 \mathrm{~km}, T=686.97 d, v=24.13 \mathrm{~km} / \mathrm{s}$ jupiter $r=778.57 e 9 m, T=4336.7 d, v=13.06 \mathrm{~km} / \mathrm{s}$ pluto $r=5.90638 e 12 m, T=90613.4 d, v=4.74 \mathrm{~km} / \mathrm{s}$
2.2.4 Angular momentum

By linking all respective mass-states between 2 orbiting objects with units of momentum $\hbar$ the total orbit momentum becomes the sum of the underlying momentum.
2.2.4.1 Orbital angular momentum $L_{\text {oam }}$

$$
\begin{gather*}
L_{\text {oam }}=2 \pi \frac{M r^{2}}{T}=N_{\text {gravitons }} n \frac{h}{2 \pi} \sqrt{2 \alpha}, \frac{\mathrm{kgm}^{2}}{\mathrm{~s}}  \tag{15}\\
N_{\text {gravitons }}=\frac{M_{\text {sun }} m_{\text {planet }}}{m_{P}^{2}} \tag{16}
\end{gather*}
$$

Orbital angular momentum $L_{\text {oam }}$ of the planets;
mercury $=.9153 \times 10^{39}(\mathrm{n}=378.2733)$
venus $=.1844 \times 10^{41}(\mathrm{n}=517.0853)$
earth $=.2662 \times 10^{41}(\mathrm{n}=607.9927)$
mars $=.3530 \times 10^{40}(\mathrm{n}=750.4850)$
jupiter $=.1929 \times 10^{44}(\mathrm{n}=1387.0157)$
pluto $=.365 \times 10^{39}(\mathrm{n}=3820.2628)$

Mean orbital velocity $v_{g}$
mercury $=47.87 \mathrm{~km} / \mathrm{s}(47.87 \mathrm{~km} / \mathrm{s}[4])$
venus $=35.02 \mathrm{~km} / \mathrm{s}(35.02 \mathrm{~km} / \mathrm{s}[4])$
earth $=29.78 \mathrm{~km} / \mathrm{s}(29.78 \mathrm{~km} / \mathrm{s}[4])$
mars $=24.13 \mathrm{~km} / \mathrm{s}(24.13 \mathrm{~km} / \mathrm{s}[4])$
jupiter $=13.06 \mathrm{~km} / \mathrm{s}(13.07 \mathrm{~km} / \mathrm{s}[4])$
pluto $=4.74 \mathrm{~km} / \mathrm{s}(4.72 \mathrm{~km} / \mathrm{s}[4])$
The angular momentum term $L_{\text {oam }}$ depends on $n$ leading to the dilemma whereby infinite distance results in infinite angular momentum. The orbital velocity $v_{g}$ decreases proportionately suggesting the graviton combines angular momentum with velocity. From;

$$
L_{o a m} v_{g}=\left(m_{P} v_{g} r_{g}\right)\left(v_{g}\right)
$$

$$
\begin{equation*}
L_{o a m} v_{g}=\left(m_{P} \frac{c}{\sqrt{2 \alpha} n} \alpha n^{2} 2 l_{p}\right)\left(\frac{c}{\sqrt{2 \alpha} n}\right) \frac{1}{2 \pi}=\frac{h c}{2 \pi} \tag{17}
\end{equation*}
$$

### 2.2.4.2 Rotational angular momentum $L_{\text {ram }}$

The rotational angular momentum contribution to planet rotation.

$$
\begin{gathered}
T_{\text {rot }}=\frac{2 \pi 2 \alpha^{2} n^{3} \lambda}{c} \\
v_{r o t}=\frac{c}{2 \alpha n} \\
r_{\text {orbital }}=\alpha n^{2} \lambda \\
L_{\text {ram }}=\left(\frac{2}{5}\right) \frac{2 \pi M r^{2}}{T}=\left(\frac{2}{5}\right) N_{\text {orbitals }} n \frac{h}{2 \pi}, \frac{\mathrm{kgm}^{2}}{s} \\
\left(m_{P} \frac{c}{2 \alpha n} \alpha n^{2} 2 l_{p}\right)\left(\frac{c}{2 \alpha n}\right) \frac{1}{2 \pi}=\frac{h c}{2 \pi 2 \alpha} \\
n_{\text {earth }}=2289.4(\mathrm{r}=6371 \mathrm{~km}) \\
T_{\text {rot }}=83847.7 s \quad(86400)
\end{gathered}
$$

$$
\begin{gathered}
v_{\text {rot }}=477.8 \mathrm{~m} / \mathrm{s} \quad(463.3) \\
L_{\text {ram }}=.727 \times 10^{34} \frac{\mathrm{kgm}^{2}}{\mathrm{~s}}(.705) \\
n_{\text {mars }}=5094.7(\mathrm{r}=3390 \mathrm{~km}) \\
T_{\text {rot }}=99208 \mathrm{~s} \quad(88643) \\
v_{\text {rot }}=214.7 \mathrm{~m} / \mathrm{s}(240.29) \\
L_{\text {ram }}=.187 \times 10^{33} \frac{\mathrm{kgm}^{2}}{\mathrm{~s}}(.209)
\end{gathered}
$$

