# The Hilbert Book Model GAME 

This e-book is still in development phase.
This e-book is the successor of Physics of the Hilbert Book Model http://vixra.org/abs/1307.0106 and it is a companion of the YouTube video http://youtu.be/yUXoN7oF2uQ

## Colophon

Written by Ir J.A.J. van Leunen
The subject of this book is a new model of fundamental physics.

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The current model is far from complete.
It will never reach completion.
The human brain is not capable to catch the full extent of the complexity of physical reality.

## Preface

I started the Hilbert Book Model project during my studies in physics in the sixties on the Technical University of Eindhoven (TUE).
In the first two years the lectures concerned only classical physics. In the third year quantum physics was introduced. I had great difficulty in understanding why the methodology of doing physics changed drastically. So I went to the teacher, which was an old nearly retired and very wise professor and asked him:
"Why is quantum mechanics done so differently from classical mechanics?".
His answer was short. He stated":
"The reason is that quantum mechanics is based on the superposition principle".

I quickly realized that this was part of the methodology and could not be the reason of the difference in methodology'. So I went back and told him my concern. He told me that he could not give me a better answer and if I wanted a useful answer I should research that myself. So, I first went to the library, but the university was quite new and its library only contained rather old sec-

[^0]ond hand books, which they got as a gift from other institutions. Next I went to the city's book shops. I finally found a booklet from P. Mittelstaedt: (Philosophische Probleme der modernen Physik, BI Hochschultaschenbücher, Band 50, 1963) that contained a chapter on quantum logic.
It learned me that small particles appear to obey a kind of logic that differs from classical logic. As a result their dynamic behavior differs from the behavior of larger objects, which obey classical logic. I concluded that this produced the answer that I was looking for. I searched further and encountered papers from Garret Birkhoff and John von Neumann that explained the correspondence between quantum logic and separable Hilbert spaces. That produced a more conclusive answer to my question.
Retrospectively my old professor was partly right. On its own, quantum logic does not provide the answer. The superposition principle also plays a crucial role.

The lectures also told me that observables were related to eigenvalues of Hermitian operators. These eigenvalues are real numbers. However, it was clearly visible that nature has a $3+1 D$ structure. So I tried to solve that discrepancy as well. After a few days of puzzling I discovered a number system that had this 3+1D structure and I called them compound numbers. I went back to
my professor and asked him why such compound numbers were not used in physics. Again he could not give a reasonable answer.
When I asked the same question to a much younger assistant professor he told me that these numbers were discovered more than a century earlier by William Rowan Hamilton when he was walking with his wife over a bridge in Dublin. He was so glad about his discovery that he carved the formula that treats the multiplication of these numbers into the sidewall of the bridge. The inscription has faded away, but it is now molded in bronze and fixed to the same wall by Hamilton's students.


The numbers are known as quaternions. So, I went to the library and searched for papers on quaternions. In those years Constantin Piron wrote his papers on the number systems that can be used by Hilbert spaces. Piron discovered that only members of suitable division rings can be used as coefficients in linear combinations of Hilbert vectors in separable Hilbert spaces. Later this was affirmed more thoroughly by Maria Pia Solèr. Division rings comprise real numbers, complex numbers and quaternions. That information completed my insight in this subject.
I finalized my physics study with an internal paper on quaternionic Hilbert spaces.

The university was specialized in applied physics and not in theoretical physics. This did not stimulate me to proceed with the subject. Next, I went into a career in industry where I used my knowledge of physics in helping to analyze intensified imaging and in assisting with the design of night vision equipment and X-ray image intensifiers. That put me with my nose on the notion of quanta.
The output window of image intensifiers did not show impinging radiation waves. Instead they showed clouds of impinging quanta. In those times I had not much opportunity to deliberate on that fact. However, after my retirement I started to rethink the matter. That was the instant that the Hilbert Book Model was revived.

In 2009 at the age of 68 I restarted the Hilbert Book Model project. The HBM is a very simple model of physics that is completely deduced and only covers the lowest levels of fundamental physics. For that reason it is strictly based on a solid foundation. For that foundation I choose the lattice structure of traditional quantum logic. The lattice structure of this logic system is isomorphic to the lattice structure of the set of closed subspaces of a separable Hilbert space.
Since neither the logic system nor the Hilbert space can represent dynamics, a full dynamic model is based on an ordered sequence of such static sub-models. This sequence shows great similarity with the set of pages of a book. This has led to the name "Hilbert Book Model"

Thus, in a few words: The Hilbert Book Model tries to explain the existence of quanta. It does that by starting from traditional quantum logic.

During deliberations in discussion groups on the internet I encountered great resistance against largely or completely deduced theories and models of physics. The Hilbert Book Model is completely deduced.
For that reason I reformatted the project in the form of a game. The game starts at well selected first principles. From this foundation a model is generated. That model
is not presented as a model of physics. However, the target of the game is a model that shows many features and phenomena that resemble features and phenomena that we know from our knowledge that we gathered by observing physical reality. When that target is reached, then it can be discussed whether the produced model represents a useful theory of physics.

You will find the achieved model to be in many aspects controversial and non-conventional. That is why the author took great efforts in order to keep the model selfconsistent.

Due to a series of new concepts that are introduced by the HBM and the fact that they lend themselves for a rather pictorial description, will physicists that support conventional physics experience the HBM as a kind of Alice's wonderland. In this manuscript, all of these new concepts will be introduced in a cautious and trustworthy manner. The methodology will directly or indirectly base on the selected foundation.

Some readers have criticized me for lack of formulas, because one formula can say more than a thousand words. Restating formulas that you can find in any physical textbook is not the purpose of this book. On the other hand this manuscript contains formulas that you will not find elsewhere. An important example is the
coupling equation. Another example is the definition of the blurred allocation function. For those who are interested in related formulas the e-paper Q-FORMULÆ contains formulas that are difficult to find in literature.

The main purpose of the Hilbert Book Model is to get insight into the possibilities of the physical toolkit.

Each time that I read this book I encounter small and sometimes big inconsistencies. When I see them I repair them. Due to my sloppy nature there must still be a lot of them left. I apologize to the reader for this inconvenience. I do not consider myself a good and precise mathematician and I consider myself as a horrible physicist. The Great Creator must be a lot better. For a better manuscript you better invite Him. He constructed this structure.

出

If a mathematical theory is self-consistent, then there is a realistic chance that nature somewhere somehow uses it.

If that theory is compatible with traditional quantum logic, then there is a much larger chance that nature will use it.

This drives my intuition.

This manuscript does not offer another physical reality. The Hilbert Book Model offers an alternative view on physical reality.

That view differs from the view that is offered by contemporary physics.

In this way the manuscript can offer new insights.

No model of physics can change physical reality. Any view on physical reality involves a model. Drastically different models can still be consistent in themselves.

The Hilbert Book Model is a simple self-consistent model. This model steps with universe-wide progression steps from one submodel to the next one. Each of these sub-models represents a static status quo of its universe. The sub-models are strictly based on traditional quantum logic

The HBM is a pure quaternion based model.
Conventional physics is spacetime based and uses complex numbers.
When both models are compared, then the progression quantity
(which represents the page number in the Hilbert Book model) corresponds to proper time in conventional physics.

In the HBM all proper time clocks are synchronized. The length of a smallest quaternionic space-progression step in the HBM corresponds to an "infinitesimal" observer's time step in conventional physics.

HvL

Ir J.A.J. van Leunen

## The Hilbert Book Model <br> GAME

## Acknowledgements

I thank my wife Albertine, who tolerated me to work days and nights on a subject that can only be fully comprehended by experts in this field. For several years she had to share me with my text processor. She stimulated me to bring this project to a feasible temporary end, because this project is in fact a never ending story.

I also have to thank my friends and discussion partners that listened to my lengthy deliberations on this non society chitchat suitable subject and patiently tolerated that my insights changed regularly.

## Details

The Hilbert Book Model is the result of a still ongoing research project.
That project started in 2009.
The continuing status of the project can be followed at http://www.e-physics.eu

The author's e-print site is:
http://vixra.org/author/j_a_j_van_leunen.
This book is accompanied by a slide show at The HBM_Game
Use a PowerPoint viewer for this .pptx file Or the corresponding YouTube video The_Hilbert_Book_Model_Game

The nice thing about laws of physics is that they repeat themselves. Otherwise they would not be noticed. The task of physicists is to notice the repetition.

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## 1 The game

The Hilbert Book Model project is shaped as an instance of a category of modelling games. This category is characterized by the fact that the game starts with well selected first principles. This foundation is expanded by a trustworthy deduction process such that finally a model is obtained that shows many features and phenomena that we know from physical reality.

The model is shaped in the form of a game because most physicists mistrust completely or largely deduced models.
A completely deduced model endangers to become the victim of the extreme fantasy of the model designer. That is why many physicist are suspicious about completely or largely deduced models and require experimental verification of every relevant physical statement. Often they do not even consider deduced models as a valid scientific approach.

This hampers the serious designers of completely deduced models, because these models withdraw from experimental verification.
One of the counter measures is to base the deduced model on a well-accepted foundation or on smartly selected first principles.

By pretending that we play a game, we cheat the criticizers. We do not call the project a model of physics. Instead we configure the project as a modeling game. At the advance of the game the model is not considered as a model of physics. If the game succeeds, then the final model will resemble physical reality and that model can be considered as a model of physics.

However, one of the main problems will be the prove of the isomorphism between the aspects of the model and corresponding aspects of physical reality.
In fact contemporary physics encounters the same problem. In contemporary physics this inconsistency is ignored. Only philosophers bother about this mapping problem.

Thus, we postpone the discussion whether we produced a model of physics to the end of the game. If the newly generated model uncovers new science, then it has reached its purpose.

## 2 First principles

### 2.1 Model creation

The creator of the model must first resolve his most acute problem. He is confronted with a very complicated task and he must take measures that this complexity stays within manageable bounds. He wants to create a universe that resembles physical reality. If he cannot keep complexity within reasonable bounds, then dynamic chaos will result.

### 2.1.1 What determines complexity?

Complexity depends on the number and the diversity of the relations that exist between objects that play a role in the considered system
Potential complexity of a set of objects is a measure that is defined by the number of potential relations that exist between the members of that set.
If there are n elements in the set, then there exist $\mathrm{n}(\mathrm{n}-1)$ potential relations.
Actual complexity of a set of objects is a measure that is defined by the number of relevant relations that exist between the members of the set.
Relational complexity is the ratio of the number of relevant relations divided by the number of potential relations.

### 2.2 Rules restrict complexity

Physical reality appears to apply restricting rules for relational structures that it accepts. These rules intent to reduce the complexity of these relational structures. The gamer must solve the same basic problems that physical reality encounters. As a consequence he must install similar rules.

### 2.2.1 Modularization

Modularization is a very powerful influencer. Together with the corresponding encapsulation it reduces the relational complexity of the ensemble of objects on which modularization works. The encapsulation keeps most relations internal to the module.

### 2.2.1.1 The basic modularization law

The model is completely deduced. For that reason the model is founded on well selected first principles. In the Hilbert Book Model Game, these first principles are taken from the definition of traditional quantum logic. Quantum logic can be considered as a logic system, but it appears that for application in model generation it can better be considered as part of a recipe for modular construction.

Quantum logic represents a skeleton relational structure that acts as a skeleton for modular construction. In this usage the elements of the skeleton relational structure are not propositions. Instead these elements are building blocks and composites of building blocks.
The HBM keeps the name quantum logic in order to honor its discoverers Garret Birkhoff and John von Neumann that gave the skeleton relational structure its

> All discrete constructs in physical reality are supposed to expose the skeleton relational structure that is defined by quantum logic

name. We can phrase a first version of the basic modularization law as follows:
For this reason, the axioms that define the selected skeleton relational structure of quantum logic are used as the first principles on which the Hilbert Book Model Game will be based. Together these axioms define part of the basic modularization law.
On itself the skeleton relational structure does not provide a recipe for modular construction. A mechanism that binds building blocks into composites and that encapsulates relations that are internal to the composites must complete the recipe.

Specifying a suitable skeleton relational structure and adding the mechanism can drastically reduce the complexity of a model. The HBM selects quantum logic as its skeleton relational structure.
It may not immediately become clear that this structure installs a modular way of building systems and subsystems. For example it is not directly clear how this structure binds building blocks into composites and how it encapsulates relations inside these composites.

These capabilities of the skeleton relational structure become more apparent when the lattice isomorphism between the elements of quantum logic and the set of closed subspaces of an infinite dimensional separable Hilbert space is considered. Via this isomorphism the superposition principle enters the model. The superposition principle is the driving force behind binding building blocks into composites and behind encapsulating relations within that composite.
How this is achieved is a delicate process that will be explained in the course of the development of the model. Here we just state that superposition implements the required functionality.

The full basic modularization law can be formulated in the realm of this Hilbert space.

> At any progression instant, all discrete constructs in physical reality can be represented by a closed subspace of a single separable Hilbert space

The full basic modularization law only concerns discrete objects and its describes the situation in a static status quo of the universe.

So now we have a two stage model. The first stage delivers the skeleton relational structure and the second stage adds the superposition principle. However, the two stage model only describes a static status quo and it does not describe continuums.


Further the second law talks in terms of subspaces. These subspaces are spanned by Hilbert vectors. So, the subspaces contain more detail than the elements of quantum logic.

The first version of our model is very simple and needs extension. Especially the Hilbert vectors that span the subspaces need interpretation. These Hilbert vectors can take the role of eigenvectors of linear operators. What will be the role of the corresponding eigenvalues? They represent the way that geometry emerges into the model!

Quantum logic and the lattice isomorphism with a separable Hilbert space was uncovered in a paper written by Garret Birkhoff and John von Neumann.

The number systems that can be used by the separable Hilbert space are confined to members of a suitable division ring. This was already mentioned in the 1936 paper of Birkhoff and von Neumann and was affirmed by Constantin Piron and Maria Pia Solèr in the sixties. It means that the only suitable number systems are real numbers, complex numbers and quaternions. The eigenvalues of the linear operators introduce geometrical aspects into the model.

