

The new accelerated coordinate system by the tetrad

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ABSTRACT

In the general relativity theory, find the new accelerated system theory that used the tetrad on the new method and discover the new inverse-coordinate transformation of the new accelerated system theory and expand to be the new accelerated system theory of the accelerated observer that have the initial velocity. Therefore know that the Rindler coordinate theory is not the unique accelerated coordinate theory.

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I.Introduction

This theory is that find the new accelerated system theory that used the tetrad on the new method and expand to be the new accelerated system theory of the accelerated observer that has the initial velocity. Therefore prove that the Rindler coordinate theory is not the unique accelerated coordinate theory.

Finding the Rindler's coordinate theory , use following the formula about the constant accelerated matter.

$$x + \frac{c^2}{a_0} = \frac{c^2}{a_0} \cosh\left(\frac{a_0 \tau}{c}\right), t = \frac{c}{a_0} \sinh\left(\frac{a_0 \tau}{c}\right) \quad (1)$$

x and t is the coordinate and the time in the inertial system about the constant accelerated matter. a_0 is the constant acceleration, τ is invariable time about the constant accelerated matter, c is light speed in the inertial system in the free space-time.

In the special relativity, the formula about 2-Dimension inertial coordinate system $S(t, x)$ and $S'(t', x')$ is

$$V = \frac{u + v_0}{1 + \frac{u}{c^2} v_0}, \quad V = \frac{dx}{dt}, u = \frac{dx'}{dt'}, \quad dx = \frac{dx' + v_0 dt'}{\sqrt{1 - \frac{v_0^2}{c^2}}}, \quad dt = \frac{dt' + \frac{v_0}{c^2} dx'}{\sqrt{1 - \frac{v_0^2}{c^2}}}$$

$$a = \frac{d}{dt} \left(\frac{V}{\sqrt{1 - \frac{V^2}{c^2}}} \right), \quad a' = \frac{d}{dt'} \left(\frac{u}{\sqrt{1 - \frac{u^2}{c^2}}} \right) \quad (1-1)$$

The velocity V has the initial velocity v_0 and the velocity u is the velocity by the pure acceleration a' .

$$a = \frac{d}{dt} \left(\frac{V}{\sqrt{1 - \frac{V^2}{c^2}}} \right) = \frac{\sqrt{1 - \frac{v_0^2}{c^2}}}{1 + \frac{v_0}{c^2} u} \frac{d}{dt'} \left(\frac{u + v_0}{\sqrt{1 - \frac{v_0^2}{c^2}} \sqrt{1 - \frac{u^2}{c^2}}} \right) = \frac{1}{1 + \frac{v_0}{c^2} u} \frac{d}{dt'} \left(\frac{u + v_0}{\sqrt{1 - \frac{u^2}{c^2}}} \right)$$

$$a \left(1 + \frac{v_0}{c^2} u \right) = \frac{d}{dt'} \left(\frac{u}{\sqrt{1 - \frac{u^2}{c^2}}} \right) + \frac{d}{dt'} \left(\frac{v_0}{\sqrt{1 - \frac{u^2}{c^2}}} \right) \quad (1-2)$$

In this time , if the pure acceleration a' of the velocity u is

$$a' = \frac{d}{dt'} \left(\frac{u}{\sqrt{1 - \frac{u^2}{c^2}}} \right), \quad u = \frac{\int a' dt'}{\sqrt{1 + \frac{1}{c^2} [\int a' dt']^2}} \quad (1-3)$$

Eq(1-2) is

$$\begin{aligned}
a(1 + \frac{v_0}{c^2}u) &= \frac{d}{dt'}\left(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}\right) + \frac{d}{dt'}\left(\frac{v_0}{\sqrt{1-\frac{u^2}{c^2}}}\right) = a' + v_0 \frac{d}{dt'}\left(\sqrt{1 + \frac{1}{c^2}[\int a' dt']^2}\right) \\
&= a' + v_0 \frac{\int a' dt'}{\sqrt{1 + \frac{1}{c^2}[\int a' dt']^2}} \frac{a'}{c^2} = a'(1 + \frac{v_0}{c^2} \frac{\int a' dt'}{\sqrt{1 + \frac{1}{c^2}[\int a' dt']^2}}) \\
&= a'(1 + \frac{v_0}{c^2}u) \quad (1-4)
\end{aligned}$$

