A new Koide tuple: strange-charm-bottom.

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Abstract

With the negative sign for $\sqrt{m}_s$, the quarks strange, charm and bottom make a Koide tuple. It continues the c-b-t tuple recently found by Rodejohann and Zhang and, more peculiar, it is quasi-orthogonal to the original charged lepton triplet.

1 A history of Koide sum rule

In the late seventies, the empirical observation of a relationship between Cabibbo angle and $\sqrt{m_d/m_s}$ drove an industry of models and textures for the quark mass matrix, simultaneously to the advent of the third generation. Interesting actors here are Wilczek and Zee, Fritzsch, and particularly Harari et al. [12], who goes as far as to propose a model that also implies a direct prediction of the u, d, s masses:

$$m_u = 0,\ \frac{m_d}{m_s} = \frac{2-\sqrt{3}}{2+\sqrt{3}} \tag{1}$$

and, note, a Cabibbo angle of 15 degrees.

The model industry, in its version of preon models, was popularized in Japan by Terazawa, who also proceed to suggest some more complex sum rules between quotients of square roots, some of them coming from GUTs, some from preons, and some of them being purely empirical. And both GUT based and preon based models allowed eventually to extend the equations to the lepton sector, which was more promising, given the high precision of the mass of electron and muon.

So, in the early eighties Koide suggested some models [6, 7, 8] able to predict a Cabibbo angle exclusively from the lepton sector, and then he found that some of the equations also predicted a relationship between the three charged leptons:

$$\frac{(\sqrt{m}_e + \sqrt{m}_\mu + \sqrt{m}_\tau)^2}{m_e + m_\mu + m_\tau} = \frac{3}{2} \tag{2}$$

Really Koide's equation predicted the mass of the tau lepton before the correct measurement of its modern value, and still today it is exact inside one-sigma levels. But at the time of the proposal, the experimental measurement of tau was not so good and it was mistaken in some percentage. Thus the "sum rule" was kept sleep until a re-evaluation of the tau mass vindicated it. At

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that time, in the mid nineties, the field has evolved and the original goals were not high in the check lists, so the formula got a new start: not a final product anymore, but a point of departure guiding model builders.

Foot [11] suggested to read the relationship more geometrically, as if asking the triple of square roots to keep an angle of 45 degrees with the triple $(1, 1, 1)$. And Esposito and Santorelli [3] did an analysis of the renormalization running, and a first approach to the question of fitting neutrinos, if massive, into a similar formula.

Later in 2005, an informal online working group[1] took the task of evaluating low energy mass formulae and its current validity, and of course Koide formula emerged again here. An incomplete review of the formula was done in [14], addressing the case of zero mass, where the relationships (1) are recovered. The neutrino case was reevaluated with most modern data [1, 2]. Eventually the reference to Koide sum rule for neutrinos did its way into the standard literature on PMNS parameters.

A byproduct of the online effort was to rewrite again the equation following Foot's idea, allowing a phase angle to parametrize the rotation around $(1, 1, 1)$.

$$m_k = M(1 + \sqrt{2}\cos(2k\pi/3 + \delta_0))^2 \tag{3}$$

It is usual to absorb the permutation ambiguity of Foot's cone in this phase δ_0, by the combination of change of sign, plus rotations of 120 degrees.

It is intriguing that for charged leptons $M \approx 313$ MeV, typical of constituent quarks or of QCD strings. But more important is that this parametrization clarifies the use of negative signs in the square roots of the masses. This was a key to build the neutrino tuples, and it is relevant for the new tuple that we are presenting in this paper.

Inspecting (3), you can see that there are two ways to produce a degenerated pair: with $\delta = 0$ or with $\delta = \pm\pi/12$, and you can use them to produce one or other hierarchy of neutrinos. Being the phase of the charged leptons δ_l, C Brannen proposed [1] a phase $\pi/12 + \delta_l$ to match known bounds, and M. D. Sheppeard proposed [16] $-\pi/12 + \delta_l$ as a match to the results of MINOS experiment.

The PhD thesis of François Goffinet [4], in 2008, revisits most of the old concepts, and then including neutrinos and the possibility of negative roots, as well as the idea of generalizing the formula to all the six quarks in a single sum. This last possibility has been also reviewed by [5].