First we will study the skeleton relational structures and the separable Hilbert spaces.

## 3 Skeleton relational structures

### 3.1 Quantum logic

Elementary particles behave non-classical. They can present themselves either as a particle or as a wave. A measurement of the particle properties of the object destroys the information that was obtained from an earlier measurement of the wave properties of that object. With elementary particles it becomes clear that that nature obeys a different logic than our old trusted classical logic. The difference resides in the modularity axiom. That axiom is weakened. Classical logic is congruent to an orthocomplemented modular lattice.
Quantum logic is congruent to an orthocomplemented weakly modular lattice. Another name for that lattice is orthomodular lattice.

### 3.1.1 Lattices

A subset of the axioms of the logic characterizes it as a half ordered set. A larger subset defines it as a lattice. A lattice is a set of elements $a, b, c$, ...that is closed for the connections $\cap$ and $\mathbf{U}$.
$\cap$ is called conjunction ${ }^{2}$.


U is called disjunction.


These connections obey:

- The set is partially ordered. With each pair of elements $a, b$ belongs an element $c$, such that $a \subset$ $c$ and $b \subset c$.
- The set is a $\cap$ half lattice if with each pair of elements $a, b$ an element $c$ exists, such that $c=$ $a \cap b$.

[^1]- The set is a $\mathbf{U}$ half lattice if with each pair of elements $a, b$ an element $c$ exists, such that $c=$ $a \cup b$.
- The set is a lattice if it is both a $\cap$ half lattice and a $\mathbf{U}$ half lattice.

The following relations hold in a lattice:

$$
\begin{align*}
& a \cap b=b \cap a  \tag{1}\\
& (a \cap b) \cap c=a \cap(b \cap c)  \tag{2}\\
& a \cap(a \cup b)=a  \tag{3}\\
& a \cup b=b \cup a  \tag{4}\\
& (a \cup b) \cup c=a \cup(b \cup c)  \tag{5}\\
& a \cup(a \cap b)=a \tag{6}
\end{align*}
$$

The lattice has a partial order inclusion $\subset$ :

$$
\begin{equation*}
\mathrm{a} \subset \mathrm{~b} \Leftrightarrow \mathrm{a} \subset \mathrm{~b}=\mathrm{a} \tag{7}
\end{equation*}
$$

A complementary lattice contains two elements $n$ and $e$ with each element a an complementary element a' such that:

$$
\begin{align*}
& a \cap a^{\prime}=n  \tag{8}\\
& a \cap n=n  \tag{9}\\
& a \cap e=a  \tag{10}\\
& a \cup a^{\prime}=e  \tag{11}\\
& a \cup e=e  \tag{12}\\
& a \cup n=a \tag{11}
\end{align*}
$$

$e$ is the unity element; $n$ is the null element of the lattice

An orthocomplemented lattice contains two elements $n$ and $e$ and with each element $a$ an element $a$ " such that:

$$
\begin{align*}
& a \cup a^{\prime \prime}=e  \tag{14}\\
& a \cap a^{\prime \prime}=n  \tag{15}\\
& \left(a^{\prime \prime}\right)^{\prime \prime}=a  \tag{1}\\
& a \subset b \Leftrightarrow b^{\prime \prime} \subset a^{\prime \prime} \tag{17}
\end{align*}
$$

A distributive lattice supports the distributive laws:

$$
\begin{align*}
& a \cap(b \cup c)=(a \cap b) \cup(a \cap c)  \tag{18}\\
& a \cup(b \cap c)=(a \cup b) \cap(a \cup c) \tag{19}
\end{align*}
$$

A modular lattice supports:

$$
\begin{align*}
(a \cap b) \cup & (a \cap c)  \tag{20}\\
& =a \cap(b \cup(a \cap c))
\end{align*}
$$

A weak modular lattice supports instead:
There exists an element $d$ such that

$$
\begin{align*}
a & \subset c \Leftrightarrow(a \cup b) \cap c  \tag{21}\\
& =a \cup(b \cap c) \cup(d \cap c)
\end{align*}
$$

where $d$ obeys:

$$
\begin{align*}
& (a \cup b) \cap d=d  \tag{22}\\
& a \cap d=n  \tag{23}\\
& b \cap d=n  \tag{24}\\
& {[(a \subset g) \text { and }(b \subset g) \Leftrightarrow d \subset g} \tag{25}
\end{align*}
$$

In an atomic lattice holds

$$
\begin{align*}
& \exists_{p \in L} \forall_{x \in L}\{x \subset p \Rightarrow x=n\}  \tag{26}\\
& \forall_{a \in L} \forall_{x \in L}\{(a<x<a \cap p)  \tag{27}\\
& \quad \Rightarrow(x=a \text { or } x=a \cap p)\}
\end{align*}
$$

$p$ is an atom
Both the set of elements of quantum logic and the set of subspaces of a separable Hilbert space $\mathbf{H}$ have the structure of an orthomodular lattice. In this respect these sets are congruent.
In Hilbert space, an atom is a pure state (a ray spanned by a single vector).

Classical logic has the structure of an orthocomplemented distributive modular and atomic lattice.

Quantum logic has the structure of an orthomodular lattice. That is an orthocomplented weakly modular and atomic lattice.

The set of closed subspaces of a Hilbert space also has that structure.

### 3.1.2 Lattice elements

Lattice elements can, but must not be propositions. In logic systems the elements are considered as propositions. This is why the name quantum logic has confused many physicists. For the purpose of model generation the elements of this structure can better be interpreted as modular construction elements.

Thus quantum logic is an exception. At has a treacherous name. In this skeleton relational structure the elements can be interpreted as building blocks and composites of building blocks. With this interpretation the skeleton relational structure becomes a recipe for modular construction.

### 3.1.2.1 Propositions

In Aristotelian logic a proposition is a particular kind of sentence, one which affirms or denies a predicate of a subject. Propositions have binary values. They are either true or they are false.
Propositions take forms like "This is a particle or a wave". In mathematical logic, propositions, also called "propositional formulas" or "statement forms", are statements that do not contain quantifiers. They are composed of well-formed formulas consisting entirely of
atomic formulas, the five logical connectives ${ }^{3}$, and symbols of grouping (parentheses etc.). Propositional logic is one of the few areas of mathematics that is totally solved, in the sense that it has been proven internally consistent, every theorem is true, and every true statement can be proved. Predicate logic is an extension of propositional logic, which adds variables and quantifiers.
Predicates may accept attributes and quantifiers. The predicate logic is also called first order logic. A dynamic logic can handle the fact that predicates may influence each other when atomic predicates are exchanged.

### 3.2 Hilbert space

The set of closed subspaces of an infinite dimensional separable Hilbert space is lattice isomorphic with the set of elements of a quantum logic system.

### 3.3 Hilbert logic

The set of elements of traditional quantum logic is lattice isomorphic with the set of closed subspaces of a separable Hilbert space. However there exist still significant differences between this logic system and the Hilbert space.

[^2]This gap can be closed by refining the specification of quantum logic.

Step 1: Add to each element as an extra attribute a numeric value that gets the name relevance factor.
Step 2: Require that linear combinations of atomic elements also belong to the logic system.
Step 3: Introduce the notion of a relational coupling measure between two linear elements. This measure has properties that are similar to the properties of the inner product of Hilbert space vectors.
Step 4: Close the subsets of the new logic system with respect to this relational coupling measure.

The relevance factor and the relational coupling measure can have values that are taken from a suitable division ring ${ }^{4}$. The resulting logic system will be called Hilbert logic.

### 3.4 Similarity with Hilbert space

The addition of the relevance factor installs the superposition principle. A linear combination of linear element is again a linear element.

[^3]In this way the Hilbert logic is lattice isomorphic as well topological isomorphic with the corresponding Hilbert space.
Due to this similarity the Hilbert logic will also feature linear operators.
In a Hilbert logic linear operators can be defined that have linear atoms as their eigen-elements. The eigenspace of these operators is countable. The eigenvalues are numbers that introduce geometry into the model. Linear elements are the equivalents of Hilbert vectors. General basic modularization structure elements are the equivalents of (closed) subspaces of a Hilbert space. The measure of the relational coupling between two linear elements is the equivalent of the inner product between two Hilbert vectors.

### 3.5 Free elements

The relevance factor can be used to define the notion of free elements.
If the modulus of the relevance factor is maximized at a fixed value, for example unity, then a free element can be defined as an element for which this maximum is reached. It means that free elements cannot be considered as members of a superposition. On the other hand free elements can be superpositions of bounded elements.
When used in this way the relevance factor takes the role of a probability amplitude.

### 3.6 Binding building blocks and encapsulating their relations.

The superposition principle is the driving force behind the binding of building blocks into composites. Composites are construction elements that can be written as linear combinations of more basic building blocks. Thus, the constituents of composites are not free elements. Instead they are bounded elements. The constituents lose their individuality and the relations in which these constituents play a role become less apparent. With other words, the superposition principle installs the relational encapsulation of the constituents of the composite.

### 3.7 Restrictions

The skeleton relational structures can only model countable sets of discrete elements. The structures offer no means for modeling continuums.
The axioms that define these structures specify relations between the elements. These axioms do not specify the content of the elements.
The axioms do not provide a means to implement dynamics. The skeleton relational structures can only model a static status quo.
These restrictions also hold for separable Hilbert spaces.

## 4 Modelling continuums

In the sixties Gelfand introduced a way to model continuums via an extension of the separable Hilbert space via a so called Gelfand triple. The Gelfand triple often gets the name rigged Hilbert space, which is confusing, because this construct is not a Hilbert space.
The Gelfand triple offers linear operators that have continuums as eigenspaces. The restrictions that hold for the number systems that can be applied for the separable Hilbert space, also hold for the Gelfand triple.


In this stage the model introduces geometry via the eigenspaces of linear operators and it offers the possibility to embed discrete objects in a continuum.
In this phase the model cannot implement dynamics. Still, like the separable Hilbert space, the Gelfand triple can only represent a static status quo.

### 4.1 Hilbert space and its Gelfand triple

Start with a separable Hilbert space that contains a normal operator that uses all rational numbers of a selected division ring as its eigenvalues.
Take the set of eigenvectors of this operator. Now define vectors that are linear combinations of these eigenvectors and that have a norm that is equal to unity. Next take series of such constructs that converge to sets in which all pairs of vectors have inner products, which approach to unity.
Finally close this space such that all limits of the series are included.
Now the space includes vectors that are eigenvectors with eigenvectors that are limits of converging series of rational numbers. Thus all real numbers have an eigenvector in the new construct.
Call this new construct the Gelfand triple of the original separable Hilbert space.

The resulting construct is a vector space that possesses continuum vector spaces.

The construct is the Gelfand triple of the Hilbert space. Sometimes it is falsely called a rigged Hilbert space ${ }^{5}$.

The selected operator in the Hilbert space and the corresponding operator in the Gelfand triple are each other's brothers. The eigenvectors of the operator in the Gelfand triple that have rational eigenvalues have a one to one correspondence to the eigenvectors of the brother operator in the separable Hilbert space. Later we use the selected operators as reference operators that deliver flat parameter spaces.

The Hilbert Book Model uses the eigenspaces of these operators as parameter spaces.

[^4]
## 5 Modelling dynamics

From observations we know that we live in a dynamical environment. Thus the target model must implement dynamics.

A very simple and efficient way to convert an essentially static model into a dynamic model is the conversion of the model into an ordered sequence of the static sub-models. These static sub-models must slightly differ from each other such that sufficient coherence exists between subsequent members of the sequence, otherwise dynamical chaos will result. On the other hand the coherence must not be too stiff, otherwise no dynamics will take place.

The static sub-models offer no built-in support for ensuring this coherence. This means that the model must be extended with an external mechanism that takes care of the coherence between subsequent steps. This mechanism acts as a kind of operating system.

The resulting model proceeds with model wide steps from each static status quo to the next static status quo. As a consequence a model wide clock resides in the model that ticks with the highest frequency that can oc-
cur in the model. The clock tick corresponds to a progression step and its "reading" represents a progression parameter.

The resulting model looks like the set of pages of a book where the progression parameter plays the role of the page number. Each page describes a static status quo of the geometry of the model. The countable eigenspaces of linear operators in the separable Hilbert spaces and the continuum eigenspaces of linear operators in the Gelfand triple act as storage places for geometrical data.

At each progression step the separable Hilbert space is regenerated. The Gelfand triple and its operators may survive this regeneration step. This means that progression steps along separable Hilbert spaces and flows in the Gelfand triple. This fact is used by the operating system in order to establish the required dynamical coherence between subsequent progression steps.

### 5.1 Paginated space progression model

The resulting model is a paginated space-progression model. Here space is represented by eigenspaces of a series of linear operators in the Gelfand triple. Another eigenspace acts as a static and flat parameter space that is formed by a quaternionic number system.

This space-progression model features a Euclidean signature. Progression cannot be measured. It proceeds so fast that no observable readings can be provided. In the paginated space progression model the observer does not play an essential role.

### 5.2 Spacetime model

At this stage the model deviates significantly from the spacetime model that is used in contemporary physics. The spacetime model features a Minkowski signature, while the paginated model features a Euclidean signature.
The difference between the spacetime model and the paginated space-progression model is located in the role of the observer. In the spacetime model the observer plays an essential role.
The progression parameter of the paginated space progression model has much in common with the proper time concept in the space time model. Proper time is ticking on a virtual clock that ticks at the location of the observed item. In the paginated space-progression model all proper time clocks are synchronized. In the spacetime model proper time may be measured by a suitable clock that is located at the position of the observer.
The spacetime model offers another notion of time. This is the coordinate time. Usually coordinate time is measured using a clock at the location of the observer. This
makes coordinate time an easily observable value. The virtual proper time at the location of the observed item is deduced from this measurement and the knowledge about the information path that runs from the observed item to the observer. The value includes the clock ticks that pass during the travel of the information from the observed item until it reaches the observer. That travel is supposed to occur with the highest possible speed. Thus, coordinate time is a mixed concept. It includes the information path that runs from observed item to the observer. In arbitrary curved space this path is not exactly known.