Therefore, the acceleration a about the accelerated matter that has the initial velocity v_0 in 2-Dimension inertial coordinate system $S(t, x)$ and the other acceleration a' about the accelerated matter that has not the initial velocity v_0 in 2-Dimension inertial coordinate system $S'(t', x')$ are same. In this time, if the acceleration a' is the constant acceleration a_0 , the inertial acceleration in 2-Dimension inertial coordinate system $S(t, x)$ and in 2-Dimension inertial coordinate system $S'(t', x')$ is the constant acceleration a_0 .

$$a_0 = a' = \frac{d}{dt'}\left(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}\right) = a = \frac{d}{dt}\left(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}\right) \quad (1-5)$$

Hence, Eq(1) is in the 2-Dimension inertial coordinate system $S'(t', x')$

$$\begin{aligned}
x' &= \frac{c^2}{a_0}(\cosh(\frac{a_0 \tau}{c}) - 1) \\
t' &= \frac{c}{a_0} \sinh(\frac{a_0 \tau}{c}) \quad (1-6)
\end{aligned}$$

Therefore, in the 2-Dimension inertial coordinate system $S(t, x)$

$$\begin{aligned}
t &= \gamma(t' + \frac{v_0}{c^2}x') = \gamma\left(\frac{c}{a_0} \sinh(\frac{a_0 \tau}{c}) + \frac{v_0}{a_0}(\cosh(\frac{a_0 \tau}{c}) - 1)\right) \\
x &= \gamma(x' + v_0 t') = \gamma\left(\frac{c^2}{a_0}(\cosh(\frac{a_0 \tau}{c}) - 1) + \frac{v_0 c}{a_0} \sinh(\frac{a_0 \tau}{c})\right), \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}} \quad (1-7)
\end{aligned}$$

$$dt = \gamma\left(\cosh(\frac{a_0}{c}\tau) + \frac{v_0}{c} \sinh(\frac{a_0}{c}\tau)\right) d\tau,$$

$$dx = \gamma\left(c \sinh(\frac{a_0}{c}\tau) + v_0 \cosh(\frac{a_0}{c}\tau)\right) d\tau,$$

$$V = \frac{dx}{dt} = (c \tanh(\frac{a_0}{c} \tau) + v_0) / (1 + \frac{v_0}{c} \tanh(\frac{a_0}{c} \tau)), \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}} \quad (1-8)$$

II. Additional chapter-I

The tetrad e_a^μ is the unit vector that is each other orthographic and it use the following formula.

$$e_a^\mu e_b^\nu g_{\mu\nu} = \eta_{ab} \quad (2)$$

e^a_μ is

$$e^a_\mu = \eta^{ab} g_{\mu\nu} e_b^\nu \quad (3)$$

and it is e_a^μ 's inverse-matrix. And it is

$$e^a_\mu e_b^\mu = \delta^a_b, \quad e^a_\mu e_a^\nu = \delta_\mu^\nu$$

$$e^a_\mu e^b_\nu \eta_{ab} = g_{\mu\nu} \quad (4)$$

The $e^\alpha_\mu(\tau)$ is the tetrad that if $\xi^1 = \xi^2 = \xi^3 = 0, d\xi^1 = d\xi^2 = d\xi^3 = 0$. It is not the accelerated system and it is that the point's the accelerated motion. In this time, in Eq(4) it does $g_{\mu\nu} = \eta_{\mu\nu}$.

Therefore, Eq(4) is

$$\eta_{\alpha\beta} e^{\alpha_0}(\tau) e^{\beta_0}(\tau) = \eta_{00} = -1$$

$$d\tau^2 = -\frac{1}{c^2} \eta_{\alpha\beta} dx^\alpha dx^\beta$$

$$\rightarrow -1 = \eta_{\alpha\beta} \left(\frac{1}{c} \frac{dx^\alpha}{d\tau}\right) \left(\frac{1}{c} \frac{dx^\beta}{d\tau}\right) = \eta_{\alpha\beta} e^{\alpha_0}(\tau) e^{\beta_0}(\tau) \quad (4-1)$$

