# Quantum gravity in Planck units with gravitational orbitals (gravitons) as units of $\hbar c$ 

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#### Abstract

Listed are gravitational formulas in terms of Planck units and the fine structure constant alpha. Gravity, as with mass, is not assigned as a constant property but instead is treated as a discrete event defined by a particle (Planck) mass point-state. The rotational velocity around each pointstate in an object is summed to give the velocity of gravitational orbits of that object. Orbital angular momentum of the planetary orbits derives from the sum of the planet-sun particle-particle orbital angular momentum irrespective of the angular momentum of the sun itself and the rotational angular momentum of a planet includes particle-particle rotational angular momentum. Angular momentum and orbit velocity combine to form discrete units of $\hbar c$ irrespective of orbit distance. As this approach presumes a digital time, it is suitable for use in programming Simulation Hypothesis models.


1 Introduction
A method for programming the Planck units for mass, length, time and charge from a mathematical electron has been proposed [1]. This approach uses frequencies (the frequency of occurrence of an event at unit Planck time) instead of probabilities, particles are treated as oscillations between an electric wave-state (over time) to a discrete unit of Planck-mass (at unit Planck-time) mass point-state. Mass is therefore not a constant property of the particle, consequently for objects whose mass is less than Planck mass there will be units of Planck time when the object has no particles in the point-state and so no gravitational interactions. Gravity, as with mass, is not treated as a constant property but rather as a discrete event.

Rotation around the point-state sums within an object to give the gravitational orbit velocities.

Every particle that is in the mass point-state per unit of Planck time is directly linked to every other particle in the universe by a 'physical link' of angular momentum, orbitals are assigned as physical structures and not probability regions. This momentum combined with orbit velocity reduces to a unit of $\hbar c$ irrespective of orbit distance.

## 2 Gravitational coupling constant

The gravitational coupling constant $\alpha_{G}$ characterizes the gravitational attraction between a given pair of elementary particles in terms of the electron mass to Planck mass ratio;

$$
\begin{equation*}
\alpha_{G}=\frac{G m_{e}^{2}}{\hbar c}=\frac{m_{e}^{2}}{m_{P}^{2}}=1.75 \ldots x 10^{-45} \tag{1}
\end{equation*}
$$

If particles oscillate between an electric wave-state to Planck-mass (for 1 unit of Planck-time) point-state then at any discrete unit of Planck time $t$ a number of particles in the universe will simultaneously be in the mass
point-state. For example a 1 kg satellite orbits the earth, for any $t$, satellite (A) will have $1 \mathrm{~kg} / m_{P}=45.9 \times 10^{6}$ particles in the point-state. The earth (B) will have 5.97 $x 10^{24} \mathrm{~kg} / m_{P}=0.274 \times 10^{33}$ particles in the point-state. For any given unit of Planck time the gravitational coupling links between the earth and the satellite will sum to;

$$
\begin{equation*}
N_{\text {links }}=\frac{m_{A} m_{B}}{m_{P}^{2}}=0.126 \times 10^{41} \tag{2}
\end{equation*}
$$

If A and B are respectively Planck mass particles then $N=1$. If A and B are respectively electrons then the probability that any 2 electrons are simultaneously in the mass point-state for any chosen unit of Planck time $t, N=\alpha_{G}$ and so a gravitational interaction between these 2 electrons will occur only once every $10^{45}$ units of Planck time.

## 3 Planck unit gravitational formulas

(inverse) fine structure constant $\alpha=137.03599 \ldots$
$n_{p}=$ number of Planck units
$\lambda_{\text {object }}=$ Schwarzschild radius
distance from a point mass

$$
\begin{equation*}
r=\alpha n_{p} 2 l_{p} \tag{3}
\end{equation*}
$$

rotation around a point mass

$$
\begin{equation*}
v=\frac{c}{\sqrt{2 \alpha n_{p}}} \tag{4}
\end{equation*}
$$