### 5.3 The Hilbert Book Model

Due to the great resemblance of the members of the sequence with the pages of a book and its relation with Hilbert spaces, the model is baptized "The Hilbert Book Model".

## 6 The embedding process

### 6.1 Embedding a building block

The members of the sequence are recreated at every progression step. Thus the embedding of a building block is a recurrent operation. Since the subsequent elements of the sequence must not differ much, the embedding will differ not much of the same embedding in the previous member. For example the location of the embedding must be nearly the same. After a series of progression steps, the locations used by the embedding of the considered building block form a swarm. This swarm forms a coherent set of used locations.

The mechanism that takes care of the dynamic coherence uses the density distribution of the swarm in order to regulate this coherence. This means that this density distribution is a continuous function. Since the locations are points in a continuum the location density distribution can also be characterized as a probability density distribution. That probability density distribution is a continuous function that defines the probability of finding the building block at the location that is given by the value of the parameter of the building block.

### 6.2 Swarming conditions

It is suggested here that the mechanism, which installs dynamic coherence installs four swarming conditions.

1. The locations that are used by the embedding of a single building block form a coherent set.
2. The set is characterized by continuous location density function
3. The set can also be interpreted as a probability density distribution
4. The probability density distribution has a Fourier transform
The reason for this suggestion is the great resemblance of the probability density distribution of the building block with the squared modulus of the wave function of elementary particles.

The fourth condition has some strong consequences.

1. The fact that the swarm has a Fourier transform means that it owns a displacement generator.
2. The fact that the swarm owns a displacement generator means that at first approximation the swarm moves as one unit
3. The fact that the probability density distribution has a Fourier transform means
that it can be interpreted as a wave package
4. As a consequence interfering swarms can form detection patterns in the form of interference patterns.

We add the following suggestions:

1. The swarm is prepared in advance
2. This preparation is done on a flat (noncurved) continuum.
3. The planned swarm contains a fixed number of potential locations.
4. After preparation the planned swarm is mapped onto the actual (eventually curved) continuum
5. At each progression step, after the mapping onto the target continuum, via random choice, a single location is selected as the current location.
6. Each location in the planned swarm is used only once.
7. After using all locations a new planned swarm is generated
8. The planned swarm characterizes the type of the building block

### 6.3 Micro-path

The hopping of the building block along the locations that are selected from the swarm can be interpreted as the hopping along a stochastic micro-path.
This hopping also occurs when the swarm stays at the same location.
If the swarm moves, then the micro-path stretches along the movement path of the swarm.
If the swarm oscillates with small enough amplitudes, then the oscillation will be drowned in the micro-path.
The micro-path may implement a quasi-rotation.
The micro-path may be walked in one of two directions.
We add the following suggestions:

1. Like the swarm the micro-path contains a fixed number of hops.
2. The micro-path is not a closed loop.
3. Thus, at each cycle the swarm has taken a small step.

As a consequence the non-moving swarms jitter.

## 7 The mapping process

The embedding of a building block can be considered as a mapping process. First the operating system prepares a planned swarm in a flat continuum. Next this planned swarm is mapped onto the target continuum, which is curved. At each progression step one location is selected via a random choice out of the mapped swarm. At this location the building block resides during this progression step.
The preparation of the planned swarm can be considered as a stochastic process, that consists of two separate stochastic processes. The first process is a Poisson process that produces a germ location. The second process is a binomial process that is implemented by a 3D spread function whose parameter is formed by the germ. We will call the combination of the two stochastic processes a stochastic spatial spread function.
The result of the stochastic spatial spread function is mapped onto the target continuum. This map can be implemented by a continuous quaternionic allocation function that has a flat parameter space that is spanned by a quaternionic number system.
This quaternionic function defines the target embedding continuum.
The total map can be considered as the convolution of the stochastic spatial spread function and the continuous quaternionic allocation function.

### 7.1 Mapping quality

The Fourier transform of the location density distribution that describes the planned swarm acts as a quality characteristic of the mapping process that is installed by the mechanism that controls the dynamic coherence. This quality characteristic works together with the Optical Transfer Function that acts as quality characteristic of linear imaging equipment.
It also corresponds to the frequency characteristic that characterizes the quality of linear operating communication equipment

## 8 Quaternions and quaternionic functions

### 8.1 Number systems

The numbers that can be used as coefficients in superpositions and as eigenvalues of linear operators in separable Hilbert spaces and their Gelfand triple must be elements of a suitable division ring. Only three suitable division rings exist:

- The real numbers
- The complex numbers
- The quaternions

The complex numbers comprise the real numbers and the quaternions comprise the complex numbers. Thus the quaternions form the most elaborate choice. An N -dimensional number system exist in $2^{\mathrm{N}}$ versions that only differ in their discrete symmetries. This means that the quaternionic number system exist in 16 symmetry flavors.

### 8.2 Quaternions

### 8.2.1 Quaternion symmetry flavors

Quaternionic number systems exist in 16 versions that differ in their symmetry flavor

- If the real part is ignored, then still $\mathbf{8}$ symmetry flavors result
- They are marked by special indices $\boldsymbol{a}^{(4)}$
- $\boldsymbol{a}^{(0}$ is the reference symmetry flavor
- They are also marked by colors $N, R, G, B, \bar{B}, \bar{G}, \bar{R}, \bar{N}$
- Half of them is right handed $\mathbf{R}$,
- The other half is left handed $\mathbf{L}$

| H1 | $\boldsymbol{a}^{(0)}{ }_{N}$ |
| :---: | :---: |
| H | $\boldsymbol{a}^{(1)} R \boldsymbol{L}$ |
| H1H | $a^{(2)} G L$ |
| H | $a^{(3)} B L$ |
| HII | $\boldsymbol{a}^{4}{ }^{(4)} \boldsymbol{R}$ |
| H15 | $\boldsymbol{a}^{(5)} \bar{G} \boldsymbol{R}$ |
| H | $a^{(6)} \bar{R} \boldsymbol{R}$ |
| III | $a^{(7)} W_{L}$ |

### 8.2.2 Quaternion geometry and arithmetic

Quaternions can be considered as the combination of a real scalar and a 3D vector that has real coefficients. This vector forms the imaginary part of the quaternion. Quaternionic number systems are division rings. Biquaternions exist whose imaginary part exists of a 3D vector that has complex coefficients. Biquaternions do not form division rings.

### 8.2.2.1 Notation

We indicate the real part of quaternion $a$ by the suffix $a_{0}$.
We indicate the imaginary part of quaternion $a$ by bold face $\boldsymbol{a}$.

$$
\begin{equation*}
a=a_{0}+\boldsymbol{a} \tag{1}
\end{equation*}
$$

### 8.2.2.2 Sum

$$
\begin{align*}
& c=c_{0}+\boldsymbol{c}=a+b  \tag{1}\\
& c_{0}=a_{0}+b_{0}  \tag{2}\\
& \boldsymbol{c}=\boldsymbol{a}+\boldsymbol{b} \tag{3}
\end{align*}
$$

### 8.2.2.3 Product

$$
\begin{equation*}
f=f_{0}+\boldsymbol{f}=d e \tag{1}
\end{equation*}
$$

$$
\begin{align*}
& f_{0}=d_{0} e_{0}-\langle\boldsymbol{d}, \boldsymbol{e}\rangle  \tag{2}\\
& \boldsymbol{f}=d_{0} \boldsymbol{e}+e_{0} \boldsymbol{d}+\boldsymbol{d} \times \boldsymbol{e} \tag{3}
\end{align*}
$$

8.2.2.4 Norm

$$
\begin{equation*}
|a|=\sqrt{a_{0} a_{0}+\langle\boldsymbol{a}, \boldsymbol{a}\rangle} \tag{1}
\end{equation*}
$$

### 8.3 Discrete quaternionic distributions

A distribution of quaternionic geometric location and displacement data can be stored in a discrete quaternionic distribution.
The locations are stored in the real part of the discrete quaternionic distribution and the displacements are stored in the imaginary part of the discrete quaternionic distribution.

Swarms are candidates for such descriptors. The locations and the corresponding hops form the elements of the discrete quaternionic distribution and progression acts as an enumerator of these elements. Together with this enumerator, the discrete quaternionic distribution defines a path.

If the discrete data belong to the same symmetry flavor and the density of these discrete data forms a sufficiently smooth function, then the discrete distribution can be described by a continuous quaternionic density distribution.

Due to the fact that the locations are points, the real part of this continuous density distribution can be considered as a probability density distribution.

### 8.4 Quaternionic functions

### 8.4.1 Quaternionic function symmetry flavors

Continuous quaternionic functions do not switch to other symmetry flavors. Thus they also exist in 16 versions that only differ in their symmetry flavor

- If the real part is ignored, then still 8 symmetry flavors result
- They are marked by special indices $\boldsymbol{\psi}^{(4)}$
- $\boldsymbol{\psi}^{(0}$ is the reference symmetry flavor
- They are also marked by colors $N, R, G, B, \bar{B}, \bar{G}, \bar{R}, \bar{N}$
- Half of them is right handed $\mathbf{R}$,
- The other half is left handed $\mathbf{L}$

| $\cdots \boldsymbol{\psi}^{(0)}{ }_{N} \boldsymbol{R}$ |
| :---: |
| $\cdots \boldsymbol{\psi}^{(1)} R \boldsymbol{L}$ |
| $\cdots \boldsymbol{\psi}^{(2)} G \boldsymbol{L}$ |
| $\square \boldsymbol{\psi}^{(3)} B \boldsymbol{L}$ |
| $\cdots \boldsymbol{\psi}^{(4)} \bar{B} \boldsymbol{R}$ |
| $\cdots \boldsymbol{\psi}^{(5)} \bar{G} \boldsymbol{R}$ |
| $\cdots \boldsymbol{\psi}^{(6)} \bar{R} \boldsymbol{R}$ |
| $\cdots \boldsymbol{\psi}^{(7)} W \boldsymbol{L}$ |

### 8.4.1.1 Symmetry flavor bundle

Mostly continuous functions are functions that are continuous apart from a finite number of singular points. Mostly continuous quaternionic functions exist in 16 different symmetry flavors.
Mostly continuous quaternionic functions exist in bundles that contains all symmetry flavors of that function Such bundles are called symmetry flavor bundles.

### 8.4.2 Algebra

We use similar notations as for quaternionic numbers. Without further notice, we suppose that the parameter space is formed by a selected quaternionic number system.

### 8.4.3 Notation

$$
\begin{equation*}
g(q)=g_{0}(q)+\boldsymbol{g}(q) \tag{1}
\end{equation*}
$$

$g$ is a quaternionic function.
$g_{0}$ is a real scalar function.
$\boldsymbol{g}$ is a real vector function. It is the imaginary part of $g$.
$q$ is a quaternion that resides in the parameter space.

$$
\begin{align*}
q= & \{\tau, x, y, z\}=\left\{q_{0}, q_{1}, q_{2}, q_{3}\right\}  \tag{2}\\
& =q_{0}+\boldsymbol{q}=q_{0}+q_{1} \boldsymbol{I}+q_{2} \boldsymbol{J}+q_{3} \boldsymbol{K}
\end{align*}
$$

If $g$ is differentiable then the quaternionic nabla $\nabla g$ of $g$ exists.
The quaternionic nabla $\nabla$ is a shorthand for $\nabla_{0}+\nabla$

$$
\begin{align*}
& \nabla_{0}=\frac{\partial}{\partial \tau}  \tag{3}\\
& \boldsymbol{\nabla}=\left\{\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right\} \tag{4}
\end{align*}
$$

$$
\begin{align*}
& h=h_{0}+\boldsymbol{h}=\nabla g  \tag{4}\\
& h_{0}=\nabla_{0} g_{0}-\langle\boldsymbol{\nabla}, \boldsymbol{g}\rangle  \tag{5}\\
& \boldsymbol{h}=\nabla_{0} \boldsymbol{g}+\boldsymbol{\nabla} g_{0}+\boldsymbol{\nabla} \times \boldsymbol{g} \tag{6}
\end{align*}
$$

### 8.4.4 Norm

Square-integrable functions are normalizable. The norm is defined by:

$$
\begin{align*}
& \|\psi\|=\int_{V}|\psi|^{2} d V  \tag{1}\\
& =\int_{V}\left\{\left|\psi_{0}\right|^{2}+|\boldsymbol{\psi}|^{2}\right\} d V \\
& =\left\|\psi_{0}\right\|+\|\boldsymbol{\psi}\|
\end{align*}
$$

### 8.4.4.1 Sum

$g$ can be a quaternion or a function $g(q)$.
$h$ can be a quaternion or a function $h(q)$.

$$
\begin{align*}
& k=k_{0}+\boldsymbol{k}=g+h  \tag{1}\\
& k_{0}=g_{0}+h_{0} \tag{2}
\end{align*}
$$

$$
\begin{equation*}
k=g+h \tag{3}
\end{equation*}
$$

### 8.4.4.2 Product

$d$ can be a quaternion or a function $d(q)$. $e$ can be a quaternion or a function $e(q)$.

$$
\begin{align*}
& f=f_{0}+\boldsymbol{f}=d e  \tag{1}\\
& f_{0}=d_{0} e_{0}-\langle\boldsymbol{d}, \boldsymbol{e}\rangle  \tag{2}\\
& \boldsymbol{f}=d_{0} \boldsymbol{e}+e_{0} \boldsymbol{d}+\boldsymbol{d} \times \boldsymbol{e} \tag{3}
\end{align*}
$$

### 8.4.5 The coupling equation

The coupling equation follows from peculiar properties of the differential equation. We start with two normalized functions $\psi$ and $\varphi$.

$$
\begin{align*}
& \|\psi\|=\|\varphi\|=1  \tag{1}\\
& \Phi=\nabla \psi=m \varphi  \tag{2}\\
& \Phi=\nabla \psi \text { defines the differential equation. }  \tag{3}\\
& \nabla \psi=\Phi \text { formulates a continuity equation. } \tag{4}
\end{align*}
$$

$\nabla \psi=m \varphi$ formulates the coupling equation. It couples $\psi$ to $\varphi$, where $\varphi$ is located in the embedding continuum and acts as source.
$m$ is the coupling factor.