The Rydberg formula with the length term;

$$
\begin{equation*}
E=\frac{h c}{2 \pi 2 \alpha} \frac{1}{\alpha \lambda_{\text {orbital }}}\left(\frac{1}{n_{i}^{2}}-\frac{1}{n_{f}^{2}}\right) \tag{23}
\end{equation*}
$$

2.3. Time dilation.
2.3.1. Velocity: In the article 'Programming Relativity in a Planck unit Universe', a model of a virtual hypersphere universe expanding in Planck steps was proposed [2]. In that model the universe hyper-sphere expands in all directions evenly, objects are pulled along by the expansion of the hyper-sphere irrespective of any motion in 3-D space. As such, while B (satellite) has a circular orbit in 3-D space co-ordinates it has a cylindrical orbit around the A (planet) time-line axis in the hyper-sphere co-ordinates with orbital period $T_{g} c\left(\right.$ from $B^{1}$ to $B^{2}$ ) at radius $r_{g}$ and orbital velocity $v_{g}$. If A is moving with the universe expansion (albeit stationary in 3-D space) then the orbital time $t_{g}$ alongside the A time-line axis (fig. 1) becomes;

$$
\begin{equation*}
t_{g}=\sqrt{\left(T_{g} c\right)^{2}-\left(2 \pi r_{g}\right)^{2}}=\left(T_{g} c\right) \sqrt{1-\frac{v_{g}^{2}}{c^{2}}} \tag{24}
\end{equation*}
$$



Fig. 1: orbit relative to A timeline axis
2.3.2. Gravitational:

$$
\begin{gather*}
v_{s}=v_{\text {escape }}=\sqrt{2} \cdot v_{g}  \tag{25}\\
\sqrt{1-\frac{2 G M}{r_{g} c^{2}}}=\sqrt{1-\frac{v_{s}^{2}}{c^{2}}} \tag{26}
\end{gather*}
$$

2.4. Binding energy in the nucleus can be simplified using the same approach.

$$
\begin{align*}
m_{n u c} & =m_{p}+m_{n}  \tag{27}\\
\lambda_{s} & =\frac{l_{p} m_{P}}{m_{n u c}} \tag{28}
\end{align*}
$$

$$
\begin{gather*}
r_{0}=\sqrt{\alpha} \lambda_{s}  \tag{29}\\
R_{s}=\alpha \lambda_{s}  \tag{30}\\
v_{s}^{2}=\frac{c^{2}}{\alpha} \tag{31}
\end{gather*}
$$

The gravitational binding energy $(\mu)$ is the energy required to pull apart an object consisting of loose material and held together only by gravity.

$$
\begin{equation*}
\mu_{G}=\frac{3 G m_{n u c}^{2}}{5 R_{s}}=\frac{3 m_{n u c} c^{2}}{5 \alpha}=\frac{3 m_{n u c} v_{s}^{2}}{5} \tag{32}
\end{equation*}
$$

Nuclear binding energy is the energy required to split a nucleus of an atom into its component parts. The electrostatic coulomb constant;

$$
\begin{gather*}
a_{c}=\frac{3 e^{2}}{20 \pi \epsilon r_{0}}  \tag{33}\\
E=\sqrt{( } \alpha) a_{c}=\frac{3 m_{n u c} c^{2}}{5 \alpha}=\frac{3 m_{n u c} v_{s}^{2}}{5}=\mu_{G} \tag{34}
\end{gather*}
$$

Average binding energy in nucleus;
$\mu_{G}=8.22 \mathrm{MeV} /$ nucleon

### 2.5. Anomalous precession

semi-minor axis: $b=\alpha l^{2} \lambda_{\text {sun }}$
semi-major axis: $a=\alpha n^{2} \lambda_{\text {sun }}$
radius of curvature L

$$
\begin{gather*}
L=\frac{b^{2}}{a}=\frac{a l^{4} \lambda_{\text {sun }}}{n^{2}}  \tag{35}\\
\frac{3 \lambda_{\text {sun }}}{2 L}=\frac{3 n^{2}}{2 \alpha l^{4}} \tag{36}
\end{gather*}
$$

$$
\begin{equation*}
\text { precession }=\frac{3 n^{2}}{2 \alpha l^{4}} \cdot 1296000 \cdot\left(100 T_{\text {earth }} / T_{\text {planet }}\right) \tag{37}
\end{equation*}
$$

| Table 1 | GR [3] | Observed |
| ---: | :---: | :---: |
| Mercury $=42.9814$ | 42.9195 | $43.1 \pm 0.5$ |
| Venus $=8.6248$ | 8.6186 | $8.4 \pm 4.8$ |
| Earth $=3.8388$ | 3.8345 | $5.0 \pm 1.2$ |
| Mars $=1.3510$ | 1.3502 |  |
| Jupiter $=0.0623$ | 0.0623 |  |
|  |  |  |

