The curvature tensor of the stationary accelerated frame in the gravity field

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ABSTRACT

In the general relativity theory, we define the accelerated frame that moves in $\hat{r}$ -axis in the curved time-space. And we calculate the curvature tensor of the stationary accelerated frame in the gravity field. In this time, the curvature tensor divide the observational curvature tensor of the people and the curvature tensor of the people's self on the planet in the gravity field.

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1.Introduction

This theory's object is that defines the accelerated frame that moves in $\hat{r}$ -axis in the curved space-time.

The Schwarzschild solution is

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}[\frac{dr^2}{1-\frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] \qquad (1)$$

In this time, a moving matter's acceleration is a in the Schwarzschild space-time.

$$a = a_{inertial} - g = \frac{d}{dt}(\frac{u}{\sqrt{1-\frac{2GM}{rc^2}-\frac{u^2}{c^2}}}), u = \frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}\frac{dr}{dt} \qquad (2)$$

$a_{inertial}$ is the inertial acceleration, g is the pure gravity acceleration.

If $a_0 = a/\sqrt{1-\frac{2GM}{rc^2}} = -g/\sqrt{1-\frac{2GM}{rc^2}}$, $a_{inertial} = 0$ is

$$a_0 = \frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}\frac{d}{dt}(\frac{u}{\sqrt{1-\frac{2GM}{rc^2}-\frac{u^2}{c^2}}}) = \frac{d}{d\hat{t}}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}),$$

$$V = \frac{d\hat{r}}{d\hat{t}} = \frac{dr}{dt}\frac{1}{(1-\frac{2GM}{rc^2})}, \quad d\hat{t} = dt\sqrt{1-\frac{2GM}{rc^2}}, \quad d\hat{r} = \frac{dr}{\sqrt{1-\frac{2GM}{rc^2}}}$$

$$a_0\hat{t} = \frac{V}{\sqrt{1-\frac{V^2}{c^2}}}, \quad V = \frac{a_0\hat{t}}{\sqrt{1+\frac{a_0^2\hat{t}^2}{c^2}}},$$ V is the $\hat{r}$ -axis's velocity (3)

If $\frac{d\theta}{dt} = \frac{d\phi}{dt} = 0$, the solution is

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}\frac{dr^2}{1-\frac{2GM}{rc^2}} = d\hat{t}^2 - \frac{1}{c^2}d\hat{r}^2 = d\hat{t}^2(1-\frac{V^2}{c^2})$$

$$= \frac{d\hat{t}^2}{\sqrt{1+\frac{a_0^2\hat{t}^2}{c^2}}} \qquad (4)$$

In this time,

$$\tau = \int d\tau = \int \frac{d\hat{t}}{\sqrt{1+\frac{{a_0}^2\hat{t}^2}{c^2}}} = \frac{c}{a_0}\sinh^{-1}(\frac{a_0}{c}\hat{t}),$$

$$\hat{t} = \frac{c}{a_0}\sinh(\frac{a_0\tau}{c}), \quad \hat{r} = \int V d\hat{t} = \int \frac{a_0\hat{t}d\hat{t}}{\sqrt{1+\frac{{a_0}^2\hat{t}^2}{c^2}}} = \frac{c^2}{a_0}\sqrt{1+\frac{{a_0}^2\hat{t}^2}{c^2}}$$

$$= \frac{c^2}{a_0}\cosh(\frac{a_0\tau}{c})$$

$$\frac{d\hat{t}}{d\tau} = \cosh(\frac{a_0}{c}\tau), \ \frac{1}{c}\frac{d\hat{r}}{d\tau} = \sinh(\frac{a_0}{c}\tau) \quad (5)$$

2. The tetrad in the curved space-time

The tetrad $e^{\hat{\alpha}}{}_{\hat{\mu}}$ is the unit vector defined by the following formula.

$$\eta_{\hat{\alpha}\hat{\beta}}e^{\hat{\alpha}}{}_{\hat{\mu}}e^{\hat{\beta}}{}_{\hat{\upsilon}} = g_{\hat{\mu}\hat{\upsilon}} \quad (6)$$

In this time, if a matter moves in $\hat{r}$ -axis in the curved space-time,

$$\eta_{\hat{\alpha}\hat{\beta}}e^{\hat{\alpha}}{}_{\hat{\mu}}(\tau)e^{\hat{\beta}}{}_{\hat{\upsilon}}(\tau) = g_{\hat{\mu}\hat{\upsilon}} = \eta_{\hat{\mu}\hat{\upsilon}}, \quad g_{\hat{\mu}\hat{\upsilon}} = \eta_{\hat{\mu}\hat{\upsilon}} \quad (7)$$

Hence, Eq(6),Eq(7) is

$$\eta_{\hat{\alpha}\hat{\beta}}e^{\hat{\alpha}}{}_{\hat{0}}(\tau)e^{\hat{\beta}}{}_{\hat{0}}(\tau) = \eta_{\hat{0}\hat{0}} = -1 \quad (8)$$

$$d\tau^2 = -\frac{1}{c^2}\eta_{\hat{\alpha}\hat{\beta}}d\hat{x}^{\alpha}d\hat{x}^{\beta}$$

$$\rightarrow \ -1 = \eta_{\hat{\alpha}\hat{\beta}}(\frac{1}{c}\frac{d\hat{x}^{\alpha}}{d\tau})(\frac{1}{c}\frac{d\hat{x}^{\beta}}{d\tau}) = \eta_{\hat{\alpha}\hat{\beta}}e^{\hat{\alpha}}{}_{\hat{0}}(\tau)e^{\hat{\beta}}{}_{\hat{0}}(\tau)$$

$$\hat{x}^{\alpha} = (c\hat{t}, \hat{r}, \hat{\theta}, \hat{\phi}) \quad (9)$$

According to Eq(5),Eq(9)

$$e^{\hat{\alpha}}{}_{\hat{0}}(\tau) = \frac{1}{c}\frac{d\hat{x}^{\alpha}}{d\tau} = (\cosh(\frac{a_0\tau}{c}), \sinh(\frac{a_0\tau}{c}), 0, 0) \quad (10)$$