### 8.4.5.1 In Fourier space

The Fourier transform of the coupling equation is:

$$
\begin{equation*}
\mathcal{M} \tilde{\psi}=m \tilde{\varphi} \tag{1}
\end{equation*}
$$

$\mathcal{M}$ is the displacement generator

### 8.4.5.2 The Dirac equation

The Dirac equation runs:

$$
\begin{equation*}
\nabla_{0}[\psi]+\boldsymbol{\nabla} \boldsymbol{\alpha}[\psi]=m \beta[\psi] \tag{1}
\end{equation*}
$$

$[\psi]$ is a Spinor.
$\alpha$ and $\beta$ are Dirac matrices

| Dirac matrices | Pauli matrices |
| :---: | :---: |
| $\alpha_{1} \equiv\left[\begin{array}{cc}0 & \boldsymbol{i} \\ -\boldsymbol{i} & 0\end{array}\right]$ | $\boldsymbol{i} \mapsto \sigma_{1} \equiv\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right]$ |
| $\alpha_{2} \equiv\left[\begin{array}{cc}0 & \boldsymbol{j} \\ -\boldsymbol{j} & 0\end{array}\right]$ | $\boldsymbol{j} \mapsto \sigma_{2} \equiv\left[\begin{array}{cc}0 & -i \\ i & 0\end{array}\right]$ |
| $\alpha_{3} \equiv\left[\begin{array}{cc}0 & \boldsymbol{k} \\ -\boldsymbol{k} & 0\end{array}\right]$ | $\boldsymbol{k} \mapsto \sigma_{3} \equiv\left[\begin{array}{cc}1 & 0 \\ 0 & -1\end{array}\right]$ |
| $\beta \equiv\left[\begin{array}{cc}0 & 1 \\ 1 & 0\end{array}\right]$ | $1 \mapsto I \equiv\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right]$ |

The $2 \times 2$ matrices combine into $4 \times 4$ matrices
Split into right handed and left handed spinors

$$
\begin{align*}
& \nabla_{0} \psi_{R}+\nabla \psi_{R}=m \psi_{L}  \tag{2}\\
& \nabla_{0} \psi_{L}-\nabla \psi_{L}=m \psi_{R} \tag{3}
\end{align*}
$$

### 8.4.5.3 Dirac equation in quaternionic format

$\nabla \psi=m \psi^{*}$ holds for the electron.
$\nabla^{*} \psi^{*}=m \psi$ holds for the positron.
$\psi^{*}$ is the quaternionic conjugate from $\psi$.
Thus the electron and the positron couple to another member of the same symmetry flavor bundle to which the quantum state function belongs.

The positron does obey the conjugate of the coupling equation. It looks as if no difference exists between the two equations. However, the parameters of the functions are not touched by the conjugation.
$\nabla \psi=m \psi^{*}$ is a special form of the coupling equation. Here the source function $\varphi$ equals $\psi^{*}$. The quantum state function $\psi$ acts as a drain.

## 9 Elementary particle hypothesis

### 9.1 Elementary coupling equation

The coupling equation for the electron and the positron as it is depicted by Paul Dirac, gives us a lead how to interpret this equation for other types of elementary particles.

The electron is an elementary particle and the question arises whether this particular form of the coupling equation is characteristic for elementary particles.

The HBM suggests that elementary particles are the result of couplings between the members of the symmetry flavor bundle that constitutes the Palestra.

One of these members is the reference member and has the same symmetry flavor as the parameter space of the bundle has.

One of the member fields plays the role of the quantum state function of the particle. The other field plays the role of the embedding field.

The operating system maps the quantum state function on the embedding field. Only the partial mirror of the quantum state function in this embedding field is used in the coupling.

Thus the coupling equation for elementary particles will be:

$$
\begin{equation*}
\nabla \psi^{x}=m \psi^{y} \tag{1}
\end{equation*}
$$

The elementary antiparticle equation would be.

$$
\begin{equation*}
\nabla^{*}\left(\psi^{x}\right)^{*}=m\left(\psi^{y}\right)^{*} \tag{2}
\end{equation*}
$$

Here the functions $\psi^{x}$ and $\psi^{y}$ are two members of the symmetry flavor bundle that constitutes the Palestra. This limits the choice to functions that have the same real part. This choice enables the existence of $8 \times 8=64$ different types of particles whose properties depend on the coupled symmetry flavors.

We consider two categories of particles.
The $\boldsymbol{F}$-category couples the quantum state function to the reference member of the Palestra.

Other particles belong to the $\boldsymbol{B}$-category.
Contemporary physics has shown that a significant difference exists between these two categories.

### 9.2 Elementary particle characteristics

Now we have collected sufficient ammunition in order to tackle what coupling means. In the elementary coupling equation, one of the pair $\left\{\psi^{x}, \psi^{y}\right\}$ will act as the source and the other $\psi^{x}$ will act as the drain.

The dimensions in which the symmetry flavors differ determine what the meaning of source and drain will be.

### 9.2.1 Categories and spin

We will give the reference symmetry flavor a preference status.
This will single out a category of elementary particles that can be characterized by the pair $\left\{\psi^{x}, \psi^{(0)}\right\}$. We call this category the F-category ${ }^{6}$. Alternative particles belong to the B-category ${ }^{7}$.

Anti-particles obey the antiparticle coupling equation. There the F-category is characterized by the pair
$\left\{\left(\psi^{x}\right)^{*},\left(\psi^{0}\right)^{*}\right\}=\left\{\left(\psi^{x}\right)^{*}, \psi^{7}\right\}$.
The F-category is generated in a way that differs from the way that the B-category is generated. As a consequence these categories show different spins.
The F-category particles have half integer spin.
The B-category particles have integer spin.
Later a suggestion is given why that is the case.
The standard model contains only particle types that can be discerned by measuring results.

[^5]The HBM scheme discerns on properties and does not mind whether they can be discerned by measurements.

### 9.2.2 e-charge

The dimensions in which the symmetry flavors differ determine the other characteristics of the elementary particle.
The HBM uses one third of the electric charge of the positron as its e-charge unit.

The e-charge equals the number of different dimensions between the $x$-symmetry flavor and the $y$-symmetry flavor. The sign of the e-charge count is switched if the handedness differs ${ }^{8}$.

### 9.2.3 c-charge

The color of the participating fields results in the coloring for the resulting particle. Since the embedding field for F-category is neutral the coloring of the particle types is straight forward for members of this category.

B-category types can be multi-colored.

[^6]
### 9.3 The HBM scheme

### 9.3.1 F-category

We do not yet treat generations.
According to the standard model, the F-category exists in three generations.
The F-category concerns 8 particle types and 8 anti-particle types.

### 9.3.1.1 Leptons

Leptons are color neutral.

### 9.3.1.1.1 Electrons and positrons

The electrons and positrons are easily comprehensible. They are isotropic (color neutral) particles.

| Pair | s- <br> type | e- <br> charge | c- <br> charge | Hand- <br> ed- <br> ness | SM <br> Name |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\left\{\psi^{(7)}, \psi^{(0)}\right\}$ | fer- <br> mion | -3 | N | LR | electron |
| $\left\{\psi^{(0}, \psi^{(7)}\right\}$ | Anti- <br> fer- <br> mion | +3 | W | $\mathbf{R L}$ | positron |

### 9.3.1.1.2 Neutrinos

Neutrinos are the most difficult to understand elementary particles. Their quantum state function has the same symmetry flavor as the embedding continuum has.

| type | s-type | e- <br> char <br> ge | c- <br> charg <br> e | Hand- <br> edness | SM <br> Name |
| :--- | :--- | :---: | :---: | :---: | :---: |
| $\left\{\psi^{(7)}, \psi^{(7)}\right\}$ | fer- <br> mion | 0 | NN | RR | neu- <br> trino |
| $\left\{\psi^{(0)}, \psi^{(0}\right\}$ | anti- <br> fer- <br> mion | 0 | WW | LL | neu- <br> trino |

### 9.3.1.2 Quarks

| Pair | s-type | e- <br> charge | c- <br> charge | Handed <br> -ness | SM <br> Name |
| :---: | :---: | :--- | :---: | :---: | :---: |
| $\left\{\psi^{(1)}, \psi^{(0)}\right.$ | fermion | -1 | R | $\mathbf{L R}$ | down- <br> quark |
| $\left\{\psi^{(6)}, \psi^{(7)}\right.$ | anti-fer- <br> mion | +1 | $\overline{\mathrm{R}}$ | $\mathbf{R L}$ | Anti- <br> down- <br> quark |
| $\left\{\psi^{(2)}, \psi^{(0)}\right.$ | fermion | -1 | G | $\mathbf{L R}$ | down- <br> quark |
| $\left\{\psi^{(5)}, \psi^{(7)}\right.$ | anti-fer- <br> mion | +1 | $\overline{\mathrm{G}}$ | $\mathbf{R L}$ | Anti- <br> down- <br> quark |
| $\left\{\psi^{(3)}, \psi^{(0)}\right.$ | fermion | -1 | B | $\mathbf{L R}$ | down- <br> quark |
| $\left\{\psi^{(4)}, \psi^{(7)}\right.$ | anti-fer- <br> mion | +1 | $\overline{\mathrm{~B}}$ | $\mathbf{R L}$ | Anti- <br> down- <br> quark |
| $\left\{\psi^{(4)}, \psi^{(0)}\right.$ | fermion | +2 | $\overline{\mathrm{~B}}$ | $\mathbf{R R}$ | up- <br> quark |
| $\left\{\psi^{(3)}, \psi^{(7)}\right.$ | anti-fer- <br> mion | -2 | B | $\mathbf{L L}$ | Anti- <br> up- <br> quark |
| $\left\{\psi^{(5)}, \psi^{(0)}\right.$ | fermion | +2 | $\overline{\mathrm{G}}$ | $\mathbf{R R}$ | up- <br> quark |


| $\left\{\psi^{(2)}, \psi^{(7)}\right.$ | Anti-fer- mion | -2 | G | LL | $\begin{aligned} & \text { Anti- } \\ & \text { up- } \\ & \text { quark } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\left\{\psi^{(6)}, \psi^{(0)}\right.$ | fermion | +2 | $\overline{\mathrm{R}}$ | RR | $\begin{gathered} \text { up- } \\ \text { quark } \end{gathered}$ |
| $\left\{\psi^{(1)}, \psi^{(7)}\right.$ | anti-fermion | -2 | R | LL | $\begin{aligned} & \text { Anti- } \\ & \text { up- } \\ & \text { quark } \end{aligned}$ |

### 9.3.2 B-category

### 9.3.2.1 W-particles

The W-particles are multi-colored.
In the HBM scheme exist 4 different $W_{+}$-like particles and 4 corresponding anti-particles. and 8 different $W_{-}$-like particles.
The standard discerns only one $W_{+}$-like particle and one $W_{-}$-like particle.

| Pair | s- <br> type | e- <br> charge | c- <br> charge | Hand- <br> ed- <br> ness | SM <br> Name |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\left\{\psi^{(6)}, \psi^{(1)}\right\}$ | boson | -3 | $\overline{\mathrm{R}} \mathrm{R}$ | $\mathbf{R L}$ | $W_{-}$ |
| $\left\{\psi^{(1)}, \psi^{(6)}\right\}$ | anti- <br> boson | +3 | $R \overline{\mathrm{R}}$ | $\mathbf{L R}$ | $W_{+}$ |


| $\left\{\psi^{(6}, \psi^{(2)}\right\}$ | boson | -3 | $\overline{\mathrm{R}} \mathrm{G}$ | $\mathbf{R L}$ | $W_{-}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\left\{\psi^{(2)}, \psi^{6}\right\}$ | anti- <br> boson | +3 | $G \overline{\mathrm{R}}$ | $\mathbf{L R}$ | $W_{+}$ |
| $\left\{\psi^{(6}, \psi^{(3)}\right\}$ | boson | -3 | $\overline{\mathrm{R}} \mathrm{B}$ | $\mathbf{R L}$ | $W_{-}$ |
| $\left\{\psi^{(3)}, \psi^{6}\right\}$ | anti- <br> boson | +3 | $B \overline{\mathrm{R}}$ | $\mathbf{L R}$ | $W_{+}$ |
| $\left\{\psi^{(5)}, \psi^{(1)}\right\}$ | boson | -3 | $\overline{\mathrm{G} G}$ | $\mathbf{R L}$ | $W_{-}$ |
| $\left\{\psi^{(1)}, \psi^{(5)}\right\}$ | anti- <br> boson | +3 | $\mathrm{G} \overline{\mathrm{G}}$ | $\mathbf{L R}$ | $W_{+}$ |
| $\left\{\psi^{(5)}, \psi^{(2)}\right\}$ | boson | -3 | $\overline{\mathrm{G} G}$ | $\mathbf{R L}$ | $W_{-}$ |
| $\left\{\psi^{(2)}, \psi^{(5)}\right\}$ | anti- <br> boson | +3 | $\mathrm{G} \overline{\mathrm{G}}$ | $\mathbf{L R}$ | $W_{+}$ |
| $\left\{\psi^{(5)}, \psi^{(3)}\right\}$ | boson | -3 | $\overline{\mathrm{~GB}}$ | $\mathbf{R L}$ | $W_{-}$ |
| $\left\{\psi^{(3)}, \psi^{(5)}\right\}$ | anti- <br> boson | +3 | $\mathrm{~B} \overline{\mathrm{G}}$ | $\mathbf{L R}$ | $W_{+}$ |
| $\left\{\psi^{(4)}, \psi^{(1)}\right\}$ | boson | -3 | $\overline{\mathrm{~B}} \mathrm{R}$ | $\mathbf{R L}$ | $W_{-}$ |
| $\left\{\psi^{(1)}, \psi^{(4)}\right\}$ | anti- <br> boson | +3 | $\mathrm{R} \overline{\mathrm{B}}$ | $\mathbf{L R}$ | $W_{+}$ |
| $\left\{\psi^{(4)}, \psi^{(2)}\right\}$ | boson | -3 | $\overline{\mathrm{~B}} \mathrm{G}$ | $\mathbf{R L}$ | $W_{-}$ |
| $\left\{\psi^{(2)}, \psi^{(4)}\right\}$ | anti- <br> boson | +3 | $\mathrm{G} \mathrm{\bar{B}}$ | $\mathbf{L R}$ | $W_{+}$ |
| $\left\{\psi^{(4)}, \psi^{(3)}\right\}$ | boson | -3 | $\overline{\mathrm{~B} B}$ | $\mathbf{R L}$ | $W_{-}$ |
| $\left\{\psi^{(3)}, \psi^{(4)}\right\}$ | anti- <br> boson | +3 | $\mathrm{~B} \overline{\mathrm{~B}}$ | $\mathbf{L R}$ | $W_{+}$ |