rotational period

$$
\begin{equation*}
T=\frac{2 \pi r}{v} \tag{5}
\end{equation*}
$$

number of particles in the point-state per unit of Planck time per object mass M

$$
\begin{equation*}
N_{p o i n t s}=\frac{M}{m_{P}} \tag{6}
\end{equation*}
$$

gravitational analogue to the principal quantum number

$$
\begin{equation*}
n=\sqrt{\frac{n_{p}}{N_{\text {points }}}} \tag{7}
\end{equation*}
$$

distance from a center of mass

$$
\begin{equation*}
r_{g}=\alpha n_{p} 2 l_{p}=\alpha n^{2} \lambda_{M} \tag{8}
\end{equation*}
$$

gravitational orbit velocity from summed point velocities

$$
\begin{equation*}
v_{g}=\sqrt{N_{\text {points }}} v=\frac{c}{\sqrt{2 \alpha} n} \tag{9}
\end{equation*}
$$

gravitational acceleration

$$
\begin{equation*}
a_{g}=\frac{v_{g}^{2}}{r_{g}} \tag{10}
\end{equation*}
$$

gravitational orbital period

$$
\begin{equation*}
T_{g}=\frac{2 \pi r_{g}}{v_{g}} \tag{11}
\end{equation*}
$$

### 3.1. Example - Earth orbits

$$
N_{\text {points }}=M_{\text {earth }} / m_{P}
$$

Earth surface orbits

$$
\begin{aligned}
& r_{g}=6371.0 \mathrm{~km} \\
& a_{g}=9.820 \mathrm{~m} / \mathrm{s}^{2} \\
& T_{g}=5060.837 \mathrm{~s} \\
& v_{g}=7909.792 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Geosynchronous orbit

$$
\begin{aligned}
& r_{g}=42164.0 \mathrm{~km} \\
& a_{g}=0.2242 \mathrm{~m} / \mathrm{s}^{2} \\
& T_{g}=86163.6 \mathrm{~s} \\
& v_{g}=3074.666 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Moon orbit ( $\mathrm{d}=84600 \mathrm{~s}$ )

$$
\begin{aligned}
& r_{g}=384400 \mathrm{~km} \\
& a_{g}=.0026976 \mathrm{~m} / \mathrm{s}^{2} \\
& T_{g}=27.4519 \mathrm{~d} \\
& v_{g}=1.0183 \mathrm{~km} / \mathrm{s}
\end{aligned}
$$

### 3.2. Example - Planetary orbits

$$
N_{\text {points }}=M_{\text {sun }} / m_{P}
$$

mercury $r_{g}=57\left(10^{9}\right) m, T_{g}=87.969 d, v_{g}=47.87 \mathrm{~km} / \mathrm{s}$ venus $r_{g}=108\left(10^{9} m, T_{g}=224.698 d, v_{g}=35.02 \mathrm{~km} / \mathrm{s}\right.$ earth $r_{g}=149\left(10^{9}\right) m, T_{g}=365.26 d, v_{g}=29.78 \mathrm{~km} / \mathrm{s}$ mars $r_{g}=227\left(10^{9}\right) m, T_{g}=686.97 d, v_{g}=24.13 \mathrm{~km} / \mathrm{s}$ jupiter $r_{g}=778\left(10^{9}\right) m, T_{g}=4336.7 d, v_{g}=13.06 \mathrm{~km} / \mathrm{s}$ pluto $r_{g}=5.9\left(10^{12}\right) m, T_{g}=90613.4 d, v_{g}=4.74 \mathrm{~km} / \mathrm{s}$

The energy required to lift a 1 kg satellite into a geosynchronous orbit is the difference between the energy of each of the 2 orbits (geosynchronous and earth).

$$
\begin{gather*}
E_{\text {orbital }}=\frac{h c}{2 \pi r_{6371}}-\frac{h c}{2 \pi r_{42164}}  \tag{12}\\
N_{\text {links }}=\left(M_{\text {earth }} m_{\text {satellite }}\right) / m_{P}^{2}=0.126 \times 10^{41} \\
E_{\text {total }}=E_{\text {orbital }} N_{\text {links }}=53 \mathrm{MJ} / \mathrm{kg}
\end{gather*}
$$