About $\hat{\theta}$ -axis's and $\hat{\phi}$ -axis's orientation

$$\eta_{\hat{2}\hat{2}}e^{\hat{2}}{}_{\hat{2}}(\tau)e^{\hat{2}}{}_{\hat{2}}(\tau) = \eta_{\hat{2}\hat{2}} = 1, \qquad e^{\hat{\alpha}}{}_{\hat{2}}(\tau) = (0,0,1,0)$$

$$\eta_{\hat{3}\hat{3}} e^{\hat{3}}{}_{\hat{3}}(\tau) e^{\hat{3}}{}_{\hat{3}}(\tau) = \eta_{\hat{3}\hat{3}} = 1, \quad e^{\hat{\alpha}}{}_{\hat{3}}(\tau) = (0,0,0,1) \tag{11}$$

And the other vector $e^{\hat{\alpha}}{}_{\hat{1}}(\tau)$ has to satisfy the tetrad condition, Eq (6),Eq(7)

$$e^{\hat{\alpha}}{}_{\hat{1}}(\tau) = (\sinh(\frac{a_0 \tau}{c}), \cosh(\frac{a_0 \tau}{c}), 0, 0) \tag{12}$$

In this time,

$$\bar{e}_{\hat{t}}{}^{\rho} = (1/\sqrt{1-\frac{2GM}{rc^2}}, 0,0,0), \bar{e}_{\hat{r}}{}^{\rho} = (0, \sqrt{1-\frac{2GM}{rc^2}}, 0,0)$$

$$\bar{e}_{\hat{\theta}}{}^{\rho} = (0,0,1/r,0) \quad , \quad \bar{e}_{\hat{\phi}}{}^{\rho} = (0,0,0,1/r\sin\theta)$$

$$g_{\rho\sigma} \bar{e}_{\hat{\alpha}}{}^{\rho} \bar{e}_{\hat{\beta}}{}^{\sigma} = \eta_{\hat{\alpha}\hat{\beta}} \tag{13}$$

$$\frac{a_0}{c}\hat{t} = \sinh(\frac{a_0}{c}\tau) = \frac{v/c}{\sqrt{1-v^2/c^2}}, \sqrt{1+\frac{a_0{}^2 \hat{t}^2}{c^2}} = \cosh(\frac{a_0}{c}\tau) = \frac{1}{\sqrt{1-v^2/c^2}} \tag{14}$$

Therefore, the Lorentz transformation $B^{\hat{\alpha}}{}_{\hat{\mu}}(v)$ is

$$B^{\hat{\alpha}}{}_{\hat{\mu}}(v) = \begin{pmatrix} \frac{1}{\sqrt{1-v^2/c^2}} & \frac{v/c}{\sqrt{1-v^2/c^2}} & 0 & 0 \\ \frac{v/c}{\sqrt{1-v^2/c^2}} & \frac{1}{\sqrt{1-v^2/c^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= e^{\hat{\alpha}}{}_{\hat{\mu}}(\tau) = \begin{pmatrix} \cosh(\frac{a_0}{c}\tau) & \sinh(\frac{a_0}{c}\tau) & 0 & 0 \\ \sinh(\frac{a_0}{c}\tau) & \cosh(\frac{a_0}{c}\tau) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{15}$$

$$\bar{e}'_{\hat{\mu}}{}^{\rho} = B^{\hat{\alpha}}{}_{\hat{\mu}}(v) \bar{e}_{\hat{\alpha}}{}^{\rho} = e^{\hat{\alpha}}{}_{\hat{\mu}}(\tau) \bar{e}_{\hat{\alpha}}{}^{\rho} \tag{16}$$

Hence,

$$g_{\rho\sigma} \bar{e}_{\hat{\alpha}}{}^{\rho} \bar{e}_{\hat{\beta}}{}^{\sigma} = \eta_{\hat{\alpha}\hat{\beta}}$$

$$g_{\rho\sigma} B^{\hat{\alpha}}{}_{\hat{\mu}}(v) \bar{e}_{\hat{\alpha}}{}^{\rho} B^{\hat{\beta}}{}_{\hat{\upsilon}}(v) \bar{e}_{\hat{\beta}}{}^{\sigma} = g_{\rho\sigma} \bar{e}'_{\hat{\mu}}{}^{\rho} \bar{e}'_{\hat{\upsilon}}{}^{\sigma} = \eta_{\hat{\alpha}\hat{\beta}} e^{\hat{\alpha}}{}_{\hat{\mu}}(\tau) e^{\hat{\beta}}{}_{\hat{\upsilon}}(\tau) = \eta_{\hat{\mu}\hat{\upsilon}} \tag{17}$$