According to Eq(1-7), Eq(4-1)

$$e^{\alpha_0}(\tau) = \frac{1}{c} \frac{dx^\alpha}{d\tau}$$

$$= (\gamma \cosh(\frac{a_0}{c} \tau) + \frac{v_0}{c} \gamma \sinh(\frac{a_0}{c} \tau), 0, 0), \gamma \sinh(\frac{a_0}{c} \tau) + \frac{v_0}{c} \gamma \cosh(\frac{a_0}{c} \tau), 0, 0), \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}}$$

(5)

About y -axis's and z -axis's orientation

$$e^{\alpha_2}(\tau) = (0, 0, 1, 0) \quad (6), \quad e^{\alpha_3}(\tau) = (0, 0, 0, 1) \quad (7)$$

And the other unit vector $e^{\alpha}_1(\tau)$ has to satisfy the tetrad condition, Eq (4)

$$e^{\alpha}_1(\tau) = (\gamma \sinh(\frac{a_0}{c} \tau) + \frac{v_0}{c} \gamma \cosh(\frac{a_0}{c} \tau),$$

$$\gamma \cosh(\frac{a_0}{c} \tau) + \frac{v_0}{c} \gamma \sinh(\frac{a_0}{c} \tau), 0, 0), \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}} \quad (8)$$

III. Additional chapter-II

According to the tetrad e^a_{μ} , in the flat Minkowski space, 4-Dimension inertial coordinate system $S(t, x, y, z)$ transform the 4-Dimension accelerated system $\xi(\xi^0, \xi^1, \xi^2, \xi^3)$. In this time, the accelerated observer of the 4-Dimension accelerated system $\xi(\xi^0, \xi^1, \xi^2, \xi^3)$ and the accelerated matter that has the initial velocity v_0 in 2-Dimension inertial coordinate system $S(t, x)$ are same. Therefore

$$d\tau^2 = dt^2 - \frac{1}{c^2} [dx^2 + dy^2 + dz^2]$$

$$= -\frac{1}{c^2} \eta_{ab} \frac{\partial x^a}{\partial \xi^\mu} \frac{\partial x^b}{\partial \xi^\nu} d\xi^\mu d\xi^\nu \quad (9)$$

$$= -\frac{1}{c^2} \eta_{ab} e^a_{\mu} e^b_{\nu} d\xi^\mu d\xi^\nu = -\frac{1}{c^2} g_{\mu\nu} d\xi^\mu d\xi^\nu \quad (10)$$

$$e^a_{\mu} = \frac{\partial x^a}{\partial \xi^\mu} \quad (11)$$

For saving the new accelerated system theory, the $e^{\alpha}_{\mu}(\xi^0)$ is used by Eq(5), Eq (6), Eq (7), Eq(8) that

used ξ^0 instead of τ and $e^{\alpha}_0(\tau)$'s term multiply $e^{\frac{a_0 \xi^1}{c^2}}$.

The vector $e^{\alpha}_0(\xi^0)$

$$e^{\alpha}_0(\xi^0) = \frac{\partial x^{\alpha}}{c \partial \xi^0}$$

$$= (e^{\frac{a_0 \xi^1}{c^2}} \gamma \cosh(\frac{a_0}{c} \xi^0) + e^{\frac{a_0 \xi^1}{c^2}} \frac{v_0}{c} \gamma \sinh(\frac{a_0}{c} \xi^0),$$

$$e^{\frac{a_0 \xi^1}{c^2}} \gamma \sinh(\frac{a_0}{c} \xi^0) + e^{\frac{a_0 \xi^1}{c^2}} \frac{v_0}{c} \gamma \cosh(\frac{a_0}{c} \xi^0), 0, 0), \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}} \quad (11-1)$$

$$\frac{\partial e^{\alpha}_0(\xi^0)}{\partial \xi^1} = \frac{\partial^2 x^{\alpha}}{\partial \xi^1 c \partial \xi^0} = \frac{\partial e^{\alpha}_1(\xi^0)}{c \partial \xi^0} \quad (11-2)$$