4 Angular momentum

$$
\begin{gather*}
N_{\text {sun }}=\frac{M_{\text {sun }}}{m_{P}}  \tag{13}\\
N_{\text {planet }}=\frac{M_{\text {planet }}}{m_{P}}  \tag{14}\\
N_{\text {links }}=N_{\text {sun }} N_{\text {planet }} \tag{15}
\end{gather*}
$$

4.1 Orbital angular momentum $L_{\text {oam }}$

$$
\begin{align*}
L_{\text {oam }}= & 2 \pi \frac{M r^{2}}{T}=N_{\text {sun }} N_{\text {planet }} \frac{h}{2 \pi} \sqrt{\frac{2 \alpha n_{p}}{N_{\text {sun }}}} \\
& =N_{\text {links }} n \frac{h}{2 \pi} \sqrt{2 \alpha}, \frac{\mathrm{kgm}^{2}}{\mathrm{~s}} \tag{16}
\end{align*}
$$

Orbital angular momentum of the planets;
mercury $=.9153 \times 10^{39}$
venus $=.1844 \times 10^{41}$
earth $=.2662 \times 10^{41}$
mars $=.3530 \times 10^{40}$
jupiter $=.1929 \times 10^{44}$
pluto $=.365 \times 10^{39}$
Angular momentum combined with orbit velocity reduces to a unit of $\hbar c$ irrespective of distance between the orbiting bodies.

$$
\begin{equation*}
L_{o a m} v_{g}=N_{\text {links }} \frac{h c}{2 \pi}, \frac{\mathrm{kgm}^{3}}{s^{2}} \tag{17}
\end{equation*}
$$

4.2 Rotational angular momentum $L_{\text {ram }}$

$$
\begin{equation*}
N_{\text {links }}=\left(N_{\text {planet }}\right)^{2} \tag{18}
\end{equation*}
$$

Rotational angular momentum contribution to planet rotation.

$$
\begin{gather*}
v_{r o t}=\sqrt{N_{\text {points }}} \frac{c}{2 \alpha \sqrt{n_{p}}}=\frac{c}{2 \alpha n}  \tag{19}\\
T_{r o t}=\frac{2 \pi r}{v_{r o t}} \tag{20}
\end{gather*}
$$

$L_{\text {ram }}=\left(\frac{2}{5}\right) \frac{2 \pi M r^{2}}{T}=\left(\frac{2}{5}\right) N_{\text {links }} n \frac{\mathrm{~h}}{2 \pi}, \frac{\mathrm{kgm}^{2}}{\mathrm{~s}}$
$n_{\text {earth }}=2289.4($ radius $=6371 \mathrm{~km})$
$T_{\text {rot }}=83847.7 s \quad(86400)$

$$
\begin{align*}
& v_{\text {rot }}=477.8 \mathrm{~m} / \mathrm{s}(463.3) \\
& L_{\text {ram }}=.727 \times 10^{34} \frac{\mathrm{kgm}^{2}}{\mathrm{~s}}(.705) \\
& n_{\text {mars }}=5094.7(\text { radius }=3390 \mathrm{~km}) \\
& T_{\text {rot }}=99208 \mathrm{~s}(88643) \\
& v_{\text {rot }}=214.7 \mathrm{~m} / \mathrm{s}(240.29) \\
& L_{\text {ram }}=.187 \times 10^{33} \frac{\mathrm{kgm}^{2}}{\mathrm{~s}}(.209) \\
& \quad L_{\text {oam }} v_{\text {rot }}=\left(\frac{2}{5}\right) N_{\text {links }} \frac{\mathrm{hc}}{2 \pi 2 \alpha}, \frac{\mathrm{kgm}^{3}}{\mathrm{~s}^{2}} \tag{22}
\end{align*}
$$