3. The accelerated frame in the curved space-time.

About the accelerated frame $\hat{\xi}$ in the curved space-time,

$$d\tau^2 = -\frac{1}{c^2}\eta_{\hat{\alpha}\hat{\beta}} d\hat{x}^\alpha d\hat{x}^\beta = -\frac{1}{c^2}\eta_{\hat{\alpha}\hat{\beta}} \frac{\partial \hat{x}^\alpha}{\partial \hat{\xi}^\mu}\frac{\partial \hat{x}^\beta}{\partial \hat{\xi}^\upsilon} d\hat{\xi}^\mu d\hat{\xi}^\upsilon$$

$$= -\frac{1}{c^2}\eta_{\hat{\alpha}\hat{\beta}} e^{\hat{\alpha}}{}_{\hat{\mu}} e^{\hat{\beta}}{}_{\hat{\upsilon}} d\hat{\xi}^\mu d\hat{\xi}^\upsilon$$

$$= -\frac{1}{c^2} g_{\hat{\mu}\hat{\upsilon}} d\hat{\xi}^\mu d\hat{\xi}^\upsilon \quad (18)$$

$$e^{\hat{\alpha}}{}_{\hat{\mu}} = \frac{\partial \hat{x}^\alpha}{\partial \hat{\xi}^\mu} \;,\; \frac{\partial e^{\hat{\alpha}}{}_{\hat{0}}}{\partial \hat{\xi}^1} = \frac{\partial^2 \hat{x}^\alpha}{c\partial \hat{\xi}^0 \partial \hat{\xi}^1} = \frac{\partial e^{\hat{\alpha}}{}_{\hat{1}}}{c\partial \hat{\xi}^0} \quad (19)$$

3-1.Case-1. Rindler-coordinate

In this time, in Eq(10),Eq(11),Eq(12), if uses $\hat{\xi}^0$ instead of τ ,

$$e^{\hat{\alpha}}{}_{\hat{0}}(\hat{\xi}^0) = \frac{1}{c}\frac{\partial \hat{x}^\alpha}{\partial \hat{\xi}^0} = ((1+\frac{a_0\hat{\xi}^1}{c^2})\cosh(\frac{a_0\hat{\xi}^0}{c}),(1+\frac{a_0\hat{\xi}^1}{c^2})\sinh(\frac{a_0\hat{\xi}^0}{c}),0,0) \quad (20)$$

$$e^{\hat{\alpha}}{}_{\hat{1}}(\hat{\xi}^0) = \frac{\partial \hat{x}^\alpha}{\partial \hat{\xi}^1} = (\sinh(\frac{a_0\hat{\xi}^0}{c}),\cosh(\frac{a_0\hat{\xi}^0}{c}),0,0) \quad (21)$$

$$e^{\hat{\alpha}}{}_{\hat{2}}(\hat{\xi}^0) = \frac{\partial \hat{x}^\alpha}{\partial \hat{\xi}^2} = (0,0,1,0), e^{\hat{\alpha}}{}_{\hat{3}}(\hat{\xi}^0) = \frac{\partial \hat{x}^\alpha}{\partial \hat{\xi}^3} = (0,0,1,0) \quad (22)$$

$$d\hat{x}^\alpha = \frac{\partial \hat{x}^\alpha}{\partial \hat{\xi}^\mu} d\hat{\xi}^\mu = e^{\hat{\alpha}}{}_{\hat{0}}(\hat{\xi}^0)cd\hat{\xi}^0 + e^{\hat{\alpha}}{}_{\hat{1}}(\hat{\xi}^0)d\hat{\xi}^1 + e^{\hat{\alpha}}{}_{\hat{2}}(\hat{\xi}^0)d\hat{\xi}^2 + e^{\hat{\alpha}}{}_{\hat{3}}(\hat{\xi}^0)d\hat{\xi}^3 \quad (23)$$

Hence,

$$cd\hat{t} = cdt\sqrt{1-\frac{2GM}{rc^2}} = (1+\frac{a_0\hat{\xi}^1}{c^2})\cosh(\frac{a_0\hat{\xi}^0}{c})cd\hat{\xi}^0 + \sinh(\frac{a_0\hat{\xi}^0}{c})d\hat{\xi}^1$$

$$d\hat{r} = \frac{dr}{\sqrt{1-\frac{2GM}{rc^2}}} = (1+\frac{a_0\hat{\xi}^1}{c^2})\sinh(\frac{a_0\hat{\xi}^0}{c})cd\hat{\xi}^0 + \cosh(\frac{a_0\hat{\xi}^0}{c})d\hat{\xi}^1$$

$$d\hat{\theta} = d\hat{\xi}^2 \quad , \quad d\hat{\phi} = d\hat{\xi}^3 \quad (24)$$

$$d\tau^2 = (1 - \frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}[\frac{dr^2}{1 - \frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2]$$

$$= d\hat{t}^2 - \frac{1}{c^2}[d\hat{r}^2 + d\hat{\theta}^2 + d\hat{\phi}^2]$$

$$= (1 + \frac{a_0\hat{\xi}^1}{c^2})^2(d\hat{\xi}^0)^2 - \frac{1}{c^2}[(d\hat{\xi}^1)^2 + (d\hat{\xi}^2)^2 + (d\hat{\xi}^3)^2] \quad (25)$$

The coordinate transformation is

$$c\hat{t} = (\frac{c^2}{a_0} + \hat{\xi}^1)\sinh(\frac{a_0\hat{\xi}^0}{c}), \hat{r} = (\frac{c^2}{a_0} + \hat{\xi}^1)\cosh(\frac{a_0\hat{\xi}^0}{c}) - \frac{c^2}{a_0}$$

$$\hat{\theta} = \hat{\xi}^2, \hat{\phi} = \hat{\xi}^3 \quad (26)$$

The inverse-transformation is

$$\hat{\xi}^0 = \frac{c}{a_0}\tanh^{-1}(\frac{c\hat{t}}{\hat{r} + \frac{c^2}{a_0}}), \ \hat{\xi}^1 = \sqrt{(\hat{r} + \frac{c^2}{a_0})^2 - c^2\hat{t}^2} - \frac{c^2}{a_0}$$

$$\hat{\xi}^2 = \hat{\theta}, \hat{\xi}^3 = \hat{\phi} \quad (27)$$

If we calculate the curvature tensor $R_{\hat{\mu}\hat{\upsilon}\hat{\rho}\hat{\lambda}}(\hat{\xi})$,