Therefore, the vector $e^{\alpha}_1(\xi^0)$ is

$$e^{\alpha}_1(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^1} = (e^{\frac{a_0 \xi^1}{c^2}} \gamma \sinh(\frac{a_0}{c} \xi^0) + e^{\frac{a_0 \xi^1}{c^2}} \frac{v_0}{c} \gamma \cosh(\frac{a_0}{c} \xi^0), \\ e^{\frac{a_0 \xi^1}{c^2}} \gamma \cosh(\frac{a_0}{c} \xi^0) + e^{\frac{a_0 \xi^1}{c^2}} \frac{v_0}{c} \gamma \sinh(\frac{a_0}{c} \xi^0), 0, 0), \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}} \quad (12)$$

About y -axis's and z -axis's orientation, the unit vector $e^{\alpha}_2(\xi^0)$ and $e^{\alpha}_3(\xi^0)$ is

$$e^{\alpha}_2(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^2} = (0, 0, 1, 0) \quad (13), \quad e^{\alpha}_3(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^3} = (0, 0, 0, 1) \quad (14)$$

The differential coordinate transformation is

$$dx^\alpha = \frac{\partial x^\alpha}{\partial \xi^\mu} d\xi^\mu = \frac{\partial x^\alpha}{\partial \xi^0} cd\xi^0 + \frac{\partial x^\alpha}{\partial \xi^1} d\xi^1 + \frac{\partial x^\alpha}{\partial \xi^2} d\xi^2 + \frac{\partial x^\alpha}{\partial \xi^3} d\xi^3 \\ = e^{\alpha}_0(\xi^0) cd\xi^0 + e^{\alpha}_1(\xi^0) d\xi^1 + e^{\alpha}_2(\xi^0) d\xi^2 + e^{\alpha}_3(\xi^0) d\xi^3 \\ cdt = \gamma [e^{\frac{a_0 \xi^1}{c^2}} \{ \cosh(\frac{a_0 \xi^0}{c}) + \frac{v_0}{c} \sinh(\frac{a_0 \xi^0}{c}) \} cd\xi^0 \\ + e^{\frac{a_0 \xi^1}{c^2}} \{ \sinh(\frac{a_0 \xi^0}{c}) + \frac{v_0}{c} \cosh(\frac{a_0 \xi^0}{c}) \} d\xi^1] \quad (15)$$

$$dx = \gamma [e^{\frac{a_0 \xi^1}{c^2}} \{ \sinh(\frac{a_0 \xi^0}{c}) + \frac{v_0}{c} \cosh(\frac{a_0 \xi^0}{c}) \} cd\xi^0 \\ + e^{\frac{a_0 \xi^1}{c^2}} \{ \cosh(\frac{a_0 \xi^0}{c}) + \frac{v_0}{c} \sinh(\frac{a_0 \xi^0}{c}) \} d\xi^1] \quad (16), \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}}, dy = d\xi^2, dz = d\xi^3 \quad (17)$$

Therefore if Eq(15), Eq(16) and Eq(17) integrate, finally the new accelerated system's coordinate transformation of the accelerated observer with the initial velocity is found.

$$ct = \gamma \frac{c^2}{a_0} e^{\frac{a_0 \xi^1}{c^2}} \{ \sinh(\frac{a_0 \xi^0}{c}) + \frac{v_0}{c} \cosh(\frac{a_0 \xi^0}{c}) \} - \gamma \frac{v_0 c}{a_0} \quad (18)$$

$$x = \gamma \frac{c^2}{a_0} e^{\frac{a_0 \xi^1}{c^2}} \{ \cosh(\frac{a_0 \xi^0}{c}) + \frac{v_0}{c} \sinh(\frac{a_0 \xi^0}{c}) \} - \gamma \frac{c^2}{a_0} \quad (19),$$

$$y = \xi^2, z = \xi^3 \quad (20), \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}}$$

