

# **The spherical solution of the quantum gravity**

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## **ABSTRACT**

In the general relativity theory, using Einstein's gravity field equation, discover the spherical solution of the quantum gravity. The careful point is that this theory is different from the other quantum theory. This theory is made by the Einstein's field equation.

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## I. Introduction

This theory is that it discovers the spherical solution of the quantum gravity.

Think that use following the formula.

$$\alpha = \frac{hc}{GM^2} \text{ is non-Dimension number. } \alpha \text{ 's Dimension is } \frac{J \cdot s \cdot m / s}{N \cdot m^2 \cdot kg^2 / kg^2} = \frac{J \cdot m}{J \cdot m} = 1$$

$h$  is the plank constant,  $c$  is the light speed,  $G$  is the gravity constant,  $M$  is the matter's mass.

The spherical solution (The Schwarzschild solution) of the general relativity is

$$d\tau^2 = \left(1 - \frac{2GM}{rc^2}\right) dt^2 - \frac{1}{c^2} \left[ \frac{dr^2}{1 - \frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] \quad (1)$$

## II. Additional chapter-I

In this theory, the general relativity theory's field equation is written completely.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu} \quad (2)$$

Eq (2) multiply  $g^{\mu\nu}$  and does contraction,

$$\begin{aligned} g^{\mu\nu} R_{\mu\nu} - \frac{1}{2} g^{\mu\nu} g_{\mu\nu} R \\ = -R = -\frac{8\pi G}{c^4} T^{\lambda}_{\lambda} \end{aligned} \quad (3)$$

Therefore, Eq (2) is

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \frac{8\pi G}{c^4} T^{\lambda}_{\lambda} &= -\frac{8\pi G}{c^4} T_{\mu\nu} \\ R_{\mu\nu} &= -\frac{8\pi G}{c^4} \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^{\lambda}_{\lambda} \right) \end{aligned} \quad (4)$$

In this time, the spherical coordinate system's vacuum solution is by  $T_{\mu\nu} = 0$

$$R_{\mu\nu} = 0 \quad (5)$$

The spherical coordinate system's invariant time is

$$d\tau^2 = A(t, r) dt^2 - \frac{1}{c^2} [B(t, r) dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2] \quad (6)$$

Using Eq(6)'s metric, save the Riemannian-curvature tensor, and does contraction, save Ricci-tensor.

$$R_{tt} = -\frac{A''}{2B} + \frac{A'B'}{4B^2} - \frac{A'}{Br} + \frac{A'^2}{4AB} + \frac{\ddot{B}}{2B} - \frac{\dot{B}^2}{4B^2} - \frac{\dot{A}\dot{B}}{4AB} = 0 \quad (7)$$

$$R_{rr} = \frac{A''}{2A} - \frac{A'^2}{4A^2} - \frac{A'B'}{4AB} - \frac{B'}{Br} - \frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB} = 0 \quad (8)$$

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = 0 \quad (9)$$

$$R_{\phi\phi} = R_{\theta\theta} \sin^2 \theta = 0 \quad (10)$$

$$R_{rr} = -\frac{\dot{B}}{Br} = 0$$

$$R_{t\theta} = R_{t\phi} = R_{r\theta} = R_{r\phi} = R_{\theta\phi} = 0 \quad (11)$$

In this time,  $' = \frac{\partial}{\partial r}$ ,  $\dot{\phantom{x}} = \frac{1}{c} \frac{\partial}{\partial t}$

By Eq(11),

$$\dot{B} = 0 \quad (12)$$

By Eq(7) and Eq(8),

$$\frac{R_{tt}}{A} + \frac{R_{rr}}{B} = -\frac{1}{Br} \left( \frac{A'}{A} + \frac{B'}{B} \right) = -\frac{(AB)'}{rAB^2} = 0 \quad (13)$$

Therefore,

$$A = \frac{1}{B} \quad (14)$$

If Eq(9) is inserted by Eq(14),

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = -1 + \left( \frac{r}{B} \right)' = 0 \quad (15)$$

If solve Eq(15)

$$\frac{r}{B} = r + C \rightarrow \frac{1}{B} = 1 + \frac{C}{r} \quad (16)$$

In this time, be able to think following the formula.

$$C = -\frac{2GM}{c^2} \exp \left[ -\alpha_1 \left( \frac{hc}{GM^2} \right)^{\beta_1} - \alpha_2 \left( \frac{hc}{GM^2} \right)^{\beta_2} - \dots - \alpha_i \left( \frac{hc}{GM^2} \right)^{\beta_i} \dots - \alpha_n \left( \frac{hc}{GM^2} \right)^{\beta_n} \right]$$

$$N_i > \alpha_i \geq 0, \beta_i \geq 1 - \varepsilon_{0i}, \quad |\varepsilon_{0i}| \ll 1$$

$$\alpha_1, \alpha_2, \dots, \alpha_n, \beta_1, \beta_2, \dots, \beta_n \text{ are real numbers.}$$

$$N_i \text{ is the large number. } \varepsilon_{0i} \text{ is the smallest number.}$$

(17)

The reason of  $\beta_i \geq 1 - \varepsilon_{0i}$  is that if  $0 \leq \beta_i < 1 - \varepsilon_{0i}$ , in this case, it isn't able to be the real gravity situation. In this time, the large number  $N_i$  is the number that is befitted the real gravity situation.