$$R_{\hat{\mu}\hat{\upsilon}\hat{\rho}\hat{\lambda}}(\hat{\xi}) = \frac{\partial\hat{X}^\alpha}{\partial\hat{\xi}^\mu}\frac{\partial\hat{X}^\beta}{\partial\hat{\xi}^\upsilon}\frac{\partial\hat{X}^\gamma}{\partial\hat{\xi}^\rho}\frac{\partial\hat{X}^\delta}{\partial\hat{\xi}^\lambda}R_{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\delta}}(\hat{X})$$

$$= e^{\hat{\alpha}}{}_{\hat{\mu}}(\hat{\xi}^0)e^{\hat{\beta}}{}_{\hat{\upsilon}}(\hat{\xi}^0)e^{\hat{\gamma}}{}_{\hat{\rho}}(\hat{\xi}^0)e^{\hat{\delta}}{}_{\hat{\lambda}}(\hat{\xi}^0)R_{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\delta}}(\hat{X}) \quad (28)$$

$$R_{\hat{t}\hat{r}\hat{t}\hat{r}} = -R_{\hat{t}\hat{r}\hat{r}\hat{t}} = R_{\hat{r}\hat{t}\hat{r}\hat{t}} = -R_{\hat{r}\hat{t}\hat{t}\hat{r}} = \frac{2GM}{r^3c^2},$$

$$R_{\hat{t}\hat{\theta}\hat{t}\hat{\theta}} = -R_{\hat{t}\hat{\theta}\hat{\theta}\hat{t}} = R_{\hat{\theta}\hat{t}\hat{\theta}\hat{t}} = -R_{\hat{\theta}\hat{t}\hat{t}\hat{\theta}} = -\frac{GM}{r^3c^2} = R_{\hat{t}\hat{\phi}\hat{t}\hat{\phi}} = -R_{\hat{t}\hat{\phi}\hat{\phi}\hat{t}} = R_{\hat{\phi}\hat{t}\hat{\phi}\hat{t}} = -R_{\hat{\phi}\hat{t}\hat{t}\hat{\phi}}$$

$$R_{\hat{\theta}\hat{\phi}\hat{\theta}\hat{\phi}} = -R_{\hat{\theta}\hat{\phi}\hat{\phi}\hat{\theta}} = R_{\hat{\theta}\hat{\phi}\hat{\theta}\hat{\phi}} = -R_{\hat{\theta}\hat{\phi}\hat{\phi}\hat{\theta}} = -\frac{2GM}{r^3c^2}$$

$$R_{\hat{r}\hat{\theta}\hat{r}\hat{\theta}} = -R_{\hat{r}\hat{\theta}\hat{\theta}\hat{r}} = R_{\hat{\theta}\hat{r}\hat{\theta}\hat{r}} = -R_{\hat{\theta}\hat{r}\hat{r}\hat{\theta}} = \frac{GM}{r^3c^2} = R_{\hat{r}\hat{\phi}\hat{r}\hat{\phi}} = -R_{\hat{r}\hat{\phi}\hat{\phi}\hat{r}} = R_{\hat{\phi}\hat{r}\hat{\phi}\hat{r}} = -R_{\hat{\phi}\hat{r}\hat{r}\hat{\phi}}$$

(29)

Therefore,

$$e^{\hat{\alpha}}{}_{\hat{0}}(\hat{\xi}^0) = ((1+\frac{a_0\hat{\xi}^1}{c^2})\cosh(\frac{a_0\hat{\xi}^0}{c}), (1+\frac{a_0\hat{\xi}^1}{c^2})\sinh(\frac{a_0\hat{\xi}^0}{c}), 0, 0)$$

$$e^{\hat{\alpha}}{}_{\hat{1}}(\hat{\xi}^0) = (\sinh(\frac{a_0\hat{\xi}^0}{c}), \cosh(\frac{a_0\hat{\xi}^0}{c}), 0, 0)$$

$$e^{\hat{\alpha}}{}_{\hat{2}}(\hat{\xi}^0) = (0,0,1,0), e^{\hat{\alpha}}{}_{\hat{3}}(\hat{\xi}^0) = (0,0,1,0) \quad (30)$$

$$R_{\hat{0}\hat{1}\hat{0}\hat{1}}(\hat{\xi}) = \frac{2GM}{r^3c^2}(1+\frac{a_0\hat{\xi}^1}{c^2})^2, \ R_{\hat{0}\hat{2}\hat{0}\hat{2}}(\hat{\xi}) = R_{\hat{0}\hat{3}\hat{0}\hat{3}}(\hat{\xi}) = -\frac{GM}{r^3c^2}(1+\frac{a_0\hat{\xi}^1}{c^2})^2$$

$$R_{\hat{2}\hat{3}\hat{2}\hat{3}}(\hat{\xi}) = -\frac{2GM}{r^3c^2}, \ R_{\hat{1}\hat{2}\hat{1}\hat{2}}(\hat{\xi}) = R_{\hat{1}\hat{3}\hat{1}\hat{3}}(\hat{\xi}) = \frac{GM}{r^3c^2} \quad (31)$$

Specially, if $t = 0$,

$$u = \frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}\frac{dr}{dt} = 0 \rightarrow V = \frac{d\hat{r}}{d\hat{t}} = \frac{a_0\hat{t}}{\sqrt{1+\frac{a_0{}^2\hat{t}^2}{c^2}}} = \frac{dr}{dt}\frac{1}{(1-\frac{2GM}{rc^2})} = 0 \quad (32)$$

Therefore, if $t = \hat{t} = \hat{\xi}^0 = 0$, the theory treats the real situation.