4.3. Time dilation.
4.3.1. Velocity: In the article 'Programming Relativity in a Planck unit Universe', a model of a virtual hypersphere universe expanding in Planck steps was proposed [2]. In that model the universe hyper-sphere expands in all directions evenly, objects are pulled along by the expansion of the hyper-sphere irrespective of any motion in 3-D space. As such, while B (satellite) has a circular orbit in 3-D space co-ordinates it has a cylindrical orbit around the A (planet) time-line axis in the hyper-sphere co-ordinates with orbital period $T_{g} c$ (from $B^{1}$ to $B^{2}$ ) at radius $r_{g}$ and orbital velocity $v_{g}$. If A is moving with the universe expansion (albeit stationary in 3-D space) then the orbital time $t_{g}$ alongside the A time-line axis (fig. 1) becomes;

$$
\begin{equation*}
t_{g}=\sqrt{\left(T_{g} c\right)^{2}-\left(2 \pi r_{g}\right)^{2}}=\left(T_{g} c\right) \sqrt{1-\frac{v_{g}^{2}}{c^{2}}} \tag{23}
\end{equation*}
$$



Fig. 1: orbit relative to A timeline axis
4.3.2. Gravitational:

$$
\begin{align*}
& v_{s}=v_{\text {escape }}=\sqrt{2} \cdot v_{g}  \tag{24}\\
& \sqrt{1-\frac{2 G M}{r_{g} c^{2}}}=\sqrt{1-\frac{v_{s}^{2}}{c^{2}}} \tag{25}
\end{align*}
$$

4.4. Binding energy in the nucleus

$$
\begin{gather*}
m_{n u c}=m_{p}+m_{n}  \tag{26}\\
\lambda_{s}=\frac{l_{p} m_{P}}{m_{n u c}}  \tag{27}\\
r_{0}=\sqrt{\alpha} \lambda_{s}  \tag{28}\\
R_{s}=\alpha \lambda_{s} \tag{29}
\end{gather*}
$$

$$
\begin{equation*}
v_{s}^{2}=\frac{c^{2}}{\alpha} \tag{30}
\end{equation*}
$$

The gravitational binding energy $\left(\mu_{G}\right)$ is the energy required to pull apart an object consisting of loose material and held together only by gravity.

$$
\begin{equation*}
\mu_{G}=\frac{3 G m_{n u c}^{2}}{5 R_{s}}=\frac{3 m_{n u c} c^{2}}{5 \alpha}=\frac{3 m_{n u c} v_{s}^{2}}{5} \tag{31}
\end{equation*}
$$

Nuclear binding energy is the energy required to split a nucleus of an atom into its component parts. The electrostatic coulomb constant;

$$
\begin{gather*}
a_{c}=\frac{3 e^{2}}{20 \pi \epsilon r_{0}}  \tag{32}\\
E=\sqrt{( } \alpha) a_{c}=\frac{3 m_{n u c} c^{2}}{5 \alpha}=\frac{3 m_{n u c} v_{s}^{2}}{5} \tag{33}
\end{gather*}
$$

Average binding energy in nucleus;
$\mu_{G}=8.22 \mathrm{MeV} /$ nucleon
4.5. Anomalous precession
semi-minor axis: $b=\alpha l^{2} \lambda_{\text {sun }}$
semi-major axis: $a=\alpha n^{2} \lambda_{\text {sun }}$
radius of curvature $L$

$$
\begin{gather*}
L=\frac{b^{2}}{a}=\frac{a l^{4} \lambda_{\text {sun }}}{n^{2}}  \tag{34}\\
\frac{3 \lambda_{\text {sun }}}{2 L}=\frac{3 n^{2}}{2 \alpha l^{4}} \tag{35}
\end{gather*}
$$

$$
\begin{align*}
& \text { precession }=\frac{3 n^{2}}{2 \alpha l^{4}} \cdot 1296000 \cdot\left(100 T_{\text {earth }} / T_{\text {planet }}\right)  \tag{36}\\
& \text { Mercury }=42.9814 \\
& \text { Venus }=8.6248 \\
& \text { Earth }=3.8388 \\
& \text { Mars }=1.3510 \\
& \text { Jupiter }=0.0623
\end{align*}
$$

