

# **The accelerated frame in the curved time-space in the general relativity theory**

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## **ABSTRACT**

In the general relativity theory, defines the accelerated frame that moves in  $\hat{t}$ -axis in the curved time-space. And calculates the curvature tensor of the accelerated frame in the curved time-space.

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**The curved time-space,**

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## I.Introduction

This theory's object is that defines the accelerated frame that moves in  $\hat{r}$ -axis in the curved time-space.

The Schwarzschild solution is

$$d\tau^2 = \left(1 - \frac{2GM}{rc^2}\right) dt^2 - \frac{1}{c^2} \left[ \frac{dr^2}{1 - \frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] \quad (1)$$

In this time, a moving matter's acceleration is  $a$  in the Schwarzschild time-space.

$$a = \frac{d}{dt} \left( \frac{u}{\sqrt{1 - \frac{2GM}{rc^2} - \frac{u^2}{c^2}}} \right), u = \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}} \frac{dr}{dt} \quad (2)$$

If  $a_0 = a / \sqrt{1 - \frac{2GM}{rc^2}}$  is,

$$a_0 = \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}} \frac{d}{dt} \left( \frac{u}{\sqrt{1 - \frac{2GM}{rc^2} - \frac{u^2}{c^2}}} \right) = \frac{d}{d\hat{t}} \left( \frac{V}{\sqrt{1 - \frac{V^2}{c^2}}} \right),$$

$$V = \frac{d\hat{r}}{d\hat{t}} = \frac{dr}{dt} \frac{1}{\left(1 - \frac{2GM}{rc^2}\right)}, d\hat{t} = dt \sqrt{1 - \frac{2GM}{rc^2}}, d\hat{r} = \frac{dr}{\sqrt{1 - \frac{2GM}{rc^2}}}$$

$$a_0 \hat{t} = \frac{V}{\sqrt{1 - \frac{V^2}{c^2}}}, V = \frac{a_0 \hat{t}}{\sqrt{1 + \frac{a_0^2 \hat{t}^2}{c^2}}}, V \text{ is the } \hat{r}\text{-axis's velocity} \quad (3)$$

If  $\frac{d\theta}{dt} = \frac{d\phi}{dt} = 0$ , the solution is

$$\begin{aligned} d\tau^2 &= \left(1 - \frac{2GM}{rc^2}\right) dt^2 - \frac{1}{c^2} \frac{dr^2}{1 - \frac{2GM}{rc^2}} = d\hat{t}^2 - \frac{1}{c^2} d\hat{r}^2 = d\hat{t}^2 \left(1 - \frac{V^2}{c^2}\right) \\ &= \frac{d\hat{t}^2}{\sqrt{1 + \frac{a_0^2 \hat{t}^2}{c^2}}} \quad (4) \end{aligned}$$

In this time,

$$\tau = \int d\tau = \int \frac{d\hat{t}}{\sqrt{1 + \frac{a_0^2 \hat{t}^2}{c^2}}} = \frac{c}{a_0} \sinh^{-1}\left(\frac{a_0}{c} \hat{t}\right),$$

$$\begin{aligned} \hat{t} &= \frac{c}{a_0} \sinh\left(\frac{a_0 \tau}{c}\right), \quad \hat{r} = \int V d\hat{t} = \int \frac{a_0 \hat{t} d\hat{t}}{\sqrt{1 + \frac{a_0^2 \hat{t}^2}{c^2}}} = \frac{c^2}{a_0} \sqrt{1 + \frac{a_0^2 \hat{t}^2}{c^2}} \\ &= \frac{c^2}{a_0} \cosh\left(\frac{a_0 \tau}{c}\right) \\ \frac{d\hat{t}}{d\tau} &= \cosh\left(\frac{a_0}{c} \tau\right), \quad \frac{1}{c} \frac{d\hat{r}}{d\tau} = \sinh\left(\frac{a_0}{c} \tau\right) \end{aligned} \quad (5)$$

## II. The tetrad in the curved time-space

The tetrad  $e^{\hat{\alpha}}_{\hat{\mu}}$  is the unit vector defined by the following formula.