2 Current advances

Very recently Rodejohann and Zhang [15] recognized the possibility of fitting Koide formula to quark triplets not of the same charge, but of nearby mass: they suggested fits for the low mass quarks *uds* with modern values of the mass quotients, and more importantly for this note, they suggested a very good fit for the heavy quarks *cbt*. This can be readily verified from current data from [13]. With $m_t = 172.9 \pm 0.6 \pm 0.9$, $m_b = 4.19^{+0.18}_{-0.06}$, and $1.29^{+0.05}_{-0.11}$ GeV, the central

[1] Around www.physicsforums.com and some blogs, after some headstart in the s.p.r usenet newsgroup. Besides the authors of the referenced papers, other strong online contributors as Hans de Vries or Dave Look provided alternate insights and even programmatic analysis tools

values give to the LHS of (2) a value of about 1.495, very close to the required 3/2. Further analysis of it, including renormalization group, can be seen in [5].

It is interesting to try to produce all the masses from the two upper ones. We can solve (2) as

$$m_3(m_1, m_2) = \left((\sqrt{m_1} + \sqrt{m_2}) \left(2 - \sqrt{3 + 6 \frac{\sqrt{m_1 m_2}}{(\sqrt{m_1} + \sqrt{m_2})^2}} \right) \right)^2 \tag{4}$$

and use it to iterate. We get the descent:

$$\begin{array}{lcl} m_t & = & 172.9 \text{ GeV} \\ m_b & = & 4.19 \text{ GeV} \\ m_c(172.9, 4.19) & = & 1.356 \text{ GeV} \\ m_s(4.19, 1.356) & = & 92 \text{ MeV} \\ m_u(1.356, 0.092) & = & 0.036 \text{ MeV} \\ m_d(0.092, 0.000036) & = & 5.3 \text{ MeV} \end{array}$$

The main point in this descent is that we have produced a tuple not yet in the literature, the one of strange, charm, and bottom. How is it?

Closer examination shows that the reason of the miss is that in order to meet (2), the value of $\sqrt{s}$ must be taken negative. But this is a valid situation, according Foot interpretation and the parametrisation (3).

Of course, once we are considering negative roots, the equation (4) is not the only possible matching. But the possibilities are nevertheless reduced by the need of a positive discriminant in the equation and by avoiding to come back to higher values, above the mass of the bottom quark. Another problem is that, once we have recognised the sign of $\sqrt{s}$, the validity of the two next steps in the descent, up and down, is unclear. We will come back to these two quarks in the next section.

A most important observation is that $(-\sqrt{m_s}, \sqrt{m_c}, \sqrt{m_b})$ is on the opposite extreme of Foot's cone respect to $(\sqrt{m_\tau}, \sqrt{m_\mu}, \sqrt{m_e})$, making an angle of almost ninety degrees.

Furthermore, the parameters of mass and phase of this quark triple[2] seem to be three times the ones of the charged leptons: we have $M_q = 939.65$ MeV and $\delta_q = 0.666$, while in the leptons $M_l = 313.8$ MeV and $\delta_l = 0.222$, about 12.7 degrees.

We can take seriously both facts and use them to proceed in the reverse way: take as only inputs the mass of electron and muon, then recover M_l and δ_l, multiply times three to get the parameters of the opposite tuple and then the masses of strange, charm and bottom, and then use the ladder up and down to recover the previous table. It is impressive:

Inputs	Outputs
$m_e = 0.510998910 \pm 0.000000013$	$m_\tau = 1776.96894(7)$ MeV
$m_\mu = 105.6583668 \pm 0.0000038$	$m_s = 92.274758(3)$ MeV
$M_q = 3M_l$	$m_c = 1359.56428(5)$ MeV
$\delta_q = 3\delta_l$	$m_b = 4197.57589(15)$ MeV
	$m_t = 173.263947(6)$ GeV
	$m_u = 0.0356$ MeV
	$m_d = 5.32$ MeV

[2]For other quark triples we have not found any obvious hint; for the charm,bottom,top, we get $M = 29.74$ GeV, $\delta = 0.0659$

Still, the results are a bit higher respect to the descent from the top quark. Actually, we don't have an argument to keep the factor three between leptons and quarks, except that the physical situation seems to be a small perturbation respect to a case where it is a bit more justified, the full orthogonal case with $\delta_l = 15°$, $\delta_q = 45°$. Lets look at it now.