$$\hat{\xi}^1 = \sqrt{(\hat{r}+\frac{c^2}{a_0})^2 - c^2\hat{t}^2} - \frac{c^2}{a_0} = \hat{r}$$

$$d\hat{r} = \frac{dr}{\sqrt{1-\frac{2GM}{rc^2}}} \rightarrow \hat{r} = \sqrt{r}\sqrt{r-\frac{2GM}{c^2}} + \frac{2GM}{c^2}\ln|\sqrt{r}+\sqrt{r-\frac{2GM}{c^2}}|$$

$$-\sqrt{r_0}\sqrt{r_0-\frac{2GM}{c^2}} - \frac{2GM}{c^2}\ln|\sqrt{r_0}+\sqrt{r_0-\frac{2GM}{c^2}}|$$

$$a_0 = \frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}\frac{d}{dt}(\frac{u}{\sqrt{1-\frac{2GM}{rc^2}-\frac{u^2}{c^2}}}) = \frac{a}{\sqrt{1-\frac{2GM}{rc^2}}}$$

$$a = -g = \frac{d}{dt}(\frac{u}{\sqrt{1-\frac{2GM}{rc^2}-\frac{u^2}{c^2}}})$$

g is the pure gravity acceleration.

r_0 is the location of the stationary accelerated frame (33)

In this time, in the curved space-time, the curvature tensor $R_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\lambda}}(\hat{\xi})$ of the stationary accelerated frame is

$$R_{\hat{0}\hat{1}\hat{0}\hat{1}}(\hat{\xi}) = \frac{2GM}{r^3c^2}(1+\frac{a_0\hat{\xi}^1}{c^2})^2 = \frac{2GM}{r^3c^2}(1+\frac{a_0\hat{r}}{c^2})^2$$

$$= \frac{2GM}{r^3c^2}[1+\frac{a_0}{c^2}\{\sqrt{r}\sqrt{r-\frac{2GM}{c^2}}+\frac{2GM}{c^2}\ln|\sqrt{r}+\sqrt{r-\frac{2GM}{c^2}}|$$

$$-\sqrt{r_0}\sqrt{r_0-\frac{2GM}{c^2}}-\frac{2GM}{c^2}\ln|\sqrt{r_0}+\sqrt{r_0-\frac{2GM}{c^2}}|\}]^2$$

$$= \frac{2GM}{r^3c^2}[1-\frac{1}{c^2}\frac{g}{\sqrt{1-\frac{2GM}{rc^2}}}\{\sqrt{r}\sqrt{r-\frac{2GM}{c^2}}+\frac{2GM}{c^2}\ln|\sqrt{r}+\sqrt{r-\frac{2GM}{c^2}}|$$

$$-\sqrt{r_0}\sqrt{r_0-\frac{2GM}{c^2}}-\frac{2GM}{c^2}\ln|\sqrt{r_0}+\sqrt{r_0-\frac{2GM}{c^2}}|\}]^2$$

$$R_{\hat{0}\hat{2}\hat{0}\hat{2}}(\hat{\xi}) = R_{\hat{0}\hat{3}\hat{0}\hat{3}}(\hat{\xi}) = -\frac{GM}{r^3c^2}(1+\frac{a_0\hat{\xi}^1}{c^2})^2 = -\frac{GM}{r^3c^2}(1+\frac{a_0\hat{r}}{c^2})^2$$

$$= -\frac{GM}{r^3c^2}[1+\frac{a_0}{c^2}\{\sqrt{r}\sqrt{r-\frac{2GM}{c^2}}+\frac{2GM}{c^2}\ln|\sqrt{r}+\sqrt{r-\frac{2GM}{c^2}}|$$

$$-\sqrt{r_0}\sqrt{r_0-\frac{2GM}{c^2}}-\frac{2GM}{c^2}\ln|\sqrt{r_0}+\sqrt{r_0-\frac{2GM}{c^2}}|\}]^2$$

$$= -\frac{GM}{r^3c^2}[1-\frac{1}{c^2}\frac{g}{\sqrt{1-\frac{2GM}{rc^2}}}\{\sqrt{r}\sqrt{r-\frac{2GM}{c^2}}+\frac{2GM}{c^2}\ln|\sqrt{r}+\sqrt{r-\frac{2GM}{c^2}}|$$

$$-\sqrt{r_0}\sqrt{r_0-\frac{2GM}{c^2}}-\frac{2GM}{c^2}\ln|\sqrt{r_0}+\sqrt{r_0-\frac{2GM}{c^2}}|\}]^2$$

$$R_{\hat{2}\hat{3}\hat{2}\hat{3}}(\hat{\xi}) = -\frac{2GM}{r^3c^2},\quad R_{\hat{1}\hat{2}\hat{1}\hat{2}}(\hat{\xi}) = R_{\hat{1}\hat{3}\hat{1}\hat{3}}(\hat{\xi}) = \frac{GM}{r^3c^2}$$

g is the pure gravity acceleration.

r_0 is the location of the stationary accelerated frame (34)

3-2.Case-2.Marzke-Wheeler cooridnate

In this time, in Eq(10),Eq(11),Eq(12), if uses $\hat{\xi}^0$ instead of τ and multiply $\exp(\frac{a_0}{c^2}\hat{\xi}^1)$

$$e^{\hat{\alpha}}{}_{\hat{0}}(\hat{\xi}^0)=\frac{1}{c}\frac{\partial\hat{X}^\alpha}{\partial\hat{\xi}^0}=(\exp(\frac{a_0}{c^2}\hat{\xi}^1)\cosh(\frac{a_0\hat{\xi}^0}{c}),\exp(\frac{a_0}{c^2}\hat{\xi}^1)\sinh(\frac{a_0\hat{\xi}^0}{c}),0,0) \quad (35)$$

$$e^{\hat{\alpha}}{}_{\hat{1}}(\hat{\xi}^0)=\frac{\partial\hat{X}^\alpha}{\partial\hat{\xi}^1}=(\exp(\frac{a_0}{c^2}\hat{\xi}^1)\sinh(\frac{a_0\hat{\xi}^0}{c}),\exp(\frac{a_0}{c^2}\hat{\xi}^1)\cosh(\frac{a_0\hat{\xi}^0}{c}),0,0)$$