$$\eta_{\hat{\alpha}\hat{\beta}} e^{\hat{\alpha}}_{\hat{\mu}} e^{\hat{\beta}}_{\hat{\nu}} = g_{\hat{\mu}\hat{\nu}} \quad (6)$$

In this time, if a matter moves in  $\hat{r}$ -axis in the curved time-space,

$$\eta_{\hat{\alpha}\hat{\beta}} e^{\hat{\alpha}}_{\hat{\mu}}(\tau) e^{\hat{\beta}}_{\hat{\nu}}(\tau) = g_{\hat{\mu}\hat{\nu}} = \eta_{\hat{\mu}\hat{\nu}}, \quad g_{\hat{\mu}\hat{\nu}} = \eta_{\hat{\mu}\hat{\nu}} \quad (7)$$

Hence, Eq(6),Eq(7) is

$$\eta_{\hat{\alpha}\hat{\beta}} e^{\hat{\alpha}}_{\hat{0}}(\tau) e^{\hat{\beta}}_{\hat{0}}(\tau) = \eta_{\hat{0}\hat{0}} = -1 \quad (8)$$

$$\begin{aligned} d\tau^2 &= -\frac{1}{c^2} \eta_{\hat{\alpha}\hat{\beta}} d\hat{\chi}^{\hat{\alpha}} d\hat{\chi}^{\hat{\beta}} \\ \rightarrow -1 &= \eta_{\hat{\alpha}\hat{\beta}} \left(\frac{1}{c} \frac{d\hat{\chi}^{\hat{\alpha}}}{d\tau}\right) \left(\frac{1}{c} \frac{d\hat{\chi}^{\hat{\beta}}}{d\tau}\right) = \eta_{\hat{\alpha}\hat{\beta}} e^{\hat{\alpha}}_{\hat{0}}(\tau) e^{\hat{\beta}}_{\hat{0}}(\tau) \end{aligned} \quad (9)$$

According to Eq(5),Eq(9)

$$e^{\hat{\alpha}}_{\hat{0}}(\tau) = \frac{1}{c} \frac{d\hat{\chi}^{\hat{\alpha}}}{d\tau} = \left(\cosh\left(\frac{a_0 \tau}{c}\right), \sinh\left(\frac{a_0 \tau}{c}\right), 0, 0\right) \quad (10)$$

About  $\hat{\chi}^{\hat{2}}$ -axis's and  $\hat{\chi}^{\hat{3}}$ -axis's orientation

$$\eta_{\hat{2}\hat{2}} e^{\hat{2}}_{\hat{2}}(\tau) e^{\hat{2}}_{\hat{2}}(\tau) = \eta_{\hat{2}\hat{2}} = 1, \quad e^{\hat{2}}_{\hat{2}}(\tau) = (0, 0, 1, 0)$$

$$\eta_{\hat{3}\hat{3}} e^{\hat{3}}_{\hat{3}}(\tau) e^{\hat{3}}_{\hat{3}}(\tau) = \eta_{\hat{3}\hat{3}} = 1, \quad e^{\hat{3}}_{\hat{3}}(\tau) = (0, 0, 0, 1) \quad (11)$$

And the other vector  $e^{\hat{\alpha}}_{\hat{1}}(\tau)$  has to satisfy the tetrad condition, Eq (6),Eq(7)

$$e^{\hat{\alpha}}_{\hat{i}}(\tau) = (\sinh(\frac{a_0 \tau}{c}), \cosh(\frac{a_0 \tau}{c}), 0, 0) \quad (12)$$

In this time,

$$\bar{e}_i^\rho = (1/\sqrt{1-\frac{2GM}{rc^2}}, 0, 0, 0), \bar{e}_r^\rho = (0, \sqrt{1-\frac{2GM}{rc^2}}, 0, 0)$$

$$\bar{e}_\theta^\rho = (0, 0, 1/r, 0) \quad , \quad \bar{e}_\phi^\rho = (0, 0, 0, 1/r \sin \theta)$$

$$g_{\rho\sigma} \bar{e}_{\hat{\alpha}}^\rho \bar{e}_{\hat{\beta}}^\sigma = \eta_{\hat{\alpha}\hat{\beta}} \quad (13)$$

$$\frac{a_0}{c} \hat{t} = \sinh(\frac{a_0}{c} \tau) = \frac{v/c}{\sqrt{1-v^2/c^2}}, \sqrt{1+\frac{a_0^2 \hat{t}^2}{c^2}} = \cosh(\frac{a_0}{c} \tau) = \frac{1}{\sqrt{1-v^2/c^2}} \quad (14)$$