### 9.3.2.2 Z-particles

.Z-particles are multi-colored.
In the HBM scheme exist 6 different $Z$-like particles and 6 different $W_{-}$-like anti-particles.
The standard discerns only one Zlike particle and one $Z$ like anti-particle.

| Pair | s-type | e- <br> charge | c- <br> charge | Hand- <br> ed- <br> ness | SM <br> Name |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\left\{\psi^{(2)}, \psi^{(1)}\right\}$ | boson | 0 | GR | LL | Z |
| $\left\{\psi^{(5)}, \psi^{(6)}\right\}$ | anti- <br> boson | 0 | $\overline{\mathrm{G}} \overline{\mathrm{R}}$ | RR | Z |
| $\left\{\psi^{(3)}, \psi^{(1)}\right\}$ | boson | 0 | BR | $\mathbf{L L}$ | Z |
| $\left\{\psi^{(4)}, \psi^{(6)}\right\}$ | anti- <br> boson | 0 | $\overline{\mathrm{R} \bar{B}}$ | $\mathbf{R R}$ | Z |
| $\left\{\psi^{(3)}, \psi^{(2)}\right\}$ | boson | 0 | BR | $\mathbf{L L}$ | Z |
| $\left\{\psi^{(4)}, \psi^{(5)}\right\}$ | anti- <br> boson | 0 | $\overline{\mathrm{R} \bar{B}}$ | $\mathbf{R R}$ | Z |
| $\left\{\psi^{(1)}, \psi^{(2)}\right\}$ | boson | 0 | RG | $\mathbf{L L}$ | Z |
| $\left\{\psi^{(6)}, \psi^{(5)}\right\}$ | anti- <br> boson | 0 | $\overline{\mathrm{R} \bar{G}}$ | $\mathbf{R R}$ | Z |
| $\left\{\psi^{(1)}, \psi^{(3)}\right\}$ | boson | 0 | RB | $\mathbf{L L}$ | Z |


| $\left\{\psi^{(6)}, \psi^{(4)}\right\}$ | anti- <br> boson | 0 | $\overline{\mathrm{R}} \overline{\mathrm{B}}$ | $\mathbf{R R}$ | Z |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\left\{\psi^{(2)}, \psi^{(3)}\right\}$ | boson | 0 | RB | $\mathbf{L L}$ | Z |
| $\left\{\psi^{(5}, \psi^{(4)}\right\}$ | anti- <br> boson | 0 | $\overline{\mathrm{R}} \overline{\mathrm{B}}$ | $\mathbf{R R}$ | Z |

### 9.3.2.3 Other particles

Until now we have treated particles that may represent the particles that are recognized in the standard model. This set already contains more particles than the SM discerns. Since the "extra" bosons only differ in their multi-color charge, which cannot yet be measured, these particles may represent SM-particles.

However, we have only treated 8 F-category particles and 8 F-category anti-particles.
The B-category represents 9 versions of the $W_{+}$-particle and 9 versions of the $W_{-}$-particle. Further it contains 12 potential representatives of the $Z$-particle. The versions only differ in their multi-color charge.

Thus result 64-16-16-12 $=20$ "other particles". Amongst them are 12 reverse-quark bosons.

## 10 Space hypothesis

### 10.1 The Palestra

The Palestra is the name of a symmetry flavor bundle that pays the role of our living space.

This selection is raised by the fact that the Dirac equation seems to couple two members of a symmetry flavor bundle. Accepting the elementary coupling equation as a valid equivalent of the Dirac equation for other elementary particle offers the opportunity to suggest the existence of a large variety of these particles.

The elementary coupling equation can be interpreted as the description of the embedding of a free elementary particle in a member of the Palestra.

### 10.2 The space hypothesis

Now we can formulate the space hypothesis.
Our living space can be represented by a field that is represented by a symmetry flavor bundle. That field is called Palestra.

Everything in universe consists of features of the Palestra

### 10.3 The embedding of elementary particles

All artifacts in universe are free elementary particles or they are composites of elementary particles. The constituents of the composites are bounded in a modular way. The elementary particles represent locations where members of the symmetry flavor bundle couple. At those locations the elementary particles can be considered to be embedded in one of the two coupling members of the Palestra.
The locations of this embedding form a swarm. This swarm characterizes the embedded particle.
The continuous density distribution that describes the swarm conforms to the squared modulus of what we know as the wave function of the elementary particle.

## 11 Preparing the swarm

### 11.1 Storage

The HBM suggests that the whole swarm is prepared in advance of its usage. The prepared swarm is stored in a persistent location. That must be an eigenspace of an appropriate operator that resides in the Gelfand triple. We will call this operator the planned-swarm operator. The prepared swarm consists of a coherent set of locations and displacements that are generated relative to the eigenspace of an operator that provides a flat parameter space that is formed by a selected quaternionic number system ${ }^{9}$.

### 11.2 Suggested creation mechanism

The set of locations is created by a combination of a Poisson process that delivers a stochastic sequence of (real) rational numbers that act as design parameters. The Poisson process is combined with a binomial process that uses the design parameters as parameter of a 3D spread function. Together these processes produce a stochastic spatial spread function that can be stored in a discrete quaternionic distribution. On its turn this dis-

[^7]crete quaternionic distribution is stored in the eigenspace of the planned-swarm operator in the Gelfand triple.
As indicated earlier the discrete quaternionic distribution implements a path of subsequent locations.

### 11.2.1 Example distribution

As an example the combination of the Poisson process and the binomial process may generate a discrete quaternionic distribution that comes close to a 3D Gaussian distribution ${ }^{10}$.

A charge distribution in the form of this example distribution corresponds to an isotropic potential in the form

$$
\begin{equation*}
\frac{\operatorname{Erf}(r)}{r} \tag{1}
\end{equation*}
$$

This quickly reduces to the $1 / r$ form of the gravitational potential of a single charge. In contrast to the single charge potential, the potential of the charge distribution does not contain a singularity.
${ }^{10}$ A 3D normal distribution.


Due to the swarm, the potential of the particle has no longer a singularity.

### 11.3 The stochastic spatial spread function

The generator of the planned swarm will be called stochastic spatial spread function and it will be indicated with symbol $\mathcal{S}$.

The generated swarm can be represented by a continuous quaternionic density distribution, which is also a quaternionic probability density distribution. This quaternionic function is indicated with symbol $\psi$

## 12 Mapping the swarm

### 12.1 The allocation function

The planned swarm is mapped on a member of the Pa lestra that acts as the embedding continuum.
The mapping can be described by a continuous quaternionic allocation function $\wp$.
This quaternionic allocation function $\wp$ describes a path through (part of) the history of the Palestra. This path describes the live of the corresponding building block. The allocation function has a flat parameter space, which is formed by rational quaternions. The real part of these parameters represents progression. For each particle $\wp$ uses a separate parameter value. The spatial image of $\wp$ varies with this parameter value and thus with progression.

With other words $\wp$ describes the target embedding continuum and the movement of the particle in this continuum.

### 12.1.1 Metric

The differential of the allocation function describes a quaternionic metric. This metric describes the curvature of the Palestra.

$$
\begin{gathered}
d s(q)=d s^{v}(q) e_{v}=d \wp=\sum_{\mu=0 \ldots 3} \frac{\partial \wp}{\partial q_{\mu}} d q_{\mu} \\
=c^{\mu}(q) d q_{\mu}
\end{gathered}
$$

$q$ is the quaternionic location.
$d s$ is the metric.
$c^{\mu}$ is a quaternion.
Pythagoras:

$$
\begin{equation*}
c^{2} d t^{2}=d s d s^{*}=d q_{0}^{2}+d q_{1}^{2}+d q_{2}^{2}+d q_{3}^{2} \tag{2}
\end{equation*}
$$

Minkowski:

$$
\begin{equation*}
d q_{0}^{2}=d \tau^{2}=c^{2} t^{2}-d q_{1}^{2}-d q_{2}^{2}-d q_{3}^{2} \tag{3}
\end{equation*}
$$

In flat space:

$$
\begin{equation*}
\Delta s_{f l a t}=\Delta q_{0}+\boldsymbol{i} \Delta q_{1}+\boldsymbol{j} \Delta q_{2}+\boldsymbol{k} \Delta q_{3} \tag{4}
\end{equation*}
$$

In curved space:

$$
\begin{equation*}
\Delta s_{\S}=c^{0} \Delta q_{0}+c^{1} \Delta q_{1}+\Delta q_{2}+c^{3} \Delta q_{3} \tag{5}
\end{equation*}
$$

$d \wp$ is a quaternionic metric
It is a linear combination of 16 partial derivatives

$$
\begin{align*}
& d \wp=\sum_{\mu=0 \ldots 3} \frac{\partial \wp}{\partial q_{\mu}} d q_{\mu}=c^{\mu}(q) d q_{\mu}  \tag{6}\\
& =\sum_{\mu=0 \ldots 3} \sum_{v=0, \ldots 3} e_{v} \frac{\partial \wp_{v}}{\partial q_{\mu}} d q_{\mu}=\sum_{\mu=0 \ldots 3} \sum_{v=0, \ldots 3} e_{v} c_{v}^{\mu} d q_{\mu}
\end{align*}
$$

### 12.2 The blurred allocation function

The real part of the parameter of the allocation function $\wp$ represents progression. This progression value is used as enumerator value by the stochastic spatial spread function.
The blurred allocation function $\mathcal{P}$ describes the combined effect of the allocation function $\wp$ and the stochastic spatial spread function $\mathcal{S}$. The combination is a convolution.

$$
\begin{equation*}
\mathcal{P}=\wp \circ \mathcal{S} \tag{1}
\end{equation*}
$$

Like $\wp, \mathcal{P}$ uses for each separate particle a single rational quaternion as a parameter and produces the current location of that particle. $\wp$ produces the center location of the complete mapped swarm.

BLURRED ALLOCATION FUNCTION $\mathcal{P}$

- Function $\mathcal{P}=\wp \circ \delta \begin{gathered}\text { Curved } \\ \text { space }\end{gathered}$
- Blurred $\mathcal{P}$
- Sharp $\wp$
- Spread function $\mathcal{S}$

Curved
space

- QPDD $\psi$
- Quaternionic

Probability
Density
Distribution
$\Rightarrow$ Produces swarm $\Rightarrow$ Qtarget
$\Rightarrow$ Produces planned Qpatch
$\Rightarrow$ Produces Qpattern

### 12.3 Names giving

The planned swarm will be called Qpattern. It is produced by stochastic spatial spread function $\mathcal{S}$.
The Qpattern is mapped onto the embedding continuum by the allocation function $\wp$. There it becomes the virtual swarm.
The target of the allocation function $\wp$ for a given particle is called Qpatch.
The target for the blurred allocation function $\mathcal{P}$ is the current location of the particle and it is called Qtarget. The blurred allocation function $\mathcal{P}$ produces a virtual swarm. The Qtarget is the only actual location in this swarm.

A continuous quaternionic density distribution $\psi$ characterizes the virtual swarm. The real part of this distribution corresponds to the squared modulus of the wave function of the particle. That is why we call $\psi$ the quaternionic quantum state function of the particle.

## 13 Embedding process

The embedding of a single building block can be interpreted as the arrival of the particle at a new location. The embedding recurs at every progression step.

### 13.1 What happens?

The elementary coupling equation describes the coupling for a free elementary particle.
One member of the Palestra couples to another member. At the location of the coupling a small amount of space is exchanged from the source member to the drain member. This occurrence causes a local singularity in the Palestra.

The coupling only lasts during a single progression step. The next coupling occurs at a slightly different location. The influence of the singularity keeps spreading in the form of a wave front that moves with the highest possible speed over the Palestra.

The wave fronts combine into super-high frequency waves. The frequency of these waves is so high that they cannot be observed. Only the averaged effects of these waves can be observed. These averaged effects form the potentials of the building block


In symmetry flavor A member, space is contracting towards the worm hole. In symmetry flavor B member, space is expanding away from the worm hole. The worm hole forms a common singularity. At the instance of embedding the collected space is transported from symmetry flavor A to symmetry flavor B. At each progression step the worm hole appears at a slightly different location.