And the smallest number  $\varepsilon_{0i}$  is able to be the positive number or is able to be the negative number.

$$\frac{1}{B} = 1 - \frac{2GM}{rc^2} \Sigma(M)$$

$$\Sigma(M) = \exp[-\alpha_1 \left(\frac{hc}{GM^2}\right)^{\beta_1} - \alpha_2 \left(\frac{hc}{GM^2}\right)^{\beta_2} - \dots - \alpha_i \left(\frac{hc}{GM^2}\right)^{\beta_i} - \dots - \alpha_n \left(\frac{hc}{GM^2}\right)^{\beta_n}]$$

$$N_i > \alpha_i \geq 0, \beta_i \geq 1 - \varepsilon_{0i}, |\varepsilon_{0i}| \ll 1$$

$$\alpha_1, \alpha_2, \dots, \alpha_n, \beta_1, \beta_2, \dots, \beta_n \text{ are real numbers.}$$

$$N_i \text{ is the large number. } \varepsilon_{0i} \text{ is the smallest number.}$$
(18)

Therefore, Eq(18) is

$$A = \frac{1}{B} = 1 - \frac{2GM}{rc^2} \Sigma(M)$$

$$\Sigma(M) = \exp[-\alpha_1 \left(\frac{hc}{GM^2}\right)^{\beta_1} - \alpha_2 \left(\frac{hc}{GM^2}\right)^{\beta_2} - \dots - \alpha_i \left(\frac{hc}{GM^2}\right)^{\beta_i} - \dots - \alpha_n \left(\frac{hc}{GM^2}\right)^{\beta_n}]$$

$$N_i > \alpha_i \geq 0, \beta_i \geq 1 - \varepsilon_{0i}, |\varepsilon_{0i}| \ll 1$$

$$\alpha_1, \alpha_2, \dots, \alpha_n, \beta_1, \beta_2, \dots, \beta_n \text{ are real numbers.}$$

$$N_i \text{ is the large number. } \varepsilon_{0i} \text{ is the smallest number.}$$
(19)

To know Eq(19)'s second term, does Newton's limitation

$$\frac{d^2 x^\lambda}{d\tau^2} = -\Gamma^\lambda_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}$$

$$\frac{d^2 r}{dt^2} \approx \frac{1}{2} c^2 \frac{\partial(-A)}{\partial r} = -\frac{GM}{r^2} \exp[-\alpha_1 \left(\frac{hc}{GM^2}\right)^{\beta_1} - \alpha_2 \left(\frac{hc}{GM^2}\right)^{\beta_2} - \dots - \alpha_n \left(\frac{hc}{GM^2}\right)^{\beta_n}]$$
(20)

Therefore, by Eq(19) ,in this theory, the spherical solution of the quantum gravity is

$$d\tau^2 = \left(1 - \frac{2GM}{rc^2} \Sigma(M)\right) dt^2 - \frac{1}{c^2} \left[ \frac{dr^2}{\left(1 - \frac{2GM}{rc^2} \Sigma(M)\right)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]$$

$$\Sigma(M) = \exp[-\alpha_1 \left(\frac{hc}{GM^2}\right)^{\beta_1} - \alpha_2 \left(\frac{hc}{GM^2}\right)^{\beta_2} - \dots - \alpha_i \left(\frac{hc}{GM^2}\right)^{\beta_i} - \dots - \alpha_n \left(\frac{hc}{GM^2}\right)^{\beta_n}]$$

$$N_i > \alpha_i \geq 0, \beta_i \geq 1 - \varepsilon_{0i}, |\varepsilon_{0i}| \ll 1$$

$$\alpha_1, \alpha_2, \dots, \alpha_n, \beta_1, \beta_2, \dots, \beta_n \text{ are real numbers.}$$

$$N_i \text{ is the large number. } \varepsilon_{0i} \text{ is the smallest number.}$$
(21)