Therefore, the Lorentz transformation  $B^{\hat{\alpha}}_{\hat{\mu}}(v)$  is

$$B^{\hat{\alpha}}_{\hat{\mu}}(v) = \begin{pmatrix} \frac{1}{\sqrt{1-v^2/c^2}} & \frac{v/c}{\sqrt{1-v^2/c^2}} & 0 & 0 \\ \frac{v/c}{\sqrt{1-v^2/c^2}} & \frac{1}{\sqrt{1-v^2/c^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= e^{\hat{\alpha}}_{\hat{\mu}}(\tau) = \begin{pmatrix} \cosh(\frac{a_0}{c} \tau) & \sinh(\frac{a_0}{c} \tau) & 0 & 0 \\ \sinh(\frac{a_0}{c} \tau) & \cosh(\frac{a_0}{c} \tau) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (15)$$

$$\bar{e}^1_{\hat{\mu}}{}^\rho = B^{\hat{\alpha}}_{\hat{\mu}}(v) \bar{e}_{\hat{\alpha}}^\rho = e^{\hat{\alpha}}_{\hat{\mu}}(\tau) \bar{e}_{\hat{\alpha}}^\rho \quad (16)$$

Hence,

$$g_{\rho\sigma} \bar{e}_{\hat{\alpha}}^\rho \bar{e}_{\hat{\beta}}^\sigma = \eta_{\hat{\alpha}\hat{\beta}}$$

$$g_{\rho\sigma} B^{\hat{\alpha}}_{\hat{\mu}}(v) \bar{e}_{\hat{\alpha}}^\rho B^{\hat{\beta}}_{\hat{\nu}}(v) \bar{e}_{\hat{\beta}}^\sigma = g_{\rho\sigma} \bar{e}^1_{\hat{\mu}}{}^\rho \bar{e}^1_{\hat{\nu}}{}^\sigma = \eta_{\hat{\alpha}\hat{\beta}} e^{\hat{\alpha}}_{\hat{\mu}}(\tau) e^{\hat{\beta}}_{\hat{\nu}}(\tau) = \eta_{\hat{\mu}\hat{\nu}} \quad (17)$$

### III. The accelerated frame in the curved time-space

About the accelerated frame  $\hat{\xi}$  in the curved time-space,

$$\begin{aligned}
d\tau^2 &= -\frac{1}{c^2} \eta_{\hat{\alpha}\hat{\beta}} d\hat{\chi}^\alpha d\hat{\chi}^\beta = -\frac{1}{c^2} \eta_{\hat{\alpha}\hat{\beta}} \frac{\partial \hat{\chi}^\alpha}{\partial \hat{\xi}^\mu} \frac{\partial \hat{\chi}^\beta}{\partial \hat{\xi}^\nu} d\hat{\xi}^\mu d\hat{\xi}^\nu \\
&= -\frac{1}{c^2} \eta_{\hat{\alpha}\hat{\beta}} e^{\hat{\alpha}}_{\hat{\mu}} e^{\hat{\beta}}_{\hat{\nu}} d\hat{\xi}^\mu d\hat{\xi}^\nu \\
&= -\frac{1}{c^2} g_{\hat{\mu}\hat{\nu}} d\hat{\xi}^\mu d\hat{\xi}^\nu \tag{18}
\end{aligned}$$

$$e^{\hat{\alpha}}_{\hat{\mu}} = \frac{\partial \hat{\chi}^\alpha}{\partial \hat{\xi}^\mu}, \quad \frac{\partial e^{\hat{\alpha}}_{\hat{0}}}{\partial \hat{\xi}^1} = \frac{\partial^2 \hat{\chi}^\alpha}{c \partial \hat{\xi}^0 \partial \hat{\xi}^1} = \frac{\partial e^{\hat{\alpha}}_{\hat{1}}}{c \partial \hat{\xi}^0} \tag{19}$$