### 13.2 Wave fronts

At every arrival at a step stone, the building block emits a wave front that carries a message about it presence and its properties. These properties become visible in the potentials that are formed by the wave fronts. The origin of space curvature is the fact that the wave fronts fold and thus curve the embedding continuum. Together, these wave fronts form super-high frequency waves
The propagation of the wave fronts is controlled by Huygens principle
For isotropic wave fronts their amplitude decreases with the inverse of the distance to their source
The effect of the Huygens principle depends on the number of involved dimensions. The form of the Green's function is determined by the Huygens principle. The Green's function determines the contribution of the wave front to the potential.

Depending on dedicated Green's functions, the integral over the wave fronts constitutes a series of potentials. The Green's function describes the contribution of a wave front to a corresponding potential Gravitation potentials and electrostatic potentials have different Green's functions

### 13.3 Potentials

Potentials are the averaged effects of the wave fronts that are emitted during the coupling that forms the building block

The wave fronts and the potentials are traces of the particle and its used step stones.
The superposition of the singularities smoothens the effect of these singularities.
Neither the emitted wave fronts, nor the potentials affect the particle that emitted the wave front
Wave fronts interfere
The wave fronts modulate a field

## 14 Binding

### 14.1 Binding mechanism

The fourth swarming condition enables the application of the superposition principle in the realm of Fourier space.
In this way the superposition coefficients become displacement generators
The consequence is that these coefficients may define movements that are internal to the construction elements

### 14.2 Composites

The fact that superposition coefficients define internal movements can best be explained by formulating the definition of composites as follows.

Composites are equipped with a quantum state function whose Fourier transform at any progression step equals the superposition of the Fourier transforms of the quantum state functions of its constituents.

Now the superposition coefficients can define internal displacements. As a function of progression they define internal oscillations.

### 14.3 Internal kinetics

Within free elementary building blocks these internal movements concern the micro-paths Within composites the internal movements concern oscillations of the constituting bounded elementary building blocks

Like elementary particles, composites obey the swarming conditions
For composites the coupling equation holds

$$
\begin{equation*}
\Phi=\nabla \psi=m \varphi \tag{1}
\end{equation*}
$$

$\psi$ and $\varphi$ are normalized quaternionic functions that represent density distributions.

### 14.4 Geoditches

Each of the step stones of the swarm causes a local pinch that describes the temporary (singular) curvature of the embedding continuum. In a free elementary particle the swarm of pinches combine in a non-singular pitch.

In a composite the micro-paths of the constituting elementary particles are folded along the internal oscillation paths. In the composite the pinches combine in ditches that like the micro-paths fold along the oscillation paths. These ditches form special kinds of geodesics that we call "Geoditches".
The geoditches explain the binding effect of the superposition of the constituents.

In a small composite, such as a nucleus or an atom, the geoditches might blur the oscillations with up-anddown hops to such an extent, that the oscillations are no longer recognizable. For electrically charged constituents this means that the oscillations of the charge do not generate corresponding EM waves! However, the hops and the oscillations contribute to the internal
kinetic energy and in this way they contribute to the total energy $m$ of the free composite.

## 15 Wave front propagation

At the instant of the embedding of an elementary particle a wave front is emitted in the embedding continuum. This wave front slightly folds and thus curves that medium. These wave fronts cannot be observed. However their averaged effects can become noticeable as the potentials of the particle. In order to investigate these phenomena we must study the propagation of the wave fronts.

At every progression step, each existing elementary particle emits a wave front. Each emission departs from a different step stone. Each wave front contributes to the potentials of the particle. The contribution of the wave front to the potential is described by a dedicated Green's function. The Green's function is determined by the Huygens principle.

The HBM suggests:
Depending on the dimension of the singularity that is caused by the embedding process, the wave front can be emitted into one, two or three dimensions.

In each of these cases the Huygens principle acts differently.

### 15.1 Huygens principle

The propagation of waves is governed by the Huygens principle. The operating system uses this mechanism in order to regenerate all wave fronts at every progression step.

### 15.1.1 Huygens principle for odd and even number of spatial dimension

The following is taken from http://www.mathpages.com/home/kmath242/kmath242.htm

The spherically symmetrical wave equation in $n$ spatial dimensions can be written as

$$
\begin{equation*}
\frac{\partial^{2} \psi}{\partial r^{2}}+\frac{n-1}{r} \frac{\partial \psi}{\partial r}=\frac{\partial^{2} \psi}{\partial t^{2}} \tag{1}
\end{equation*}
$$

Now suppose we define a new scalar field $\phi$ by the relation

$$
\begin{equation*}
\phi(r, t)=r^{(\mathrm{n}-1) / 2} \psi(r, t) \tag{2}
\end{equation*}
$$

This leads to

$$
\begin{equation*}
\frac{\partial^{2} \phi}{\partial r^{2}}+\frac{(n-1)(n-3)}{4 r^{2}} \phi=\frac{\partial^{2} \phi}{\partial t^{2}} \tag{3}
\end{equation*}
$$

If n equals 1 , meaning that we have just a single space dimension, then the second term on the left hand side vanishes, leaving us with a one-dimensional wave equation, with has the well-known general solution

$$
\begin{equation*}
\psi(r, t)=f(r-t)+g(r+t) \tag{4}
\end{equation*}
$$

for arbitrary functions $f$ and $g$.
if $n$ equals 3, i.e., in the case of three spatial dimensions, the spherically symmetrical wave equation reduces again to a one-parametric wave equation, in the modified wave function $\phi=r \psi$. Hence the general solution in three space dimensions is

$$
\begin{equation*}
\psi(r, t)=\frac{f(r-t)}{r}+\frac{g(r+t)}{r} \tag{5}
\end{equation*}
$$

The fact that this solution is divided by $r$ signifies that the magnitude of the wave tends to drop as $r$ increases (unlike the one-dimensional case, in which a wave would theoretical propagate forever with non-diminished strength). Focusing on just the "retarded" component of the wave, $f(r-t) / r$, the fact that the time parameter $t$ appears only in the difference $r-t$ implies that the (attenuated) wave propagates in time with a phase velocity of precisely 1 , because for any fixed
phase $\beta$ we have $r-t=\beta$ and so $d r / d t$ for this phase point is 1 . Consequently if $f$ is a single pulse, it will propagate outward in a spherical shell at precisely the speed 1 , i.e., on the light cone. Conversely, it can be shown that the wave function at any point in space and time is fully determined by the values and derivatives of that function on the past light cone of the point.

Any wave equation for which this is true (i.e., for which disturbances propagate at a single precise speed) is said to satisfy Huygens' Principle. The connection with Huygens' original statement about secondary wavelets is that each wavelet - with the same speed as the original wave - represents a tiny light cone at that point, and Huygens' principle asserts that light is confined to those light cones.

For $n$ equals 2 the extra term in equation (3) does not vanish. We can still solve the wave equation, but the solution is not just a simple spherical wave propagating with unit velocity. Instead, we find that there are effectively infinitely many velocities, in the sense that a single pulse disturbance at the origin will propagate outward on infinitely many "light cones" (and sub-cones) with speeds ranging from the maximum down to zero. Hence if we lived in a universe with two spatial dimensions (instead of three), an observer at a fixed location from the origin of a single pulse would "see" an initial
flash but then the disturbance "afterglow" would persist, becoming less and less intense, but continuing forever, as slower and slower subsidiary branches arrive.

### 15.1.2 The case of even spatial dimensions

Now again start from equation (1) and try a solution in the form:

$$
\begin{equation*}
\psi(r, t)=f(r) g(t) \tag{1}
\end{equation*}
$$

Inserting this into the wave equation and expanding the derivatives by the product rule gives

$$
\begin{equation*}
g \frac{\partial^{2} f}{\partial r^{2}}+\frac{n-1}{r} g \frac{\partial f}{\partial r}=f \frac{\partial^{2} g}{\partial t^{2}} \tag{2}
\end{equation*}
$$

Dividing through by $f g$ gives

$$
\begin{equation*}
\frac{1}{f} \frac{\partial^{2} f}{\partial r^{2}}+\frac{n-1}{f r} \frac{\partial f}{\partial r}=\frac{1}{g} \frac{\partial^{2} g}{\partial t^{2}} \tag{3}
\end{equation*}
$$

This decouples into two equations

$$
\begin{equation*}
\frac{\partial^{2} f}{\partial r^{2}}+\frac{n-1}{r} \frac{\partial f}{\partial r}=k f \tag{3}
\end{equation*}
$$

And

$$
\begin{equation*}
\frac{\partial^{2} g}{\partial t^{2}}=k g \tag{4}
\end{equation*}
$$

If $k$ is positive or zero the right hand equation gives "run-away" solutions for $g(t)$, whereas if $k$ is negative we can choose scaling so that $k=-1$ and then $g(t)$ satisfies the simple harmonic equation, whose solutions include functions of the form $\sin (c t)$ and $\cos (c t)$. In that case equation (9) can be re-written in the form

$$
\begin{equation*}
r \frac{\partial^{2} f}{\partial r^{2}}+(n-1) \frac{\partial f}{\partial r}+r f=0 \tag{5}
\end{equation*}
$$

This is the form of a Bessel's equation. In fact for $\mathrm{n}=2$ the solution is the zero order Bessel function $J_{0}(r)$.

$$
\begin{equation*}
J_{0}(r)=\frac{2}{\pi} \int_{0}^{\infty} \sin (\cosh (\theta) r) d \theta \tag{6}
\end{equation*}
$$

A plot of $J_{0}(r)$ is shown below.


Inserting $g(t)=\sin (c t)$ gives

$$
\begin{align*}
\psi(r, t)=\frac{1}{\pi} \int_{0}^{\infty} & {[\cos (\cosh (\theta) r-c t)}  \tag{7}\\
& -\cos (\cosh (\theta) r+c t)] d \theta
\end{align*}
$$

Hence, instead of the solution being purely a function of $r \pm c t$ as in the case of odd dimensions, we find that it is an integral of functions of $\cosh (\theta) r \pm c t$. Each
value of $\theta$ corresponds to a propagation speed of $c / \cosh (\theta)$, so the speeds vary from $c$ down to zero. This signifies that the wave function at any event is correlated not just with the wave function on its "light cone", but with the wave function at every event inside its light cone.

In two dimensions the Huygens principle corresponds to a centripetal force ${ }^{11}$ with potential

$$
\begin{equation*}
V=-\frac{\hbar}{8 M r^{2}} . \tag{8}
\end{equation*}
$$

### 15.1.3 Huygens principle applied

HYPOTHESIS : Particles transmit waves in dimensions where the discrete symmetry of the quantum state function differs from the discrete symmetry of the embedding background.

The operating system uses the Huygens principle in order to restore the potentials at each progression step. The Huygens principle works differently depending on the number of dimensions in which the waves are transmitted.

[^8]The characteristics of the potentials that are emitted or absorbed by elementary particles are determined by the differences between the discrete symmetry set of the quantum state function of the particle and the symmetry set of the coupled QPDD that represents the embedding continuum. This difference determines whether the potentials act in 1,2 or 3 dimensions. In odd dimensions the persistence of the potentials can be explained by the common interpretation of the Huygens principle. This common interpretation is that at every point of each wave front new waves are generated. This does not work for particles that send their waves in two dimensions. This includes quarks, W-particles and Z-particles. The corresponding messengers are gluons. For these objects the potentials also act in two dimensions. In even dimensions the Huygens principle does not act in its normal way.
The same conditions that determine whether waves are emitted in 1,2 , or 3 dimensions also determine whether the particle has $1 / 3,2 / 3$ or $3 / 3$ integer electric charge. The re-emitted waves consist out of a retarded component and an advanced component. These components correspond to outbound interactions and inbound interactions.

### 15.2 Discussion

The particular behavior of the Huygens principle for potential contributions that cover even dimensions might
explain the exceptional strength of the corresponding strong force mechanism.

It appears that fermions with electric charges of $\pm \mathrm{n} / 3 \mathrm{e}$ produce n dimensional waves that contribute to their electrostatic potential.
For $\mathrm{n}=3$ the Green's function is of form $1 / \mathrm{r}$.
For $\mathrm{n}=2$ the Green's function is a zero order Bessel function.
For $\mathrm{n}=1$ the Green's function is a constant.
On the other hand the color confinement principle ${ }^{12}$ prevents that the even dimensional actions of the Huygens principle will ever become observable in a long lasting way.
The gravitation potential is not influenced by the discrete symmetries. The corresponding potential contributions are always transmitted isotropic in three dimensions.

The electrostatic potential is controlled by the discrete symmetry sets. Depending on the resulting electric charge of the particle the electric potential contributions are transmitted in 1, 2 or 3 dimensions.

[^9]The operating system applies the Huygens principle for the recreation in each progression step of the corresponding potentials.

### 15.3 Green's functions

No mathematical solution is known for the conversion to a rather static potential function of a super-high frequency train of wave fronts that start at slightly different positions. Normally the relation between a set of static and identical charges and a potential function is regulated by a dedicated Green's function.
We can try a similar solution by letting each of the wave fronts play the role of the potential of a single "charge". We can also use the fact that a building block contains a fixed number of step stones. Thus, instead of an integral a sum over $N_{w}$ step stones can be used.

