Abstraction and the Standard Model

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Abstract:

We study the Standard Model in light of the Zero-Postulation of the Theory of Abstraction. Yukawa Coupling, chiral superfields, the SUSY model, Interacting Boson Models (IBMs), Clebsch-Gordan coefficients, Interacting Boson-Fermion Model (IBFM), etc., are some of the concepts that we study in this paper. Non-commutative geometry seems to come very handy in describing the quantum world. Bosons and fermions both seem to be governed by the rules of such geometry. The principle of conservation of boson number inside a system is seen to follow directly from the Abstraction Model. The IBMs are seen to obey the Laws of Physical Transaction that follows from Zero-Postulation. The chaotic superfields at the requisite scalingratio yields necessary equation-parameters needed to describe them at that given scaling-ratio. This is seen to be independent of the choice of scale, but at smaller scaling-ratios, we have less loss of information. At a higher scale, we seem to have less number of parameters required to describe them.

Introduction:

In the Theory of Abstraction, we start from zero-postulation and build on to reach results that depend upon certain parameters that in turn depend on the system in question itself. Our

theory seems to fit in reasonably well in our study of systems in all scales. Starting from a basis of no postulation, we build our theory. As we go on piling up possibilities, we come to a similar basis for understanding the four non-contact forces of nature known till date. The difference in ranges of these forces is explained from this basis in previous papers. Zero postulation or abstraction as the basis of theory synthesis allows us to explore even imaginary and chaotic non-favoured solutions as possibilities. With no postulation as the fundamental basis, we are thus able to pile up postulated results or favoured results, but not the other way round. We keep describing such implications of abstraction in this paper too. We deal with the abstraction of observable parameters involved in the Standard Model.

From previous work on th theory, we know that the force that will be felt due to a quantity A' of the property A in this field of acceleration **a** is,

$$F = A' c^2 \frac{(\Delta A)^{\frac{4i}{8i-1}}}{(A)^{\frac{2}{8i-1}}} \qquad ... (1),$$

where the acceleration a is,

A ;

$$a = c^2 \frac{(\Delta A)^{\frac{\pi}{8i-1}}}{(A)^{\frac{2}{8i-1}}} \qquad ... (2).$$

The uncertainty, in turn, depends upon the Lyapunov exponents (v). Moreover, there is a stretching or shrinking of a given direction according to the factor e^{vt} , according as v being positive or negative in that direction.

This will be our starting point in our study of the Standard Model in this present body of work.

<u>Higgs Potential:</u>

When the gauge coupling constants, depending upon equation (2), are g and g' for of SU(2) and U(1), respectively, the tree-level Higgs Potential in the minimal (SUSY) Standard Model is,

$$v = \frac{g^2}{8} (\overline{H}_{1 \tau_a} H_1 + \overline{H}_{2 \tau_a} H_2)^2 + \frac{g'^2}{8} (\overline{H}_1 H_1 + \overline{H}_2 H_2)^2,$$

where (here),
$$\tau_a = \frac{A^{\frac{1}{8i-1}}}{c} \qquad \dots (3).$$

From the breaking of electroweak symmetry caused by

we get the tree level masses of the Higgs scalars as,

$$\begin{split} m_{x^{0}}^{2} &= m_{1}^{2} + m_{2}^{2}, \\ m_{x^{\pm}}^{2} &= m_{x^{0}}^{2} + m_{W^{\pm}}^{2}, \\ m_{a,b}^{2} &= \\ \frac{1}{2} [m_{x^{0}}^{2} + m_{z^{0}}^{2} \pm \sqrt{\{(m_{x^{0}}^{2} + m_{z^{0}}^{2})^{2} - 4m_{x^{0}}^{2}m_{z^{0}}^{2}\cos^{2}2\theta\}], \\ \tan \theta &= \frac{v_{2}}{v_{1}}. \end{split}$$

Interacting Boson Models:

For a Hamiltonian H(q, p) and equations of motion $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$ With D degrees of freedom, x = (q, p), $q = (q_1, q_2, q_3, \dots, q_D),$ $p = (p_1, p_2, p_3, \dots, p_D).$

The value of the Hamiltonian function at the state space point x = (q, p) is constant along the trajectory x(t). Thus the energy along the trajectory x(t) is constant,

$$\frac{d}{dt}H[q(t), p(t)] = \frac{\partial H}{\partial q_i}\dot{q}_i(t) + \frac{\partial H}{\partial p_i}\dot{p}_i(t) = \frac{\partial H}{\partial q_i}\frac{\partial H}{\partial p_i} - \frac{\partial H}{\partial p_i}\frac{\partial H}{\partial q_i}$$
$$= 0$$

The trajectories therefore lie on surfaces of constant energy or level sets of the Hamiltonian [(q, p): H(q, p) = E].