In this time, in Eq(10),Eq(11),Eq(12), if uses  $\hat{\xi}^0$  instead of  $\tau$ ,

$$e^{\hat{\alpha}}_{\hat{0}}(\hat{\xi}^0) = \frac{1}{c} \frac{\partial \hat{\chi}^\alpha}{\partial \hat{\xi}^0} = \left( \left(1 + \frac{a_0 \hat{\xi}^1}{c^2}\right) \cosh\left(\frac{a_0 \hat{\xi}^0}{c}\right), \left(1 + \frac{a_0 \hat{\xi}^1}{c^2}\right) \sinh\left(\frac{a_0 \hat{\xi}^0}{c}\right), 0, 0 \right) \tag{20}$$

$$e^{\hat{\alpha}}_{\hat{1}}(\hat{\xi}^0) = \frac{\partial \hat{\chi}^\alpha}{\partial \hat{\xi}^1} = \left( \sinh\left(\frac{a_0 \hat{\xi}^0}{c}\right), \cosh\left(\frac{a_0 \hat{\xi}^0}{c}\right), 0, 0 \right) \tag{21}$$

$$e^{\hat{\alpha}}_{\hat{2}}(\hat{\xi}^0) = \frac{\partial \hat{\chi}^\alpha}{\partial \hat{\xi}^2} = (0, 0, 1, 0), \quad e^{\hat{\alpha}}_{\hat{3}}(\hat{\xi}^0) = \frac{\partial \hat{\chi}^\alpha}{\partial \hat{\xi}^3} = (0, 0, 1, 0) \tag{22}$$

$$d\hat{\chi}^\alpha = \frac{\partial \hat{\chi}^\alpha}{\partial \hat{\xi}^\mu} d\hat{\xi}^\mu = e^{\hat{\alpha}}_{\hat{0}}(\hat{\xi}^0) c d\hat{\xi}^0 + e^{\hat{\alpha}}_{\hat{1}}(\hat{\xi}^0) d\hat{\xi}^1 + e^{\hat{\alpha}}_{\hat{2}}(\hat{\xi}^0) d\hat{\xi}^2 + e^{\hat{\alpha}}_{\hat{3}}(\hat{\xi}^0) d\hat{\xi}^3 \tag{23}$$

Hence,

$$cd\hat{t} = cdt \sqrt{1 - \frac{2GM}{rc^2}} = \left(1 + \frac{a_0 \hat{\xi}^1}{c^2}\right) \cosh\left(\frac{a_0 \hat{\xi}^0}{c}\right) c d\hat{\xi}^0 + \sinh\left(\frac{a_0 \hat{\xi}^0}{c}\right) d\hat{\xi}^1$$

$$d\hat{r} = \frac{dr}{\sqrt{1 - \frac{2GM}{rc^2}}} = \left(1 + \frac{a_0 \hat{\xi}^1}{c^2}\right) \sinh\left(\frac{a_0 \hat{\xi}^0}{c}\right) c d\hat{\xi}^0 + \cosh\left(\frac{a_0 \hat{\xi}^0}{c}\right) d\hat{\xi}^1$$

$$d\hat{\chi}^2 = d\hat{\xi}^2, \quad d\hat{\chi}^3 = d\hat{\xi}^3 \tag{24}$$

$$d\tau^2 = \left(1 - \frac{2GM}{rc^2}\right) dt^2 - \frac{1}{c^2} \left[ \frac{dr^2}{1 - \frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]$$

$$\begin{aligned}
&= d\hat{t}^2 - \frac{1}{c^2} [d\hat{r}^2 + (d\hat{x}^2)^2 + (d\hat{x}^3)^2] \\
&= (1 + \frac{a_0 \hat{\xi}^1}{c^2})^2 (d\hat{\xi}^0)^2 - \frac{1}{c^2} [(d\hat{\xi}^1)^2 + (d\hat{\xi}^2)^2 + (d\hat{\xi}^3)^2]
\end{aligned} \tag{25}$$