Therefore, the new inverse-coordinate transformation of the new accelerated system of the accelerated observer with the initial velocity is

$$\frac{(ct + \gamma \frac{v_0 c}{a_0})}{(x + \gamma \frac{c^2}{a_0})} = \frac{\tanh(\frac{a_0 \xi^0}{c}) + \frac{v_0}{c}}{1 + \frac{v_0}{c} \cdot \tanh(\frac{a_0 \xi^0}{c})}$$

$$\frac{(ct + \gamma \frac{v_0 c}{a_0})}{(x + \gamma \frac{c^2}{a_0})} - \frac{v_0}{c}$$

$$\xi^0 = \frac{c}{a_0} \tanh^{-1} \left[\frac{(ct + \gamma \frac{v_0 c}{a_0})}{(x + \gamma \frac{c^2}{a_0})} - \frac{v_0}{c} \right] (20-1), \quad \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}}$$

$$(x + \gamma \frac{c^2}{a_0})^2 - (ct + \gamma \frac{v_0 c}{a_0})^2 = \frac{c^4}{a_0^2} e^{\frac{2a_0 \xi^1}{c^2}} \gamma^2 (1 - \frac{v_0^2}{c^2}) = \frac{c^4}{a_0^2} e^{\frac{2a_0 \xi^1}{c^2}}$$

$$\xi^1 = \frac{c^2}{2a_0} \ln \left| \frac{a_0}{c^2} \sqrt{(x + \gamma \frac{c^2}{a_0})^2 - (ct + \gamma \frac{v_0 c}{a_0})^2} \right| (20-2), \quad \xi^2 = y, \xi^3 = z (20-3), \quad \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}}$$

IV. Additional chapter-III

Therefore, the invariable time $d\tau$ of the new accelerated system theory of the accelerated observer with the initial velocity is by Eq(15),Eq(16)Eq(17)

$$d\tau^2 = dt^2 - \frac{1}{c^2} [dx^2 + dy^2 + dz^2]$$

$$= e^{\frac{2a_0 \xi^1}{c^2}} (d\xi^0)^2 - \frac{1}{c^2} [e^{\frac{2a_0 \xi^1}{c^2}} (d\xi^1)^2 + (d\xi^2)^2 + (d\xi^3)^2] (21)$$

Hence, the invariable time $d\tau$ of the new accelerated system theory of the accelerated observer that has the initial velocity v_0 is not related to the initial velocity v_0 .

About x -axis's light speed ,

$$dy = d\xi^2 = dz = d\xi^3 = 0, \quad y = \xi^2 = z = \xi^3 = 0$$

$$cdt = dx, \quad ct = x, \quad cd\xi^0 = d\xi^1, \quad c\xi^0 = \xi^1 \quad (22)$$

In this time, if use the accelerated system's coordinate transformation, Eq(18),Eq(19)

$$\begin{aligned}
ct &= \gamma \frac{c^2}{a_0} e^{\frac{a_0 \xi^1}{c^2}} \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{v_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} - \gamma \frac{v_0 c}{a_0} \\
&= \gamma \frac{c^2}{a_0} e^{\frac{a_0 \xi^0}{c}} \left\{ \frac{e^{\frac{a_0 \xi^0}{c}} - e^{-\frac{a_0 \xi^0}{c}}}{2} + \frac{v_0}{c} \frac{e^{\frac{a_0 \xi^0}{c}} + e^{-\frac{a_0 \xi^0}{c}}}{2} \right\} - \gamma \frac{v_0 c}{a_0} \\
&= \gamma \frac{c^2}{a_0} \left\{ \frac{e^{\frac{2a_0 \xi^0}{c}} - 1}{2} + \frac{v_0}{c} \left(\frac{e^{\frac{2a_0 \xi^0}{c}} + 1}{2} - 1 \right) \right\} \\
&= \gamma \frac{c^2}{a_0} \left\{ \frac{e^{\frac{2a_0 \xi^0}{c}} - 1}{2} + \frac{v_0}{c} \frac{e^{\frac{2a_0 \xi^0}{c}} - 1}{2} \right\} \\
&= x = \gamma \frac{c^2}{a_0} e^{\frac{a_0 \xi^1}{c^2}} \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{v_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} - \gamma \frac{c^2}{a_0} \\
&= \gamma \frac{c^2}{a_0} e^{\frac{a_0 \xi^0}{c}} \left\{ \frac{e^{\frac{a_0 \xi^0}{c}} + e^{-\frac{a_0 \xi^0}{c}}}{2} + \frac{v_0}{c} \frac{e^{\frac{a_0 \xi^0}{c}} - e^{-\frac{a_0 \xi^0}{c}}}{2} \right\} - \gamma \frac{c^2}{a_0} \\
&= \gamma \frac{c^2}{a_0} \left\{ \left(\frac{e^{\frac{2a_0 \xi^0}{c}} + 1}{2} - 1 \right) + \frac{v_0}{c} \frac{e^{\frac{2a_0 \xi^0}{c}} - 1}{2} \right\} \\
&= \gamma \frac{c^2}{a_0} \left\{ \frac{e^{\frac{2a_0 \xi^0}{c}} - 1}{2} + \frac{v_0}{c} \frac{e^{\frac{2a_0 \xi^0}{c}} - 1}{2} \right\} \quad (23)
\end{aligned}$$