Given a smooth function g(x), the standard map is,

$$x_{n+1} = x_n + y_{n+1}$$

$$y_{n+1} = y_n + g(x_n).$$

This is an area-preserving map. The corresponding $n^{
u n}$ iterate Jacobian matrix is,

$$M^{n}(x_{0}, y_{0}) = \prod_{K=n}^{1} \begin{pmatrix} 1 + g'(x_{K}) & 1 \\ g'(x_{K}) & 1 \end{pmatrix} \qquad \dots (4).$$

The complete Hamiltonian of the IBM1 is,

$$\begin{split} H &= H^{(I)} + v_R R^2 + v_q Q^2, \\ \text{where} \quad H^{(I)} &= \varepsilon_n N + v_n N^2 + (\varepsilon_d^{'} + v_{nd} N) n_d + v_d n_d^{-2} + v_t T^2 + v_j J^2, \end{split}$$

 $|Nn_d \tau n_\Delta JM\rangle$ are eigenfunctions of the operator $H^{(I)}$, with eigenvalues $E_{N\tau Jn_d}^{(I)}$.

A Floquet multiplier $\Lambda = \Lambda(x_0, t)$ associated to a trajectory is an eigenvalue of the Jacobian matrix J and it satisfies

$$det(J - \Lambda I) = det(J^{T} - \Lambda I) = det(-\omega J^{T} \omega - \Lambda I)$$

= $det(J^{-1}) det(I - \Lambda J)$
= $\Lambda^{2D} det(J - \Lambda^{-1}I)$... (5).

This is because, $J^{-1} = -\omega J^T \omega$, J being symplectic. If Λ is an eigenvalue of J so are $\frac{1}{\Lambda}$, Λ^* and $\frac{1}{\Lambda^*}$. Real eigenvalues always come paired as Λ , $\frac{1}{\Lambda}$. The complex eigenvalues come in pairs Λ , Λ^* , $|\Lambda| = 1$, or in loxodromic quartets Λ , $\frac{1}{\Lambda}$, Λ^* and $\frac{1}{\Lambda^*}$.

For a trajectory originating near $x_0 = x(0)$ with an initial infinitesimal displacement $\delta x(0)$, the flow transports the displacement $\delta x(t)$ along the trajectory $x(x_0, t) = f^t(x_0)$.

This infinitesimal displacement is transported along the trajectory $x(x_0, t)$, with time variation given by,

$$\frac{d}{dt} \delta x_i(x_0, t) = \sum_j \frac{\partial v_i}{\partial x_j}(x) \Big|_{x=x(x_0, t)} \delta x_j(x_0, t) \qquad \dots (6).$$

For two scalar bosons $arphi_a$ and $arphi_b$ the mass-matrix is,

$$\begin{split} (\varphi_a \varphi_b) \begin{bmatrix} m_{\chi^0}^2 + m_{Z^0}^2 \sin^2 2\theta & -m_{Z^0}^2 \sin 2\theta \cos 2\theta \\ &+ \frac{1}{2} \delta \sin^2 2\theta . v^2 & + \delta \sin^2 \theta \sin 2\theta . v^2 \\ -m_{Z^0}^2 \sin 2\theta \cos 2\theta & m_{Z^0}^2 \cos^2 2\theta \\ &+ \delta \sin^2 \theta \sin 2\theta . v^2 & + 2\delta \sin^4 \theta . v^2 \end{bmatrix} \begin{pmatrix} \varphi_a \\ \varphi_b \end{pmatrix}, \\ \text{where, } \delta &\simeq 3 \left(\log \frac{m^2 + m_t^2}{m_t^2} \right) \left(\frac{h_t^2}{4\pi} \right). \end{split}$$