The coordinate transformation is

$$\begin{aligned}
c\hat{t} &= (\frac{c^2}{a_0} + \hat{\xi}^1) \sinh(\frac{a_0 \hat{\xi}^0}{c}), \hat{r} = (\frac{c^2}{a_0} + \hat{\xi}^1) \cosh(\frac{a_0 \hat{\xi}^0}{c}) - \frac{c^2}{a_0} \\
\hat{x}^2 &= \hat{\xi}^2, \hat{x}^3 = \hat{\xi}^3
\end{aligned} \tag{26}$$

The inverse-transformation is

$$\begin{aligned}
\hat{\xi}^0 &= \frac{c}{a_0} \tanh^{-1}(\frac{c\hat{t}}{\hat{r} + \frac{c^2}{a_0}}), \hat{\xi}^1 = \sqrt{(\hat{r} + \frac{c^2}{a_0})^2 - c^2 \hat{t}^2} - \frac{c^2}{a_0} \\
\hat{\xi}^2 &= \hat{x}^2, \hat{\xi}^3 = \hat{x}^3
\end{aligned} \tag{27}$$

If calculates the curvature tensor  $R_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\lambda}}(\hat{\xi})$ ,

$$\begin{aligned}
R_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\lambda}}(\hat{\xi}) &= \frac{\partial \hat{x}^\alpha}{\partial \hat{\xi}^\mu} \frac{\partial \hat{x}^\beta}{\partial \hat{\xi}^\nu} \frac{\partial \hat{x}^\gamma}{\partial \hat{\xi}^\rho} \frac{\partial \hat{x}^\delta}{\partial \hat{\xi}^\lambda} R_{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\delta}}(\hat{X}) \\
&= e^{\hat{\alpha}}_{\hat{\mu}}(\hat{\xi}^0) e^{\hat{\beta}}_{\hat{\nu}}(\hat{\xi}^0) e^{\hat{\gamma}}_{\hat{\rho}}(\hat{\xi}^0) e^{\hat{\delta}}_{\hat{\lambda}}(\hat{\xi}^0) R_{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\delta}}(\hat{X})
\end{aligned} \tag{28}$$

$$R_{\hat{t}\hat{r}\hat{t}\hat{r}} = -R_{\hat{t}\hat{r}\hat{r}\hat{t}} = R_{\hat{r}\hat{t}\hat{r}\hat{t}} = -R_{\hat{r}\hat{t}\hat{t}\hat{r}} = \frac{2GM}{r^3 c^2},$$

$$R_{\hat{t}\hat{\theta}\hat{t}\hat{\theta}} = -R_{\hat{t}\hat{\theta}\hat{\theta}\hat{t}} = R_{\hat{\theta}\hat{t}\hat{t}\hat{\theta}} = -R_{\hat{\theta}\hat{t}\hat{\theta}\hat{t}} = -\frac{GM}{r^3 c^2} = R_{\hat{t}\hat{\phi}\hat{t}\hat{\phi}} = -R_{\hat{t}\hat{\phi}\hat{\phi}\hat{t}} = R_{\hat{\phi}\hat{t}\hat{t}\hat{\phi}} = -R_{\hat{\phi}\hat{t}\hat{\phi}\hat{t}}$$

$$R_{\hat{\theta}\hat{\phi}\hat{\theta}\hat{\phi}} = -R_{\hat{\theta}\hat{\phi}\hat{\phi}\hat{\theta}} = R_{\hat{\phi}\hat{\theta}\hat{\theta}\hat{\phi}} = -R_{\hat{\phi}\hat{\theta}\hat{\theta}\hat{\phi}} = -\frac{2GM}{r^3 c^2}$$

$$R_{\hat{r}\hat{\theta}\hat{r}\hat{\theta}} = -R_{\hat{r}\hat{\theta}\hat{\theta}\hat{r}} = R_{\hat{\theta}\hat{r}\hat{r}\hat{\theta}} = -R_{\hat{\theta}\hat{r}\hat{r}\hat{\theta}} = \frac{GM}{r^3 c^2} = R_{\hat{r}\hat{\phi}\hat{r}\hat{\phi}} = -R_{\hat{r}\hat{\phi}\hat{\phi}\hat{r}} = R_{\hat{\phi}\hat{r}\hat{r}\hat{\phi}} = -R_{\hat{\phi}\hat{r}\hat{r}\hat{\phi}}$$