### 15.4 Gravity and electrostatics

Potentials depend on the Green's function that is used to convert the corresponding density distribution into a potential function. Apart from their Green's function, gravity and electrostatics can be treated by similar equations. We use the fact that charge Q is spread over $N_{w}$ step stones that have charge $q=Q / N_{w}$.

| Description | Gravity | Electrostatics |
| :---: | :---: | :---: |
| Field | $\boldsymbol{g}=-\boldsymbol{\nabla} \varphi$ | $\boldsymbol{E}=-\boldsymbol{\nabla} \varphi$ |
| Force | $\boldsymbol{F}=m \boldsymbol{g}$ | $\boldsymbol{F}=Q \boldsymbol{E}$ |


| Gauss law | $\langle\boldsymbol{\nabla}, \mathrm{g}\rangle=-4 \pi G \rho$ | $\langle\boldsymbol{\nabla}, \mathrm{E}\rangle=\frac{Q}{\varepsilon}$ |
| :---: | :---: | :---: |
| Poisson law <br> $\Delta \varphi=$ <br> $\langle\boldsymbol{\nabla}, \boldsymbol{\nabla} \varphi\rangle$ | $\Delta \varphi=4 \pi G \rho$ | $\Delta \varphi=-\frac{Q}{\varepsilon}$ |
| Greens func- <br> tion | $\frac{-\rho\left(\boldsymbol{r}^{\prime}\right)}{\left\|\boldsymbol{r}-\boldsymbol{r}^{\prime}\right\|}$ | $\frac{q}{\left\|\boldsymbol{r}-\boldsymbol{r}^{\prime}\right\|}$ |
| Single <br> charge po- <br> tential | $\varphi=-\frac{4 \pi G m}{\|\boldsymbol{r}\|}$ | $\varphi=\frac{Q}{4 \pi \varepsilon\|\boldsymbol{r}\|}$ |
| Single <br> charge field | $g=-\frac{4 \pi G m}{\|\boldsymbol{r}\|^{2}} \boldsymbol{r}$ | $\boldsymbol{E}=\frac{Q}{4 \pi \varepsilon\|\boldsymbol{r}\|^{2}} \boldsymbol{r}$ |
| Two charge <br> force | $\boldsymbol{F}=-\frac{4 \pi G m_{1} m_{2}}{\|\boldsymbol{r}\|^{3}} \boldsymbol{r}$ | $\boldsymbol{F}=\frac{Q_{1} Q_{2}}{4 \pi \varepsilon\|\boldsymbol{r}\|^{3}} \boldsymbol{r}$ |
| Mode | attracting | $\mathbf{r e p e l l i n g}$ |

The table shows that the Greens functions of both fields differ in sign. For the gravitation potential the Green's function is charged with the local "charge" density $\rho\left(\boldsymbol{r}^{\prime}\right)$. For the electrostatic potential the Green's function is charged with a (constant) electric charge $Q$. The Yukawa potential ${ }^{13}$ uses a short range Green's function:

[^10]\[

$$
\begin{equation*}
\frac{-\rho\left(\boldsymbol{r}^{\prime}\right)}{\left|\boldsymbol{r}-\boldsymbol{r}^{\prime}\right|} \exp \left(-\mu\left|\boldsymbol{r}-\boldsymbol{r}^{\prime}\right|\right) \tag{1}
\end{equation*}
$$

\]

In this example we use the gravitational Green's function.
Since the items are carriers with charge $\rho_{i}$, the density distribution $_{\mathrm{f}}(\mathbf{r})$ correspond to a potential $\varphi(\boldsymbol{r})$.
Every item contributes a term $\varphi_{i}\left(\boldsymbol{r}-\boldsymbol{r}_{\boldsymbol{i}}\right)=\frac{-\rho_{i}}{\left|\boldsymbol{r}-\boldsymbol{r}_{\boldsymbol{i}}\right|}$

$$
\varphi(\boldsymbol{r})=\sum_{i} \varphi_{i}\left(\boldsymbol{r}-\boldsymbol{r}_{\boldsymbol{i}}\right)=\sum_{i} \frac{-\rho_{i}}{\left|\boldsymbol{r}-\boldsymbol{r}_{\boldsymbol{i}}\right|}
$$

Example: If there is a static spherically symmetric Gaussian charge density

$$
\rho_{\mathrm{g}}(\mathrm{r})=\frac{\rho_{c}}{\sigma^{3} \sqrt{2 \pi}^{3}} \exp \left(\frac{-\mathrm{r}^{2}}{2 \sigma^{2}}\right)
$$

where $\rho_{c}$ is the total charge, then the solution $\varphi(r)$ of Poisson's equation,

$$
\nabla^{2} \varphi=\rho_{\mathrm{g}}
$$

is given by

$$
\varphi(\mathrm{r})=\frac{\rho_{c}}{4 \pi \varepsilon \mathrm{r}} \operatorname{erf}\left(\frac{\mathrm{r}}{\sqrt{2 \sigma}}\right)=\frac{-1}{4 \pi \varepsilon} \int \frac{\rho_{\mathrm{g}}\left(\boldsymbol{r}^{\prime}\right)}{\left|\boldsymbol{r}-\boldsymbol{r}^{\prime}\right|} d^{3} \boldsymbol{r}^{\prime}
$$

where $\operatorname{erf}(x)$ is the error function.
Note that, for $r$ much greater than $\sigma$, the erf function approaches

### 15.4.1 Interpretation

The above integral can be interpreted as a summation of influences by all step stones that constitute the micropath of the particle.
Thus the potential of the (Gaussian) particle is given by:

$$
\begin{equation*}
\varphi(\mathrm{r}) \approx \frac{\rho_{c}}{4 \pi \varepsilon \mathrm{r}} \operatorname{erf}\left(\frac{\mathrm{r}}{\sqrt{2 \sigma}}\right) \tag{1}
\end{equation*}
$$

This no longer represents a singularity.

### 15.4.2 Bertrand's theorem

Now we remember Bertrand's theorem. ${ }^{14}$ :

Bertrand's theorem states that only two types of central force potentials produce stable, closed orbits:
(1) an inverse-square central force such as the gravitational or electrostatic potential

$$
\begin{equation*}
V(r)=\frac{-k}{r} \tag{1}
\end{equation*}
$$

and

[^11](2) the radial harmonic oscillator potential
\[

$$
\begin{equation*}
V(r)=1 / 2 k r^{2} \tag{2}
\end{equation*}
$$

\]

According to this investigation it becomes acceptable to assume that the undisturbed shape of the Qpatterns can be characterized by something that comes close to a 3D Gaussian distributions. Since such a distribution produces the correct shape of the gravitation potential, the underlying mechanism would explain the origin of curvature.

### 15.5 Multiple sets of wave fronts

There are at least two kinds of potentials. This means that there are at least as many sets of wave fronts that play a role.
It is already indicated that wave fronts may depend on the number of participating dimensions. This may depend on the kind of singularity that governs the embedding process. On its turn this depends on the symmetry flavors of the fields that are coupled by the embedding process. In each embedding two fields are participating. One plays the role of the source and the other plays the role of the drain. Each of the participating field has its own set of wave fronts and each of these sets corresponds to a type of potential.

The HBM suggests that the field that characterizes the quantum state function will deliver the electrostatic potential. The embedding field delivers the gravitation potential. Both fields are members of the symmetry flavor bundle that constitutes the Palestra.

## 16 Interpreting the coupling equation.

### 16.1 Speed

The definition of the norm involves an integral.
For the continuous density distribution this means:

$$
\|\psi\|=\int_{V}|\psi|^{2} d V
$$

The coupling equation in integral format runs:

$$
\begin{aligned}
\|\phi\|= & \int_{V}|\phi|^{2} d V=\int_{V}|\nabla \psi|^{2} d V \\
& =m^{2} \int_{V}|\psi|^{2} d V=m^{2}
\end{aligned}
$$

Now let us apply this to the swarm by replacing the integral by a summation of squared hop sizes $\left|h_{i}\right|$.

$$
m^{2}=\sum_{i}^{N}\left|h_{i}\right|^{2}=N \overline{\left|h_{l}\right|^{2}}
$$

$h_{i}$ is the quaternionic value of the $i$-th hop. It is an imaginary quaternion. $N$ is the number of elements in the swarm ${ }^{15} . N$ is not affected by uniform movement.

$$
h_{i}=l_{i}-l_{i-1} ; i=1 \ldots N
$$

$l_{0}$ is the start location. $l_{i}$ is the location of the $i$-th swarm element.

The displacement of the whole swarm after a full cycle is:

$$
\Delta_{s}=\sum_{i}^{N} \boldsymbol{h}_{\boldsymbol{i}}
$$

$\Delta_{s}$ is a vector. This move takes N progression steps. Thus the swarm speed is

$$
v=\frac{\Delta_{s}}{N \Delta_{\tau}}
$$

[^12]$N \Delta_{\tau}$ is the swarm recycle period. It is the duration of walking through the full micro-path. $\boldsymbol{v}$ is a vector.
$\Delta_{\tau}$ is the progression step size.

### 16.2 Maximum speed

The micro-path wraps along the movement path of the swarm. In the most extreme case the path fits on a part of a single geodesic. In that case the swarms maximum speed is reached. The whole swarm is stretched along the geodesic. All hops are part of the geodesic.

In the case of an open micro-path:

$$
\Delta_{\text {smax }}=1 / 2 \sum_{i}^{N}\left|h_{i}\right|
$$

In the case of a closed micro-path:

$$
\Delta_{s m a x}=1 / 4 \sum_{i}^{N}\left|h_{i}\right|
$$

$\Delta_{\text {smax }}$ is a real number.
This move still takes N progression steps. Thus the maximum swarm speed $c$ is

$$
c=\frac{\Delta_{\text {smax }}}{N \Delta_{\tau}}=\frac{N \overline{\left|h_{\imath}\right|}}{n N \Delta_{\tau}}=\frac{\overline{\left|h_{\imath}\right|}}{n \Delta_{\tau}}
$$

Here $\overline{\left|h_{l}\right|}$ is the average value of the hop size $\left|h_{i}\right|$. $n$ is 2 for an open path and 4 for a closed path.

A fixed maximum speed of building blocks goes together with a fixed size of the length of the micro-path and a fixed number $N$ of elements in the swarm.

Two types of articles exist

- Open path particles
- Closed path particles

At any speed holds:

$$
N_{\text {closed }} \geq N_{\max } \geq 2 N_{\text {open }}
$$

At maximum speed no oscillations take place. In that case:

$$
N_{\text {closed }}=N_{\max } \geq 2 N_{\text {open }}
$$

An open path particle may contain less than $1 / 2 N_{\max }$ hops.

A transition from closed path in open path splits the closed path particle in two open path particles.
The converse transition combines two open path particles into a closed path particle.

The existence of a generally valid maximum speed of building blocks is the base of special relativity theory.

In contemporary physics c is considered to be equivalent to the speed of free information transfer.

### 16.2.1 Closed path particles

For closed particles hold:
$N$ depends on the oscillation mode.

$$
\begin{aligned}
& m^{2}=\sum_{i}^{N}\left|h_{i}\right|^{2}=N \overline{\left|h_{\imath}\right|^{2}} \\
& \sigma_{\left|h_{i}\right|}=\sqrt{\sum_{i}^{N}\left(\left|h_{i}\right|-\overline{\left|h_{\imath}\right|}\right)^{2}}
\end{aligned}
$$

$\sigma_{\left|h_{i}\right|}$ is the standard deviation of the step size $\left|h_{i}\right|$.

$$
\begin{aligned}
& \begin{aligned}
& \sigma_{\left|h_{i}\right|}^{2}=\operatorname{var}\left(\left|h_{i}\right|\right)=\sum_{i}^{N}\left\{\left|h_{i}\right|^{2}-\left(\overline{\left|h_{\imath}\right|}\right)^{2}\right\} \\
&=N \overline{\left|h_{\imath}\right|^{2}}-N\left(\overline{\left|h_{\imath}\right|}\right)^{2} \\
&= m^{2}-N\left(c \Delta_{\tau}\right)^{2} \\
&= m^{2}-\frac{\Delta_{s}^{2}}{N} \\
& m^{2}=\sigma_{\left|h_{i}\right|}^{2}+\frac{\Delta_{s}^{2}}{N}
\end{aligned}
\end{aligned}
$$

$N, c, \Delta_{\tau}$ and $\Delta_{s}$ are supposed to be constants. Thus $m$ depends on $\sigma_{\left|h_{i}\right|}$. For particles at rest this is also a constant.

### 16.3 Unfolding the micro-path

The micro-path can be unfolded in many different ways.

An open micro-path unfolds to a condition that leads to a maximum speed in the condition that all folds are arranged along a part of a single geodesic. In that condition it moves at maximum speed $c$.

A closed micro-path may unfold in an oscillation that keeps the micro-path a closed cycle and that follows a
closed geodesic. This may be done such that the location of the full swarm follows a part of a single open geodesic.

A closed micro-path may unfold such that in the extreme condition the closed path is aligned up and down along a single geodesic. In that condition the particle moves at maximum speed $c$.

The closed path particle resists unfolding. In its most stable condition the closed micro-path is maximally folded. In that case the swarm is at rest.

### 16.3.1 Unfolding to harmonic oscillation

An oscillation that unfolds the closed micro-path can be interpreted as an harmonic oscillation ${ }^{16}$ and will occur in a range of modes that each corresponds to a discrete oscillation energy. This can be understood when the oscillation increases the number of hops.
Adding harmonic oscillation energy to a closed path particle can occur by absorbing the energy of an open path particle that contains the corresponding number of hops.
${ }^{16}$ A quantum harmonic oscillation increases the energy of the object in a discrete fashion. Adding hops does the same. However, the energy levels can be better understood via the harmonic oscillation.

Increasing the energy higher than the maximum oscillation energy level will result in a displacement that corresponds with a linear movement of the closed path particle. That movement may eventually break the closure of the micro-path.

An oscillating swarm may also move with respect to its oscillation center. That condition can be treated as the superposition of a movement and an oscillation at a fixed center location.
The harmonic oscillation can correspond to one of a range of discrete energies. Jumping from one energy mode to another requires a special procedure, which results in the emission or absorption of a photon or gluon ${ }^{17}$.

### 16.3.2 Relativity

$m$ is the total energy. According to relativity theory the energy of the swarm at rest is $m_{r}$ where ${ }^{18}$

$$
m_{r}^{2}=m^{2}-\left(m \frac{v}{c}\right)^{2}
$$

[^13]$$
m=\frac{m_{r}}{\sqrt{1-\left(\frac{v}{c}\right)^{2}}}
$$

Here $v$ is a real number. Thus if $v$ approaches $c$, then the total energy $m$ goes to infinity.
Thus completely unfolding the micro-path is effectively made impossible by the operating system.

### 16.4 Interpretation

The swarm has a built-in mechanism that resists the unfolding of the micro-path.