One of the eigenvalues being always smaller than $m_z^2 \sigma cos^2 2\theta + 2\delta sin^4 \theta . v^2$ the mass of the lightest scalar boson φ_l is,

$$m_{l} \leq \sqrt{\{m_{z^{0}}^{2} \cos^{2} 2\theta + \frac{6}{(2\pi)^{2}} \left(\log \frac{m^{2} + m_{t}^{2}}{m_{t}^{2}}\right) \frac{m_{t}^{4}}{v^{2}}\}}$$

The equations of motion for a time-independent *D*-degrees of freedom Hamiltonian can be written as,

$$\dot{x}_i = \omega_{ij} H_j(x), \omega = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, H_j(x) = \frac{\partial}{\partial x_j} H(x);$$

where $x = (q, p) \in B$ is a phase space point. $H_K = \partial_K H$ is the column vector of partial derivatives of H, I is the $[D \times D]$ unit matrix and ω the $[2D \times 2D]$ symplectic form.

$$\omega^{\mathrm{T}} = -\omega, \omega^{2} = -1$$

The evolution of J^t is determined by the stability matrix A ,

$$\frac{d}{dt}J^{t}(x) = A(x)J^{t}(x),$$

$$A_{ij}(x) = \omega_{ik}H_{kj}(x) \qquad \dots (7),$$

where the matrix of second derivations $H_{kn} = \partial_k \partial_n H$ is the Hessian matrix. For symmetry of H_{kn} , $A^T \omega + \omega A = 0$.

The eigenenergy of the *I*-th state is $E_{NJ}^{\left(I
ight)}$ and satisfies the relation,

$$H|INMJ \rangle = E_{NJ}^{(I)}|INMJ \rangle$$

The Yukawa coupling h_t of the top quark is defined in the superpotential,

$$G=h_tarphi_{tR}arphi_{q\,L^3}arphi_{H_2}$$
 ,

where φ_{tR} and φ_{qL^3} are chiral superfields of the right-handed top quark and the lefthanded quark doublet in the third generation, respectively.

In this respect, the Yukawa coupling of the top quark is,

$$h_t = \sqrt{2}m_t / v_2$$

The Hamiltonian $H^{(1)}$ of an active boson of a single state $|b_{IM}\rangle$ is constituted by its kinetic energy $K^{(1)}$ and its potential energy $P^{(1)}$.

The IBMs are seen to obey the Laws of Physical Transaction that follows from Zero-Postulation. The chaotic superfields at the requisite scaling-ratio yields necessary equation-parameters needed to describe them at that given scaling-ratio. This is seen to be independent of the choice of scale, but at smaller scaling-ratios, we have less loss of information. At a higher scale, we seem to have less number of parameters required to describe them.

The 12 creation operators for bosons are:

$$b^+_{\pi,jm} = s^+_{\pi}, d^+_{\pi,m} \ (m = -2, -1, ..., 2)$$
 and $b^+_{v,jm} = s^+_{v}, d^+_{v,m} \ (m = -2, -1, ..., 2),$

where π is a proton and ${\mathcal V}$ is a neutron.

Thus, for these pairs,

$$H = H_{\pi} + H_{\nu} + V_{\pi\nu}$$
 , which is the usual result.

A normalized, symmetric many-boson state (with occupational numbers: N_a , N_b , ..., N_x), for creation operator b_n^+ (with index n), is represented by,

 $(N_a! N_b! \dots N_x!)^{-\frac{1}{2}} (b_a^+)^{N_a} (b_b^+)^{N_b} \dots (b_x^+)^{N_x}| \ge |N_a N_b \dots N_x >$, where the index represents the angular momentum of the single state and its projection.

Conservation of Boson Number:

Let us now consider the flow of an energy quantum, with frequency (ν) in a given direction in vacuum. The distance (D) of transport in that given direction can be considered to

be(CT); where (C) is the velocity of energy-quantum in vacuum and (T) is the time of transport.

For an empty environment in which the flow takes place, we may assume that the concerned difference in concentrations between the initial and the final points,

$$\lambda = h\nu$$

where h is Plank's constant.

Placing D = cT and $\lambda = h\nu$ in the transaction equation, we get, $F \propto \frac{\omega h\nu S}{cR}$

Again, as the energy-quantum moves from the initial to the final point completely, the concerned flow,

$$F = h\nu$$

Placing this value of F in the previous equation, we get,

$$\omega = c\left(\frac{R}{S}\right).$$

Assuming, the resistance against the concerned flow and the support towards it for the energy, i.e., considering R = S, we have,

$$\omega = c \qquad \qquad \dots (8).$$

The value of the constant ω may therefore be replaced by the speed of light in vacuum, C, in the equations concerning transport of a given physical entity.