4.6. $F_{p}=$ Planck force;

$$
\begin{gather*}
F_{p}=\frac{m_{P} c^{2}}{l_{p}} \\
M_{a}=\frac{m_{P} \lambda_{a}}{2 l_{p}}, m_{b}=\frac{m_{P} \lambda_{b}}{2 l_{p}}  \tag{37}\\
F_{g}=\frac{M_{a} m_{b} G}{R^{2}}=\frac{\lambda_{a} \lambda_{b} F_{p}}{4 R_{g}^{2}}=\frac{\lambda_{a} \lambda_{b} F_{p}}{4 \alpha^{2} n^{4}\left(\lambda_{a}+\lambda_{b}\right)^{2}} \tag{38}
\end{gather*}
$$

a) $M_{a}=m_{b}$

$$
\begin{equation*}
F_{g}=\frac{F_{p}}{\left(4 \alpha n^{2}\right)^{2}} \tag{39}
\end{equation*}
$$

b) $M_{a} \gg m_{b}$

$$
\begin{equation*}
F_{g}=\frac{\lambda_{b} F_{p}}{\left(2 \alpha n^{2}\right)^{2} \lambda_{a}}=\frac{m_{b} c^{2}}{2 \alpha^{2} n^{4} \lambda_{a}}=m_{b} a_{g} \tag{40}
\end{equation*}
$$

## 5 Orbital transition

Atomic electron transition is defined as a change of an electron from one energy level to another, theoretically this should be a discontinuous electron jump from one energy level to another although the mechanism for this is not clear. The following presumes 'physical links' instead of mathematical orbital regions.
Let us consider the Hydrogen Rydberg formula for transition between and initial $i$ and a final $f$ orbit. The incoming photon $\lambda_{R}$ causes the electron to 'jump' from the $n=i$ to $n=f$ orbit.

$$
\begin{equation*}
\lambda_{R}=R \cdot\left(\frac{1}{n_{i}^{2}}-\frac{1}{n_{f}^{2}}\right)=\frac{R}{n_{i}^{2}}-\frac{R}{n_{f}^{2}} \tag{41}
\end{equation*}
$$

The above can be interpreted as referring to 2 photons;

$$
\lambda_{R}=\left(+\lambda_{i}\right)-\left(+\lambda_{f}\right)
$$

Let us suppose a region of space between a free proton $p^{+}$ and a free electron $e^{-}$which we may define as zero. This region then divides into 2 waves of inverse phase which we may designate as photon $(+\lambda)$ and anti-photon $(-\lambda)$ whereby

$$
(+\lambda)+(-\lambda)=\text { zero }
$$

The photon $(+\lambda)$ leaves (at the speed of light), the antiphoton $(-\lambda)$ however is trapped between the electron and proton and forms a standing wave orbital. Due to the loss of the photon, the energy of $\left(p^{+}+e^{-}+-\lambda\right)<$ ( $p^{+}+e^{-}+0$ ) and so is stable.

Let us define an $(n=i)$ orbital as $\left(-\lambda_{i}\right)$. The incoming Rydberg photon $\lambda_{R}=\left(+\lambda_{i}\right)-\left(+\lambda_{f}\right)$ arrives in a 2 -step process. First the $\left(+\lambda_{i}\right)$ adds to the existing $\left(-\lambda_{i}\right)$ orbital.

$$
\left(-\lambda_{i}\right)+\left(+\lambda_{i}\right)=\text { zero }
$$

The $\left(-\lambda_{i}\right)$ orbital is canceled and we revert to the free electron and free proton; $p^{+}+e^{-}+0$ (ionization). However we still have the remaining $-\left(+\lambda_{f}\right)$ from the Rydberg formula.

$$
0-\left(+\lambda_{f}\right)=\left(-\lambda_{f}\right)
$$

From this wave addition followed by subtraction we have replaced the $n=i$ orbital with an $n=f$ orbital. The electron has not moved (there was no transition from an $n_{i}$ to $n_{f}$ orbital), however the electron region (boundary) is now determined by the new $n=f$ orbital $\left(-\lambda_{f}\right)$.

## References

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