### III. Additional chapter-II

In Eq(21), if  $h \rightarrow 0$ ,

$$\Sigma(M) = \exp[-\alpha_1 \left(\frac{hc}{GM^2}\right)^{\beta_1} - \alpha_2 \left(\frac{hc}{GM^2}\right)^{\beta_2} - \dots - \alpha_n \left(\frac{hc}{GM^2}\right)^{\beta_n}] \rightarrow 1$$

$$d\tau^2 = \left(1 - \frac{2GM}{rc^2}\right) dt^2 - \frac{1}{c^2} \left[ \frac{dr^2}{1 - \frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] \quad (22)$$

In Eq(21), if  $M \rightarrow 0$ ,

$$M \Sigma(M) = M \exp[-\alpha_1 \left(\frac{hc}{GM^2}\right)^{\beta_1} - \alpha_2 \left(\frac{hc}{GM^2}\right)^{\beta_2} - \dots - \alpha_n \left(\frac{hc}{GM^2}\right)^{\beta_n}] = 0$$

$$d\tau^2 = dt^2 - \frac{1}{c^2} [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2] \quad (23)$$

Specially, in Eq(21), if  $\alpha_1 = \beta_1 = 1, \alpha_2 = \dots = \alpha_n = 0$ , Eq(21) is

$$d\tau^2 = \left(1 - \frac{2GM}{rc^2} \Sigma(M)\right) dt^2 - \frac{1}{c^2} \left[ \frac{dr^2}{\left(1 - \frac{2GM}{rc^2} \Sigma(M)\right)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]$$

$$= \left(1 - \frac{2GM}{rc^2} \exp\left(-\frac{hc}{GM^2}\right)\right) dt^2 - \frac{1}{c^2} \left[ \frac{dr^2}{\left(1 - \frac{2GM}{rc^2} \exp\left(-\frac{hc}{GM^2}\right)\right)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]$$

$$= \left(1 - \frac{2GM'}{rc^2}\right) dt^2 - \frac{1}{c^2} \left[ \frac{dr^2}{\left(1 - \frac{2GM'}{rc^2}\right)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right],$$

$$M' = M \exp\left(-\frac{hc}{GM^2}\right) \quad (24)$$

Caution:  $M'$  isn't concerned about the real physical mass and only use for computing the other physical element.

1. First example, if  $M$  is the sun's mass, saves  $M'$

$$M \sim 1.99 \times 10^{30} \text{ kg}$$

$$\exp\left(-\frac{hc}{GM^2}\right) \sim \exp\left(-\frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s} \times 3 \times 10^8 \text{ m/s}}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \times 1.99^2 \times 10^{60} \text{ kg}^2}\right)$$

$$\sim \exp(-0.75 \times 10^{-75})$$

$$\sim 1 - 0.75 \times 10^{-75}$$

$$M' = M \exp\left(-\frac{hc}{GM^2}\right) \sim 1.99 \times 10^{30} \times (1 - 0.75 \times 10^{-75}) \text{ kg}$$

$$= (1.99 \times 10^{30} - 1.49 \times 10^{-45}) \text{ kg}$$

Therefore, in the example.1, it is enough to use the Schwarzschild solution.

2. Second example, if  $M$  is the mass of the ball that the mass is  $1kg$ , saves  $M'$

$$M = 1kg$$

$$\begin{aligned} \exp\left(-\frac{hc}{GM^2}\right) &\sim \exp\left(-\frac{6.63 \times 10^{-34} J \cdot s \times 3 \times 10^8 m/s}{6.67 \times 10^{-11} N \cdot m^2 / kg^2 \times 1kg^2}\right) \\ &\sim \exp(-2.98 \times 10^{-15}) \\ &\sim 1 - 2.98 \times 10^{-15} \end{aligned}$$

$$\begin{aligned} M' &= M \exp\left(-\frac{hc}{GM^2}\right) \sim 1 \times (1 - 2.98 \times 10^{-15})kg \\ &= (1 - 2.98 \times 10^{-15})kg \end{aligned}$$

3. Third example, if  $M$  is the electron's mass, saves  $M'$

$$M \sim 9.11 \times 10^{-31} kg$$

$$\exp\left(-\frac{hc}{GM^2}\right) \sim \exp\left(-\frac{6.63 \times 10^{-34} J \cdot s \times 3 \times 10^8 m/s}{6.67 \times 10^{-11} N \cdot m^2 / kg^2 \times 9.11^2 \times 10^{-62} kg^2}\right) \sim \exp(-3.59 \times 10^{45})$$

$$M' = M \exp\left(-\frac{hc}{GM^2}\right) \sim 9.11 \times 10^{-31} \exp(-3.59 \times 10^{45})kg$$

Therefore, in the example.3, it must use this theory that includes the quantum gravity effect.

#### IV. Conclusion

It found the spherical solution of the quantum gravity.

The careful point is that this theory is different from the other quantum theory. This theory is made by the Einstein's field equation.

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