(36)

$$e^{\hat{\alpha}}{}_{\hat{2}}(\hat{\xi}^0)=\frac{\partial\hat{X}^\alpha}{\partial\hat{\xi}^2}=(0,0,1,0),e^{\hat{\alpha}}{}_{\hat{3}}(\hat{\xi}^0)=\frac{\partial\hat{X}^\alpha}{\partial\hat{\xi}^3}=(0,0,1,0) \quad (37)$$

$$d\hat{X}^\alpha=\frac{\partial\hat{X}^\alpha}{\partial\hat{\xi}^\mu}d\hat{\xi}^\mu=e^{\hat{\alpha}}{}_{\hat{0}}(\hat{\xi}^0)cd\hat{\xi}^0+e^{\hat{\alpha}}{}_{\hat{1}}(\hat{\xi}^0)d\hat{\xi}^1+e^{\hat{\alpha}}{}_{\hat{2}}(\hat{\xi}^0)d\hat{\xi}^2+e^{\hat{\alpha}}{}_{\hat{3}}(\hat{\xi}^0)d\hat{\xi}^3 \quad (38)$$

Hence,

$$cd\hat{t}=cdt\sqrt{1-\frac{2GM}{rc^2}}=\exp(\frac{a_0}{c^2}\hat{\xi}^1)\cosh(\frac{a_0\hat{\xi}^0}{c})cd\hat{\xi}^0+\exp(\frac{a_0}{c^2}\hat{\xi}^1)\sinh(\frac{a_0\hat{\xi}^0}{c})d\hat{\xi}^1$$

$$d\hat{r}=\frac{dr}{\sqrt{1-\frac{2GM}{rc^2}}}=\exp(\frac{a_0}{c^2}\hat{\xi}^1)\sinh(\frac{a_0\hat{\xi}^0}{c})cd\hat{\xi}^0+\exp(\frac{a_0}{c^2}\hat{\xi}^1)\cosh(\frac{a_0\hat{\xi}^0}{c})d\hat{\xi}^1$$

$$d\hat{\theta}=d\hat{\xi}^2 \quad , \quad d\hat{\phi}=d\hat{\xi}^3 \quad (39)$$

$$d\tau^2=(1-\frac{2GM}{rc^2})dt^2-\frac{1}{c^2}[\frac{dr^2}{1-\frac{2GM}{rc^2}}+r^2d\theta^2+r^2\sin^2\theta d\phi^2]$$
$$=d\hat{t}^2-\frac{1}{c^2}[d\hat{r}^2+d\hat{\theta}^2+d\hat{\phi}^2]$$
$$=\exp(2\frac{a_0\hat{\xi}^1}{c^2})(d\hat{\xi}^0)^2-\frac{1}{c^2}[\exp(2\frac{a_0\hat{\xi}^1}{c^2})(d\hat{\xi}^1)^2+(d\hat{\xi}^2)^2+(d\hat{\xi}^3)^2]$$

(40)

The coordinate transformation is

$$c\hat{t} = \frac{c^2}{a_0}\exp(\frac{a_0}{c^2}\hat{\xi}^1)\sinh(\frac{a_0\hat{\xi}^0}{c}), \hat{r} = \frac{c^2}{a_0}\exp(\frac{a_0}{c^2}\hat{\xi}^1)\cosh(\frac{a_0\hat{\xi}^0}{c}) - \frac{c^2}{a_0}$$

$$\hat{\theta} = \hat{\xi}^2, \hat{\phi} = \hat{\xi}^3 \qquad (41)$$

The inverse-transformation is

$$\hat{\xi}^0 = \frac{c}{a_0}\tanh^{-1}(\frac{c\hat{t}}{\hat{r} + \frac{c^2}{a_0}}), \ \hat{\xi}^1 = \frac{c^2}{a_0}\ln\left|\frac{a_0}{c^2}\sqrt{(\hat{r} + \frac{c^2}{a_0})^2 - c^2\hat{t}^2}\right|$$

$$\hat{\xi}^2 = \hat{\theta}, \hat{\xi}^3 = \hat{\phi} \qquad (42)$$

If we calculate the curvature tensor $R_{\hat{\mu}\hat{\upsilon}\hat{\rho}\hat{\lambda}}(\hat{\xi})$,

$$R_{\hat{\mu}\hat{\upsilon}\hat{\rho}\hat{\lambda}}(\hat{\xi}) = \frac{\partial\hat{X}^\alpha}{\partial\hat{\xi}^\mu}\frac{\partial\hat{X}^\beta}{\partial\hat{\xi}^\upsilon}\frac{\partial\hat{X}^\gamma}{\partial\hat{\xi}^\rho}\frac{\partial\hat{X}^\delta}{\partial\hat{\xi}^\lambda}R_{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\delta}}(\hat{X})$$

$$= e^{\hat{\alpha}}{}_{\hat{\mu}}(\hat{\xi}^0)e^{\hat{\beta}}{}_{\hat{\upsilon}}(\hat{\xi}^0)e^{\hat{\gamma}}{}_{\hat{\rho}}(\hat{\xi}^0)e^{\hat{\delta}}{}_{\hat{\lambda}}(\hat{\xi}^0)R_{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\delta}}(\hat{X}) \qquad (43)$$

$$R_{\hat{t}\hat{r}\hat{t}\hat{r}} = -R_{\hat{t}\hat{r}\hat{r}\hat{t}} = R_{\hat{r}\hat{t}\hat{r}\hat{t}} = -R_{\hat{r}\hat{t}\hat{t}\hat{r}} = \frac{2GM}{r^3c^2},$$