Such a transfer of any physical entity as described by the transaction equation will continue until and unless the difference in concentrations concerned, i.e., λ becomes zero.

While

$$\frac{F}{T} = 2c\left(\frac{\lambda}{D}\right) \qquad \dots (9).$$

Equation (9) describes fundamentally the effect (i.e., the flow F in timeT) of two materialpoints having same factorial conditions regarding one or a number of entities. Considering a collection of such points and applying a statistical approach, the logistic equation (due to May,

1967) for
$$\left(\frac{\lambda}{T}\right)$$
 can be written as,
 $2c\left(\frac{\lambda}{D}\right)_{t+1} = 2Kc\left(\frac{\lambda}{D}\right)_t \left[1 - 2c\left(\frac{\lambda}{D}\right)_t\right]$

i.e.,

$$\left(\frac{\lambda}{D}\right)_{t+1} = K\left(\frac{\lambda}{D}\right)_t \left[1 - 2c\left(\frac{\lambda}{D}\right)_t\right]$$

where K is a constant.

Also, the quadratic map (due to Lorentz, 1987) can be written as,

$$2c\left(\frac{\lambda}{D}\right)_{t+1} = K - \left(2c\frac{\lambda}{D}\right)_{t}^{2}$$

i.e.,

$$2c\left(\frac{\lambda}{D}\right)_{t+1} = K - 4c^2\left(\frac{\lambda}{D}\right)_t^2 \qquad \dots (10)$$

Let us consider a system of n_d bosons. The parameter d represents its structural-orientation, which depends upon our experimental conditions (like the scaling-ratio used, differences in concentration, etc.). For our present scale of interest, the frequency of a boson ϑ_d represents the given difference in concentration λ_d .

Thus the total energy carried by a given boson is,

$$E = h\lambda_d$$

Also, let us have ${n_d}_1$ and ${n_d}_2$ bosons interacting, such that total energy in the system is,

$$E = h \big(\lambda_{d_1} + \lambda_{d_2} \big).$$

The principle of conservation of energy for that given system requires that the resulting system of bosons (λ_{d_3}) have an equal amount of total energy, such that,

$$\lambda_{d_3}=\frac{\lambda_{d_1}+\lambda_{d_2}}{2},$$

which, in turn, means,

$$n_{d_3} = n_{d_1} + n_{d_2}$$

All trajectories described by the quadratic map become asymptotic to $-\infty$ for K < -0.25 and K > 2 .

In the SO(6) or γ -instable limit, a linear combination of Casimir operators on the chain $U(6) \supset SO(6) \supset SO(5) \supset SO(3)$ is,

$$H = \varepsilon_n N + v_n N^2 + v_r R^2 + v_t T^2 + v_j J^2 \, .$$

For the functions $arphi_i=\lambda_i$ and $arphi_j=\lambda_j$ belonging to the same system,

$$\int \lambda_{j}^{*}(\underline{R})\lambda_{i}(\underline{R})d\underline{R} \equiv \iint \lambda_{j}^{*}(\underline{R})\delta(\underline{R}-\underline{R}')\lambda_{i}(\underline{R}')d\underline{R}d\underline{R}' = \sum_{\beta}\int \lambda_{j}^{*}(\underline{R})\lambda_{\beta}(\underline{R})d\underline{R} \cdot \int \lambda_{\beta}^{*}(\underline{R}')\lambda_{i}(\underline{R}')d\underline{R}' \qquad \dots (11).$$

Using Lyapunov exponents for a given transport, and replacing $2c\left(\frac{\lambda}{D}\right)$ by a quantity τ' , we have,

$$\frac{d}{d\tau}f^{n}(\tau) = \frac{\delta n}{\delta o}$$

$$\frac{\delta n}{\delta o} = \prod_{i=1}^{n} f'(\tau_{i}) \qquad \dots (12)$$

$$b = \frac{1}{n}\log_{e}\left(\frac{\delta n}{\delta o}\right)$$
i.e.,

$$b = \frac{1}{n} \sum_{i=1}^{n-1} \log_e |f'(\tau_i)| \qquad \dots (13),$$

where \boldsymbol{b} is a constant (the local slope of all possible routes), and

$$\Psi = \lim_{n \to \infty} \frac{1}{n} \sum_{i=0}^{n-1} \log_e |f'(\tau_i)| \qquad \dots (14),$$

where Ψ is a constant for the system.