(29)

Therefore,

$$e^{\hat{\alpha}}_{\hat{\delta}}(\hat{\xi}^0) = ((1 + \frac{a_0 \hat{\xi}^1}{c^2}) \cosh(\frac{a_0 \hat{\xi}^0}{c}), (1 + \frac{a_0 \hat{\xi}^1}{c^2}) \sinh(\frac{a_0 \hat{\xi}^0}{c}), 0, 0)$$

$$e^{\hat{\alpha}}_1(\hat{\xi}^0) = \left( \sinh\left(\frac{a_0 \hat{\xi}^0}{c}\right), \cosh\left(\frac{a_0 \hat{\xi}^0}{c}\right), 0, 0 \right)$$

$$e^{\hat{\alpha}}_2(\hat{\xi}^0) = (0, 0, 1, 0), e^{\hat{\alpha}}_3(\hat{\xi}^0) = (0, 0, 1, 0) \quad (30)$$

$$R_{\hat{0}\hat{1}\hat{0}\hat{1}}(\hat{\xi}) = \frac{2GM}{r^3 c^2} \left(1 + \frac{a_0 \hat{\xi}^1}{c^2}\right)^2, R_{\hat{0}\hat{2}\hat{0}\hat{2}}(\hat{\xi}) = R_{\hat{0}\hat{3}\hat{0}\hat{3}}(\hat{\xi}) = -\frac{GM}{r^3 c^2} \left(1 + \frac{a_0 \hat{\xi}^1}{c^2}\right)^2$$

$$R_{\hat{2}\hat{3}\hat{2}\hat{3}}(\hat{\xi}) = -\frac{2GM}{r^3 c^2}, R_{\hat{1}\hat{2}\hat{1}\hat{2}}(\hat{\xi}) = R_{\hat{1}\hat{3}\hat{1}\hat{3}}(\hat{\xi}) = \frac{GM}{r^3 c^2} \quad (31)$$

Specially, if  $t = 0$ ,

$$u = \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}} \frac{dr}{dt} = 0 \rightarrow V = \frac{d\hat{r}}{d\hat{t}} = \frac{a_0 \hat{t}}{\sqrt{1 + \frac{a_0^2 \hat{t}^2}{c^2}}} = \frac{dr}{dt} \frac{1}{\left(1 - \frac{2GM}{rc^2}\right)} = 0 \quad (32)$$

Therefore, if  $t = \hat{t} = \hat{\xi}^0 = 0$ , the theory treats the real situation.

$$\hat{\xi}^1 = \sqrt{\left(\hat{r} + \frac{c^2}{a_0}\right)^2 - c^2 \hat{t}^2} - \frac{c^2}{a_0} = \hat{r}$$

$$d\hat{r} = \frac{dr}{\sqrt{1 - \frac{2GM}{rc^2}}} \rightarrow \hat{r} = \sqrt{r} \sqrt{r - \frac{2GM}{c^2}} + \frac{2GM}{c^2} \ln \left| \sqrt{r} + \sqrt{r - \frac{2GM}{c^2}} \right|$$

$$- \sqrt{r_0} \sqrt{r_0 - \frac{2GM}{c^2}} - \frac{2GM}{c^2} \ln \left| \sqrt{r_0} + \sqrt{r_0 - \frac{2GM}{c^2}} \right|$$

$$a_0 = \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}} \frac{d}{dt} \left( \frac{u}{\sqrt{1 - \frac{2GM}{rc^2} - \frac{u^2}{c^2}}} \right) = \frac{a}{\sqrt{1 - \frac{2GM}{rc^2}}}$$

$$a = \frac{d}{dt} \left( \frac{u}{\sqrt{1 - \frac{2GM}{rc^2} - \frac{u^2}{c^2}}} \right) \quad (33)$$