$$R_{\hat{t}\hat{\theta}\hat{t}\hat{\theta}} = -R_{\hat{t}\hat{\theta}\hat{\theta}\hat{t}} = R_{\hat{\theta}\hat{t}\hat{\theta}\hat{t}} = -R_{\hat{\theta}\hat{t}\hat{t}\hat{\theta}} = -\frac{GM}{r^3c^2} = R_{\hat{t}\hat{\phi}\hat{t}\hat{\phi}} = -R_{\hat{t}\hat{\phi}\hat{\phi}\hat{t}} = R_{\hat{\phi}\hat{t}\hat{\phi}\hat{t}} = -R_{\hat{\phi}\hat{t}\hat{t}\hat{\phi}}$$

$$R_{\hat{\theta}\hat{\phi}\hat{\theta}\hat{\phi}} = -R_{\hat{\theta}\hat{\phi}\hat{\phi}\hat{\theta}} = R_{\hat{\theta}\hat{\phi}\hat{\theta}\hat{\phi}} = -R_{\hat{\theta}\hat{\phi}\hat{\phi}\hat{\theta}} = -\frac{2GM}{r^3c^2}$$

$$R_{\hat{r}\hat{\theta}\hat{r}\hat{\theta}} = -R_{\hat{r}\hat{\theta}\hat{\theta}\hat{r}} = R_{\hat{\theta}\hat{r}\hat{\theta}\hat{r}} = -R_{\hat{\theta}\hat{r}\hat{r}\hat{\theta}} = \frac{GM}{r^3c^2} = R_{\hat{r}\hat{\phi}\hat{r}\hat{\phi}} = -R_{\hat{r}\hat{\phi}\hat{\phi}\hat{r}} = R_{\hat{\phi}\hat{r}\hat{\phi}\hat{r}} = -R_{\hat{\phi}\hat{r}\hat{r}\hat{\phi}}$$

(44)

Hence,

$$e^{\hat{\alpha}}{}_{\hat{0}}(\hat{\xi}^0) = (\exp(\frac{a_0}{c^2}\hat{\xi}^1)\cosh(\frac{a_0\hat{\xi}^0}{c}), \exp(\frac{a_0}{c^2}\hat{\xi}^1)\sinh(\frac{a_0\hat{\xi}^0}{c}), 0, 0)$$

$$e^{\hat{\alpha}}{}_{\hat{1}}(\hat{\xi}^0) = (\exp(\frac{a_0}{c^2}\hat{\xi}^1)\sinh(\frac{a_0\hat{\xi}^0}{c}), \exp(\frac{a_0}{c^2}\hat{\xi}^1)\cosh(\frac{a_0\hat{\xi}^0}{c}), 0, 0)$$

$$e^{\hat{\alpha}}{}_{\hat{2}}(\hat{\xi}^0) = (0,0,1,0), e^{\hat{\alpha}}{}_{\hat{3}}(\hat{\xi}^0) = (0,0,1,0) \qquad (45)$$

$$R_{\hat{0}\hat{1}\hat{0}\hat{1}}(\hat{\xi}) = \frac{2GM}{r^3c^2}\exp(4\frac{a_0\hat{\xi}^1}{c^2}),\ R_{\hat{0}\hat{2}\hat{0}\hat{2}}(\hat{\xi}) = R_{\hat{0}\hat{3}\hat{0}\hat{3}}(\hat{\xi}) = -\frac{GM}{r^3c^2}\exp(2\frac{a_0\hat{\xi}^1}{c^2})$$

$$R_{\hat{2}\hat{3}\hat{2}\hat{3}}(\hat{\xi}) = -\frac{2GM}{r^3c^2},\ R_{\hat{1}\hat{2}\hat{1}\hat{2}}(\hat{\xi}) = R_{\hat{1}\hat{3}\hat{1}\hat{3}}(\hat{\xi}) = \frac{GM}{r^3c^2}\exp(2\frac{a_0}{c^2}\hat{\xi}^1) \qquad (46)$$

Specially, if $t = 0$,

$$u = \frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}\frac{dr}{dt} = 0 \rightarrow V = \frac{d\hat{r}}{d\hat{t}} = \frac{a_0\hat{t}}{\sqrt{1+\frac{{a_0}^2\hat{t}^2}{c^2}}} = \frac{dr}{dt}\frac{1}{(1-\frac{2GM}{rc^2})} = 0 \qquad (47)$$

Hence, if $t = \hat{t} = \hat{\xi}^0 = 0$, the theory treats the real situation.

$$\hat{\xi}^1 = \frac{c^2}{a_0}\ln|\frac{a_0}{c^2}\sqrt{(\hat{r}+\frac{c^2}{a_0})^2 - c^2\hat{t}^2}| = \frac{c^2}{a_0}\ln|(1+\frac{a_0}{c^2}\hat{r})|$$

$$\exp(\frac{a_0}{c^2}\hat{\xi}^1) = 1 + \frac{a_0}{c^2}\hat{r}$$

$$d\hat{r} = \frac{dr}{\sqrt{1-\frac{2GM}{rc^2}}} \rightarrow \hat{r} = \sqrt{r}\sqrt{r-\frac{2GM}{c^2}} + \frac{2GM}{c^2}\ln|\sqrt{r}+\sqrt{r-\frac{2GM}{c^2}}|$$

$$-\sqrt{r_0}\sqrt{r_0-\frac{2GM}{c^2}} - \frac{2GM}{c^2}\ln|\sqrt{r_0}+\sqrt{r_0-\frac{2GM}{c^2}}|$$

$$a_0 = \frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}\frac{d}{dt}(\frac{u}{\sqrt{1-\frac{2GM}{rc^2}-\frac{u^2}{c^2}}}) = \frac{a}{\sqrt{1-\frac{2GM}{rc^2}}}$$

$$a = -g = \frac{d}{dt}(\frac{u}{\sqrt{1-\frac{2GM}{rc^2}-\frac{u^2}{c^2}}})$$

g is the pure gravity acceleration.