About the curvature tensor $R^\rho{}_{\mu\nu\lambda}$,

The inertial system $S(t, x, y, z)$'s the curvature tensor $R^\delta{}_{\alpha\beta\gamma} = 0$

The new accelerated system $\xi(\xi^0, \xi^1, \xi^2, \xi^3)$'s the curvature tensor $R'^{\rho}{}_{\mu\nu\lambda}$ has to zero because it has to be in the flat Minkowski space.

$$0 = R^\delta{}_{\alpha\beta\gamma} = \frac{\partial x^\delta}{\partial \xi^\rho} \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{\partial \xi^\nu}{\partial x^\beta} \frac{\partial \xi^\lambda}{\partial x^\gamma} R'^{\rho}{}_{\mu\nu\lambda} \quad (24)$$

If compute the new accelerated system's the curvature tensor $R'^{\rho}{}_{\mu\nu\lambda}$

$$d\tau^2 = e^{\frac{2a_0 \xi^1}{c^2}} (d\xi^0)^2 - \frac{1}{c^2} [e^{\frac{2a_0 \xi^1}{c^2}} (d\xi^1)^2 + (d\xi^2)^2 + (d\xi^3)^2]$$

$$g_{00} = -e^{\frac{2a_0 \xi^1}{c^2}}, \quad g_{11} = e^{\frac{2a_0 \xi^1}{c^2}}, \quad g_{22} = g_{33} = 1$$

$$\begin{aligned}
g^{00} &= -e^{-\frac{2a_0 \xi^1}{c^2}}, \quad g^{11} = e^{-\frac{2a_0 \xi^1}{c^2}}, \quad g^{22} = g^{33} = 1 \\
\Gamma^{1}_{00} &= \frac{1}{2} g^{11} \left(-\frac{\partial g_{00}}{\partial \xi^1} \right) = \frac{a_0}{c^2}, \quad \Gamma^0_{01} = \Gamma^0_{10} = \frac{1}{2} g^{00} \left(\frac{\partial g_{00}}{\partial \xi^1} \right) = \frac{a_0}{c^2}, \quad \Gamma^1_{11} = \frac{1}{2} g^{11} \left(\frac{\partial g_{11}}{\partial \xi^1} \right) = \frac{a_0}{c^2} \\
R^{11}_{001} &= \Gamma^1_{00} \Gamma^1_{11} - \Gamma^0_{01} \Gamma^1_{00} = \frac{a_0^2}{c^4} - \frac{a_0^2}{c^4} = 0 \\
-R^{11}_{001} &= R^1_{010} = -\Gamma^1_{00} \Gamma^1_{11} + \Gamma^0_{01} \Gamma^1_{00} = -\frac{a_0^2}{c^4} + \frac{a_0^2}{c^4} = 0 \\
&\text{otherwise } R'^{\rho}_{\mu\nu\lambda} = 0
\end{aligned}$$

Therefore, new accelerated system is in the flat Minkowski space.

V. Conclusion

It found the new accelerated system theory that used the tetrad on the new method. And it expanded to be the new accelerated system theory of the accelerated observer that have the initial velocity.

Hence, we knew that the Rindler coordinate theory

$$ct = \left(\frac{c^2}{a_0} + \xi^1 \right) \sinh\left(\frac{a_0 \xi^0}{c} \right), \quad x = \left(\frac{c^2}{a_0} + \xi^1 \right) \cosh\left(\frac{a_0 \xi^0}{c} \right) - \frac{c^2}{a_0}, \quad y = \xi^2, \quad z = \xi^3 \quad (25)$$

is not the unique accelerated system theory.

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