In this time, in the curved time-space, the curvature tensor  $R_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\lambda}}(\hat{\xi})$  of the stationary accelerated frame is

$$R_{\hat{0}\hat{1}\hat{0}\hat{1}}(\hat{\xi}) = \frac{2GM}{r^3 c^2} \left(1 + \frac{a_0 \hat{\xi}^1}{c^2}\right)^2 = \frac{2GM}{r^3 c^2} \left(1 + \frac{a_0 \hat{r}}{c^2}\right)^2$$

$$\begin{aligned}
&= \frac{2GM}{r^3 c^2} \left[ 1 + \frac{a_0}{c^2} \left\{ \sqrt{r} \sqrt{r - \frac{2GM}{c^2}} + \frac{2GM}{c^2} \ln \left| \sqrt{r} + \sqrt{r - \frac{2GM}{c^2}} \right| \right. \right. \\
&\quad \left. \left. - \sqrt{r_0} \sqrt{r_0 - \frac{2GM}{c^2}} - \frac{2GM}{c^2} \ln \left| \sqrt{r_0} + \sqrt{r_0 - \frac{2GM}{c^2}} \right| \right\} \right]^2 \\
&= \frac{2GM}{r^3 c^2} \left[ 1 + \frac{1}{c^2} \frac{a}{\sqrt{1 - \frac{2GM}{rc^2}}} \left\{ \sqrt{r} \sqrt{r - \frac{2GM}{c^2}} + \frac{2GM}{c^2} \ln \left| \sqrt{r} + \sqrt{r - \frac{2GM}{c^2}} \right| \right. \right. \\
&\quad \left. \left. - \sqrt{r_0} \sqrt{r_0 - \frac{2GM}{c^2}} - \frac{2GM}{c^2} \ln \left| \sqrt{r_0} + \sqrt{r_0 - \frac{2GM}{c^2}} \right| \right\} \right]^2 \\
R_{\hat{0}\hat{2}\hat{0}\hat{2}}(\hat{\xi}) &= R_{\hat{0}\hat{3}\hat{0}\hat{3}}(\hat{\xi}) = -\frac{GM}{r^3 c^2} \left( 1 + \frac{a_0 \hat{\xi}^1}{c^2} \right)^2 = -\frac{GM}{r^3 c^2} \left( 1 + \frac{a_0 \hat{r}}{c^2} \right)^2 \\
&= -\frac{GM}{r^3 c^2} \left[ 1 + \frac{a_0}{c^2} \left\{ \sqrt{r} \sqrt{r - \frac{2GM}{c^2}} + \frac{2GM}{c^2} \ln \left| \sqrt{r} + \sqrt{r - \frac{2GM}{c^2}} \right| \right. \right. \\
&\quad \left. \left. - \sqrt{r_0} \sqrt{r_0 - \frac{2GM}{c^2}} - \frac{2GM}{c^2} \ln \left| \sqrt{r_0} + \sqrt{r_0 - \frac{2GM}{c^2}} \right| \right\} \right]^2 \\
&= -\frac{GM}{r^3 c^2} \left[ 1 + \frac{1}{c^2} \frac{a}{\sqrt{1 - \frac{2GM}{rc^2}}} \left\{ \sqrt{r} \sqrt{r - \frac{2GM}{c^2}} + \frac{2GM}{c^2} \ln \left| \sqrt{r} + \sqrt{r - \frac{2GM}{c^2}} \right| \right. \right. \\
&\quad \left. \left. - \sqrt{r_0} \sqrt{r_0 - \frac{2GM}{c^2}} - \frac{2GM}{c^2} \ln \left| \sqrt{r_0} + \sqrt{r_0 - \frac{2GM}{c^2}} \right| \right\} \right]^2 \\
R_{\hat{2}\hat{3}\hat{2}\hat{3}}(\hat{\xi}) &= -\frac{2GM}{r^3 c^2}, \quad R_{\hat{1}\hat{2}\hat{1}\hat{2}}(\hat{\xi}) = R_{\hat{1}\hat{3}\hat{1}\hat{3}}(\hat{\xi}) = \frac{GM}{r^3 c^2} \tag{34}
\end{aligned}$$

#### IV. Conclusion

In the general relativity theory, defines the accelerated frame that moves in  $\hat{r}$ -axis in the curved time-space. Specially, if  $t = \hat{t} = \hat{\xi}^0 = 0$ , this theory treats the curvature tensor of the stationary accelerated frame in the curved time-space.



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