2.6. $F_{p}=$ Planck force, $\lambda=$ Schwarzschild radius;

$$
\begin{gather*}
F_{p}=\frac{m_{P} c^{2}}{l_{p}} \\
M_{a}=\frac{m_{P} \lambda_{a}}{2 l_{p}}, m_{b}=\frac{m_{P} \lambda_{b}}{2 l_{p}}  \tag{38}\\
F_{g}=\frac{M_{a} m_{b} G}{R^{2}}=\frac{\lambda_{a} \lambda_{b} F_{p}}{4 R_{g}^{2}}=\frac{\lambda_{a} \lambda_{b} F_{p}}{4 \alpha^{2} n^{4}\left(\lambda_{a}+\lambda_{b}\right)^{2}} \tag{39}
\end{gather*}
$$

a) If $M_{a}=m_{b}$, the object mass is not required

$$
\begin{equation*}
F_{g}=\frac{F_{p}}{\left(4 \alpha n^{2}\right)^{2}} \tag{40}
\end{equation*}
$$

b) If $M_{a} \gg m_{b},\left(\lambda_{a}+\lambda_{b}=\lambda_{a}\right)$, then relative mass is used and $F_{g}=m_{b} a_{g}$

$$
\begin{gather*}
F_{g}=\frac{\lambda_{b} F_{p}}{\left(2 \alpha n^{2}\right)^{2} \lambda_{a}}  \tag{41}\\
F_{g}=\frac{m_{b} c^{2}}{2 \alpha^{2} n^{4} \lambda_{a}}=m_{b} a_{g} \tag{42}
\end{gather*}
$$

3 Orbital transition
Atomic electron transition is defined as a change of an electron from one energy level to another, theoretically this should be a discontinuous electron jump from one energy level to another although the mechanism for this is not clear. The following uses the wavelengths (frequency in units of Planck time) of the orbitals and the Rydberg formula as a means to 'time' the transition period.
Let us consider the Hydrogen Rydberg formula for transition between and initial $i$ and a final $f$ orbit. The incoming photon $\lambda_{R}$ causes the electron to 'jump' from the $n=i$ to $n=f$ orbit.

$$
\begin{equation*}
\lambda_{R}=R \cdot\left(\frac{1}{n_{i}^{2}}-\frac{1}{n_{f}^{2}}\right)=\frac{R}{n_{i}^{2}}-\frac{R}{n_{f}^{2}} \tag{43}
\end{equation*}
$$

The above can be interpreted as referring to 2 photons;

$$
\lambda_{R}=\left(+\lambda_{i}\right)-\left(+\lambda_{f}\right)
$$

Let us suppose a region of space between a free proton $p^{+}$ and a free electron $e^{-}$which we may define as zero. This region then divides into 2 waves of inverse phase which we may designate as photon $(+\lambda)$ and anti-photon $(-\lambda)$ whereby

$$
(+\lambda)+(-\lambda)=\text { zero }
$$

The photon $(+\lambda)$ leaves (at the speed of light), the antiphoton $(-\lambda)$ however is trapped between the electron and proton and forms a standing wave orbital. Due to the loss of the photon, the energy of $\left(p^{+}+e^{-}+-\lambda\right)<$ $\left(p^{+}+e^{-}+0\right)$ and so stable.

Let us define an $(n=i)$ orbital as $\left(-\lambda_{i}\right)$. The incoming Rydberg photon $\lambda_{R}=\left(+\lambda_{i}\right)-\left(+\lambda_{f}\right)$ arrives in a 2 -step process. First the $\left(+\lambda_{i}\right)$ adds to the existing $\left(-\lambda_{i}\right)$ orbital.

$$
\left(-\lambda_{i}\right)+\left(+\lambda_{i}\right)=\text { zero }
$$

The $\left(-\lambda_{i}\right)$ orbital is canceled and we revert to the free electron and free proton; $p^{+}+e^{-}+0$ (ionization).

However we still have the remaining $-\left(+\lambda_{f}\right)$ from the Rydberg formula.

$$
0-\left(+\lambda_{f}\right)=\left(-\lambda_{f}\right)
$$

From this wave addition followed by subtraction we have replaced the $n=i$ orbital with an $n=f$ orbital. The electron has not moved (there was no transition from an $n_{i}$ to $n_{f}$ orbital), however the electron region (boundary) is now determined by the new $n=f$ orbital $\left(-\lambda_{f}\right)$.

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