3 The $m_e = 0$, or m_u=0, limit

Our purpose is to think of the empirical triples as a rotation from a more symmetrical situation, keeping the sum of the masses (which is the modulus square of Foot's vector) constant. Our candidate unperturbed state is the one who has the mass of the electron equal to zero.

It is precisely when $\delta = 15°$ in the parametrisation (3), that one of the masses becomes zero and the mass tuple is

$$m_{15} = \left(3(1+\frac{\sqrt{3}}{2})M, 0, 3(1-\frac{\sqrt{3}}{2})M \right) \tag{5}$$

You can notice that this is equal to the result (1) above.

The state in the opposite generatrix of the cone is given by a phase of -165 degrees, which is, modulus the 120° and sign ambiguities, 45 degrees, ie three times the other phase. Our argument is thus that for a small perturbation this factor can be kept at least at first order, if not better.

Now, lets contemplate the components of this orthogonal state:

$$m_{45} = \left((1-\frac{\sqrt{3}}{2})M', 4M', (1+\frac{\sqrt{3}}{2})M' \right) \tag{6}$$

There are obviously orthogonal (remember that we take $-sqrt(s)$) for any values of M and N', but for $M' = 3M$ there is an extra symmetry, or an extra degeneration of levels if you wish. So again we incorporate this relationship, and its approximate validity under perturbation, as a postulate. If you look at the empirical data above, you will see that $M_q/M_l \approx 2.99$.

So we identify m_{45} as the quark triplet of strange, bottom and charm. Now we can proceed in the same way that before, to produce from bottom and charm the mass m_t, and from strange and charm the mass m_u, and next m_d. Just to put numbers in, and see how far we are from the perturbed state, lets fix the only parameter M to 313.86 MeV. Then we get $m_s = m_\mu = 126.1$ MeV, $m_c = m_\tau = 1757$ MeV, $m_b = 3766$ MeV and $m_t = 180$ GeV.

For m_u we need to use $\pm\sqrt{m_s}$, and then we have two possibilities in (4). But the alternative with the minus sign produces a zero discriminant, and then the same mass than m_b. We can either to claim a halt here, and disregard the possibility to estimate u and d, or to choose the plus sign.

In this case, the triple u, s, c has the same solution that the lepton triple, and then $m_u = 0$.

And the final ladder in the descent, d, u, s with $m_u = 0$, again does not allow for negative roots, just look at (3) to verify it. So we meet again the same proportion, and now really it is the relationship (1), with $m_d = m_s \frac{2-\sqrt{3}}{2+\sqrt{3}} = 9.05$ MeV.

The reproduction of equation (1) in this formalism justifies the descent with positive sign; but it is not fundamental for the rest of the work.

A last remark: Which is the mass of the pion in this limit? With m_d still not null and greater than the sum of perturbed $m_u + m_d$, it seems even a bit higher than usual. But it would be interesting to find an unperturbed model where $m_\pi = m_\mu$. Then, given that $m_e = 0$, the charged pion would be stable.

4 Conclusions

We have shown that by taking Koide equation, parametrised in Foot's cone, and then allowing determinate negative square roots, a new triplet of quarks can be legitimately added to the collection of fermion sum rules.

Furthermore, we have seen that this triplet, strange-charm-bottom, is almost orthogonal to the original triplet of charged leptons; we have argued that it is valid to accept the proportionality constants from this orthogonality and to translate the parameters from one triplet to another. This allows to calculate the masses of the quark triplet and then, applying Koide formula, also the mass of other quarks, and particularly [15] the mass of the top quark.

We have not done the effort of exploring the variation of the matchings along renormalisation group runnings, as the general insights from other works on Koide equation will apply here too. But we raise the suspicion that the matchings are to be interpreted in the infrared or at last at low energy, because of the conspicuous role of mass scales usually associated to QCD and chiral symmetry breaking: note the above empirical values for the basic mass of lepton and quark triplets, 313.8 and 939.7 MeV.

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