Non-commutative Operators:

Let the spectral triple $(\mathcal{A}, \mathcal{H}, \mathcal{D})$ denote a non-commutative geometry, where in Hilbert space \mathcal{H} we have an involutive geometry \mathcal{A} and a self-adjoint unbounded operator \mathcal{D} . \mathcal{D}^{-1} , the inverse of \mathcal{D} denotes infinitesimal length. $\mathcal{A} = C^{\infty}(M)$ is the algebra of smooth functions on the Riemann Manifold M. The Dirac operator of Levi-Civita spin is \mathcal{D} . The Hilbert space of L^2 -spinors is,

$$\mathcal{H}=L^2(M,S).$$

Considering x dimensions of the manifold M, a Z/2 gradient for difference in concentrations of one or a group of physical quantity or quantities λ_x in the Hilbert space \mathcal{H} satisfies:

 $\lambda_x = \lambda_x^*, \lambda_x^2 = 1, \lambda_x a = a\lambda_x \forall a \in \mathcal{A}, \lambda_x \mathcal{D} = -\mathcal{D}\lambda_x \dots (15)$. The spectrum of the operator \mathcal{D} replaces the status of points $x \in M$ in commutative geometry.

x modulo 8 determines the values of the parameters $\mathcal{E}, \mathcal{E}', \mathcal{E}''$ such that, $J^2 = \mathcal{E}, J\mathcal{D} = \mathcal{E}'\mathcal{D}J, J\lambda_x = \mathcal{E}''\lambda_x J, \ \mathcal{E}, \mathcal{E}', \mathcal{E}'' \in \{-1, 1\}$,

where J is an anti-linear isometry in the Hilbert space ${\mathcal H}$ and it denotes the real structure on ${\mathcal H}$.

For any given flow F in ${\mathcal H}$, we have,

$$\mathcal{A} = C^{\infty}(M) \otimes \mathcal{A}_{F},$$

$$\mathcal{A}_{F} = C \oplus \mathcal{H} \oplus M_{3}(C),$$

$$\mathcal{H} = \{ \begin{pmatrix} \alpha & \beta \\ -\overline{\beta} & \overline{\alpha} \end{pmatrix}; \alpha, \beta \in C \},$$

$$\mathcal{H} = L^2(M, S) \otimes \mathcal{H}_F$$
, $\mathcal{D} = \tilde{\mathfrak{d}}_M \otimes 1 + \lambda_{\chi 5} \otimes \mathcal{D}_F$.
The Dirac operator \mathcal{D}_{χ} , considering inner fluctuations, yields a 36×36 matrix for the 36 quarks. σ^{α} and ψ^i represent Pauli matrices and Gell-Mann matrices, respectively. This matrix, with Clifford algebra tensors, is,

 \mathbf{T}

$$D_{x} = \begin{bmatrix} \lambda_{x}^{\mu} \otimes \left(\mathcal{D}_{\mu} - \frac{i}{2} g_{02} A_{\mu}^{\alpha} \sigma^{\alpha} - \frac{i}{2} g_{01} B_{\mu} \otimes 1_{2} \right) \otimes 1_{3} & \lambda_{x5} \otimes K_{0}^{e} \otimes \mathcal{H}_{F} \\ \lambda_{x5} \otimes K_{0}^{e*} \otimes \mathcal{H}_{F}^{*} & \lambda_{x5}^{\mu} \otimes \left(\mathcal{D}_{\mu} + i g_{01} B_{\mu} \right) \otimes 1_{3} \end{bmatrix}$$

Conclusion:

We study the Standard Model taking into consideration the Zero-Postulation of the Theory of Abstraction. The relative difference in concentrations of any given physical entity creates a tensor-gradient that causes the varied ways of flow. The necessary complete set of parameters and the required scaling-ratio for a given set of observations describes the observations themselves. We can arrive at IBMs and IBFMs in this way. We may as well describe the basis of the Standard Model itself using the Theory of Abstraction.

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