r_0 is the location of the stationary accelerated frame (48)

In this time, in the curved space-time, the curvature tensor $R_{\hat{\mu}\hat{\upsilon}\hat{\rho}\hat{\lambda}}(\hat{\xi})$ of the stationary accelerated frame is

$$R_{\hat{0}\hat{1}\hat{0}\hat{1}}(\hat{\xi}) = \frac{2GM}{r^3c^2}\exp(4\frac{a_0\hat{\xi}^1}{c^2}) = \frac{2GM}{r^3c^2}(1+\frac{a_0}{c^2}\hat{r})^4$$

$$=\frac{2GM}{r^3c^2}[1+\frac{a_0}{c^2}\{\sqrt{r}\sqrt{r-\frac{2GM}{c^2}}+\frac{2GM}{c^2}\ln|\sqrt{r}+\sqrt{r-\frac{2GM}{c^2}}|$$

$$-\sqrt{r_0}\sqrt{r_0-\frac{2GM}{c^2}}-\frac{2GM}{c^2}\ln|\sqrt{r_0}+\sqrt{r_0-\frac{2GM}{c^2}}|\}]^4$$

$$=\frac{2GM}{r^3c^2}[1-\frac{1}{c^2}\frac{g}{\sqrt{1-\frac{2GM}{rc^2}}}\{\sqrt{r}\sqrt{r-\frac{2GM}{c^2}}+\frac{2GM}{c^2}\ln|\sqrt{r}+\sqrt{r-\frac{2GM}{c^2}}|$$

$$-\sqrt{r_0}\sqrt{r_0-\frac{2GM}{c^2}}-\frac{2GM}{c^2}\ln|\sqrt{r_0}+\sqrt{r_0-\frac{2GM}{c^2}}|\}]^4$$

$$R_{\hat{0}\hat{2}\hat{0}\hat{2}}(\hat{\xi})=R_{\hat{0}\hat{3}\hat{0}\hat{3}}(\hat{\xi})=-\frac{GM}{r^3c^2}\exp(2\frac{a_0\hat{\xi}^1}{c^2})=-\frac{GM}{r^3c^2}(1+\frac{a_0\hat{r}}{c^2})^2$$

$$=-\frac{GM}{r^3c^2}[1+\frac{a_0}{c^2}\{\sqrt{r}\sqrt{r-\frac{2GM}{c^2}}+\frac{2GM}{c^2}\ln|\sqrt{r}+\sqrt{r-\frac{2GM}{c^2}}|$$

$$-\sqrt{r_0}\sqrt{r_0-\frac{2GM}{c^2}}-\frac{2GM}{c^2}\ln|\sqrt{r_0}+\sqrt{r_0-\frac{2GM}{c^2}}|\}]^2$$

$$=-\frac{GM}{r^3c^2}[1-\frac{1}{c^2}\frac{g}{\sqrt{1-\frac{2GM}{rc^2}}}\{\sqrt{r}\sqrt{r-\frac{2GM}{c^2}}+\frac{2GM}{c^2}\ln|\sqrt{r}+\sqrt{r-\frac{2GM}{c^2}}|$$

$$-\sqrt{r_0}\sqrt{r_0-\frac{2GM}{c^2}}-\frac{2GM}{c^2}\ln|\sqrt{r_0}+\sqrt{r_0-\frac{2GM}{c^2}}|\}]^2$$

$$R_{\hat{2}\hat{3}\hat{2}\hat{3}}(\hat{\xi})=-\frac{2GM}{r^3c^2},$$

$$R_{\hat{1}\hat{2}\hat{1}\hat{2}}(\hat{\xi})=R_{\hat{1}\hat{3}\hat{1}\hat{3}}(\hat{\xi})=\frac{GM}{r^3c^2}\exp(2\frac{a_0}{c^2}\hat{\xi}^1)=\frac{GM}{r^3c^2}(1+\frac{a_0}{c^2}\hat{r})^2$$

$$=\frac{GM}{r^3c^2}[1+\frac{a_0}{c^2}\{\sqrt{r}\sqrt{r-\frac{2GM}{c^2}}+\frac{2GM}{c^2}\ln|\sqrt{r}+\sqrt{r-\frac{2GM}{c^2}}|$$

$$-\sqrt{r_0}\sqrt{r_0-\frac{2GM}{c^2}}-\frac{2GM}{c^2}\ln|\sqrt{r_0}+\sqrt{r_0-\frac{2GM}{c^2}}|\}]^2$$

$$=\frac{GM}{r^3c^2}[1-\frac{1}{c^2}\frac{g}{\sqrt{1-\frac{2GM}{rc^2}}}\{\sqrt{r}\sqrt{r-\frac{2GM}{c^2}}+\frac{2GM}{c^2}\ln|\sqrt{r}+\sqrt{r-\frac{2GM}{c^2}}|$$

$$-\sqrt{r_0}\sqrt{r_0-\frac{2GM}{c^2}}-\frac{2GM}{c^2}\ln|\sqrt{r_0}+\sqrt{r_0-\frac{2GM}{c^2}}|\}]^2$$

g is the pure gravity acceleration.

r_0 is the location of the stationary accelerated frame (49)

4. Conclusion

In the general relativity theory, we define the accelerated frame that moves in $\hat{r}$ -axis in the curved space-time. Specially, if $t=\hat{t}=\hat{\xi}^0=0$, this theory treats the curvature tensor of the stationary accelerated frame in the curved space-time in two-cases. In this time, $R_{\hat{\mu}\hat{\upsilon}\hat{\rho}\hat{\lambda}}(\hat{\xi})$ is the observational curvature tensor of the people on the planet in the gravity field but $R_{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\delta}}(\hat{X})$ is the curvature tensor of the people's self on the planet in the gravity field.

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