Breaking the micro-path oscillation cycle results in a moving object that itself can be considered as a swarm at rest or a swarm at a lower oscillation mode.

## 17 Lorentz transformation

### 17.1 Reference frames

Differences between positions in subsequent members of the sequence of HBM pages can be interpreted as displacements. The displacement is a coordinate transformation. For the properties of this transformation it does not matter where the displacement starts or in which direction it is taken.

The same holds for displacements that concern sequence members that are located further apart. The corresponding displacements form a group. The displacement is a function of both the position and the sequence number. The sequence number is represented by the progression step number.

The Lorentz transform concerns the sequence of Hilbert spaces and NOT the Gelfand triple. With other words the transform does not concern the field that represents space! The reference frames that are treated here are pairs of locations and progression step numbers. These are discrete objects. For simplicity the next section treats them as quaternions that have real, rather than rational coefficients.
The locations are locations of swarms (Qpatches) and not locations of step stones (Qtargets).

### 17.2 The transformation

The displacement $\tau, z \rightarrow \tau^{\prime}, z^{\prime}$ can be interpreted as a coordinate transformation and can be described by a matrix. Here $\tau$ is progression and it is the real part of the quaternion. $z$ represents the imaginary part of the quaternion. Both quaternions concern characteristic data of a single complete swarm. From the previous chapter we know that the dynamics of the swarm is limited by a
maximum speed $c$. The condition of the swarm at the two reference frames can be represented by a matrix.

$$
\left[\begin{array}{l}
\tau^{\prime}  \tag{1}\\
z^{\prime}
\end{array}\right]=\left[\begin{array}{ll}
\gamma & \delta \\
\beta & \alpha
\end{array}\right]\left[\begin{array}{l}
\tau \\
z
\end{array}\right]
$$

The matrix elements are interrelated. When the displacement concerns a uniform movement, the interrelations of the matrix elements become a function of the speed $v$. Here $v$ is the speed measured as displacement per progression interval. The group properties together with the isomorphism of space fix the interrelations.

$$
\left[\begin{array}{l}
\tau^{\prime}  \tag{2}\\
z^{\prime}
\end{array}\right]=1 / \sqrt{1+k v^{2}}\left[\begin{array}{cc}
1 & k v \\
-v & 1
\end{array}\right]\left[\begin{array}{l}
\tau \\
z
\end{array}\right]
$$

If $k$ is positive, then there may be transformations with $k v^{2} \gg 1$ which transform progression into a spatial coordinate and vice versa. This is considered to be unphysical. The Hilbert book model also supports that vision.

The condition $\mathrm{k}=0$ corresponds to a Galilean transformation

$$
\left[\begin{array}{l}
\tau^{\prime}  \tag{3}\\
z^{\prime}
\end{array}\right]=\left[\begin{array}{cc}
1 & 0 \\
-v & 1
\end{array}\right]\left[\begin{array}{l}
\tau \\
z
\end{array}\right]
$$

The condition $k<0$ corresponds to a Lorentz transformation. We can set $k c^{2}=-1$, where $c$ is an invariant speed that corresponds to the maximum of $v$.

$$
\begin{aligned}
& {\left[\begin{array}{l}
\tau^{\prime} \\
z^{\prime}
\end{array}\right]=1 / \sqrt{1-v^{2} / c^{2}}\left[\begin{array}{cc}
1 & -v / c^{2} \\
-v & 1
\end{array}\right]\left[\begin{array}{l}
\tau \\
z
\end{array}\right]} \\
& \tau^{\prime}=\frac{1}{\gamma}\left(\tau-\frac{v z}{c^{2}}\right) \\
& z^{\prime}=\frac{1}{\gamma}(z-v \tau) \\
& \tau=\frac{z-\gamma z^{\prime}}{v}
\end{aligned}
$$

### 17.3 Phenomena

Lorentz transformations introduce the phenomena that go together with relativity, such as length contraction, progression dilatation and relativity of simultaneity that occur when two inertial reference frames are considered. Within an inertial reference frame Newton's first law holds and in our consideration all points move at the same speed.

$$
\begin{aligned}
& \tau_{1}^{\prime}=\frac{1}{\gamma}\left(\tau_{1}-\frac{v z_{1}}{c^{2}}\right) \\
& \tau_{1}^{\prime}-\tau_{2}^{\prime}=\frac{1}{\gamma}\left(\tau_{1-} \tau_{2}-\frac{v\left(z_{1}-z_{2}\right)}{c^{2}}\right)
\end{aligned}
$$

With $z_{1}=z_{2}$ this delivers:

$$
\Delta \tau^{\prime}=\frac{\Delta \tau}{\gamma}
$$

This explains progression dilation

$$
\begin{aligned}
& z_{1}^{\prime}=\frac{1}{\gamma}\left(\mathrm{z}_{1}-v \tau_{1}\right) \\
& \Delta z^{\prime}=\frac{\Delta z}{\gamma}
\end{aligned}
$$

This explains length contraction.

These phenomena occur in the Hilbert Book Model when different members of the sequence of Hilbert spaces are compared. Usually the inertial frames are spread over a range of Hilbert book pages.

Since the members of the sequence represent static status quos and swarms are only defined within such a static status quo, the relativity of simultaneity restricts the selection of the inertial frames. Each of the inertial frames must be situated completely in a single member of the sequence.

The indicated phenomena arise when one inertial frame is observed from the other inertial frame.

### 17.4 Relativistic energy and momentum

The micro-path of the elementary particle stretches along the movement path. On the other hand, for observing objects the length in the direction of movement contracts. These effects compensate. Time dilation is not compensated.

The observing object gets its information via the wave fronts that at super-high frequency are emitted by the observed elementary particle. This super-high frequency cannot be observed.

Displacements of the considered moving particle will be observed to be smaller. Thus its momentum will be affected. Phenomena inside the realm of the particle run slower.
$\boldsymbol{p}$ and $p_{0}$ are both generators of increments of respectively $\delta \boldsymbol{q}$ and $\delta q_{0}$

$$
\delta g=p \delta q
$$

The real part of $\delta g$ is used as the generator in the unitary transform, which is invariant against inversion ${ }^{19}$.

$$
\begin{aligned}
& \delta g_{0}= p_{0} \delta q_{0}-\langle\boldsymbol{p}, \delta \boldsymbol{q}\rangle \\
&=\delta g_{0}^{\prime}=p_{0}^{\prime} \delta q_{0}^{\prime}-\left\langle\boldsymbol{p}^{\prime}, \delta \boldsymbol{q}^{\prime}\right\rangle \\
& \delta q_{0}= \frac{\delta q_{0}^{\prime}-\left\langle\delta \boldsymbol{q}^{\prime}, \frac{\boldsymbol{v}}{c^{2}}\right\rangle}{\gamma} \\
& \delta \boldsymbol{q}= \frac{\delta \boldsymbol{q}^{\prime}-\boldsymbol{v} \delta q_{0}^{\prime}}{\gamma} \\
& p_{0} \delta q_{0}-\langle\boldsymbol{p}, \delta \boldsymbol{q}\rangle
\end{aligned} \quad \begin{aligned}
& =p_{0} \frac{\delta q_{0}^{\prime}-\left\langle\delta \boldsymbol{q}^{\prime}, \frac{\boldsymbol{v}}{c^{2}}\right\rangle}{\gamma}-\left\langle\boldsymbol{p}, \frac{\delta \boldsymbol{q}^{\prime}-\boldsymbol{v} \delta q_{0}^{\prime}}{\gamma}\right\rangle
\end{aligned}
$$

${ }^{19}$ This approach was taken according to ideas of Shan Gao in his "Derivation of the energy momentum relation".

$$
\begin{aligned}
&=\left(\left\langle\frac{\boldsymbol{p}}{\gamma}, \boldsymbol{v}\right\rangle+\frac{p_{0}}{\gamma}\right) \delta q_{0}^{\prime}-\left\langle\frac{\boldsymbol{p}}{\gamma}, \delta \boldsymbol{q}^{\prime}\right\rangle+\left\langle\frac{p_{0}}{\gamma} \frac{\boldsymbol{v}}{c^{2}}, \delta \boldsymbol{q}^{\prime}\right\rangle \\
&=\frac{\langle\boldsymbol{p}, \boldsymbol{v}\rangle+p_{0}}{\gamma} \delta q_{0}^{\prime}-\left\langle\delta \boldsymbol{q}^{\prime}, \frac{\boldsymbol{p}+\frac{\boldsymbol{v}}{c^{2}}}{\gamma}\right\rangle \\
&=p_{0}^{\prime} \delta q_{0}^{\prime}-\left\langle\boldsymbol{p}^{\prime}, \delta \boldsymbol{q}^{\prime}\right\rangle
\end{aligned}
$$

$$
\begin{aligned}
& \boldsymbol{p}^{\prime}=\frac{\boldsymbol{p}+p_{0} \frac{\boldsymbol{v}}{c^{2}}}{\gamma} \\
& p_{0}^{\prime}=\frac{\langle\boldsymbol{p}, \boldsymbol{v}\rangle+p_{0}}{\gamma}
\end{aligned}
$$

These equations represent the relativistic energy and momentum equations.

The formulas can be reversed:

$$
\begin{gathered}
p_{0}=\gamma p_{0}^{\prime}-\langle\boldsymbol{p}, \boldsymbol{v}\rangle \\
\boldsymbol{p}=\gamma \boldsymbol{p}^{\prime}-p_{0} \frac{\boldsymbol{v}}{c^{2}} \\
p_{0}=\gamma p_{0}^{\prime}-\gamma\left\langle\boldsymbol{p}^{\prime}, \boldsymbol{v}\right\rangle+p_{0} \frac{v^{2}}{c^{2}} \\
p_{0}=p_{0}^{\prime}-\left\langle\boldsymbol{p}^{\prime}, \boldsymbol{v}\right\rangle \\
\boldsymbol{p}=\gamma \boldsymbol{p}^{\prime}-\left(\gamma p_{0}^{\prime}-\left\langle\boldsymbol{p}^{\prime}, \boldsymbol{v}\right\rangle\right) \frac{\boldsymbol{v}}{c^{2}}
\end{gathered}
$$

### 17.5 Analyzing the imaginary part of the generator

The imaginary part of the generator $\delta f$ might also be invariant.

$$
\begin{aligned}
& \delta f=p \delta q \\
& \delta \boldsymbol{f}=\boldsymbol{p} \delta q_{0}+p_{0} \delta \boldsymbol{q}+\boldsymbol{p} \times \delta \boldsymbol{q} \\
& \delta \boldsymbol{f}=\delta \boldsymbol{f}^{\prime}
\end{aligned}
$$

So let us investigate whether:

$$
\begin{gathered}
\boldsymbol{p} \delta q_{0}+p_{0} \delta \boldsymbol{q}+\boldsymbol{p} \times \delta \boldsymbol{q} \\
\stackrel{\partial}{=} \boldsymbol{p}^{\prime} \delta q_{0}^{\prime}+p_{0}^{\prime} \delta \boldsymbol{q}^{\prime}+\boldsymbol{p}^{\prime} \times \delta \boldsymbol{q}^{\prime} \\
=\boldsymbol{p} \frac{\delta q_{0}^{\prime}-\left\langle\delta \boldsymbol{q}^{\prime}, \frac{\boldsymbol{v}}{c^{2}}\right\rangle}{\gamma}+p_{0} \frac{\delta \boldsymbol{q}^{\prime}-\boldsymbol{v} \delta q_{0}^{\prime}}{\gamma}+\boldsymbol{p} \times \frac{\delta \boldsymbol{q}^{\prime}-\boldsymbol{v} \delta q_{0}^{\prime}}{\gamma}
\end{gathered}
$$

Or:

$$
\begin{aligned}
& \boldsymbol{p}^{\prime} \delta q_{0}^{\prime}+p_{0}^{\prime} \delta \boldsymbol{q}^{\prime}+\boldsymbol{p}^{\prime} \times \delta \boldsymbol{q}^{\prime} \\
& \stackrel{\partial}{=}\left(\gamma \boldsymbol{p}^{\prime}-\left(p_{0}^{\prime}-\left\langle\boldsymbol{p}^{\prime}, \boldsymbol{v}\right\rangle\right) \frac{\boldsymbol{v}}{c^{2}}\right) \frac{\delta q_{0}^{\prime}-\left\langle\delta \boldsymbol{q}^{\prime}, \frac{\boldsymbol{v}}{c^{2}}\right\rangle}{\gamma}
\end{aligned}
$$

$$
\begin{gathered}
+\left(p_{0}^{\prime}-\left\langle\boldsymbol{p}^{\prime}, \boldsymbol{v}\right\rangle\right) \frac{\delta \boldsymbol{q}^{\prime}-\boldsymbol{v} \delta q_{0}^{\prime}}{\gamma} \\
+\left(\gamma \boldsymbol{p}^{\prime}-\left(p_{0}^{\prime}-\left\langle\boldsymbol{p}^{\prime}, \boldsymbol{v}\right\rangle\right) \frac{\boldsymbol{v}}{c^{2}}\right) \times \frac{\delta \boldsymbol{q}^{\prime}-\boldsymbol{v} \delta q_{0}^{\prime}}{\gamma}
\end{gathered}
$$

Or:

$$
\begin{aligned}
& \left(p_{0}^{\prime}-\left\langle\boldsymbol{p}^{\prime}, \boldsymbol{v}\right\rangle\right) \frac{v}{c^{2}} \frac{\left\langle\delta \boldsymbol{q}^{\prime}, \frac{\boldsymbol{v}}{c^{2}}\right\rangle}{\gamma}+\left\langle\boldsymbol{p}^{\prime}, \boldsymbol{v}\right\rangle \frac{v \delta q_{0}^{\prime}}{\gamma} \\
& +\left(p_{0}^{\prime}-\left\langle\boldsymbol{p}^{\prime}, \boldsymbol{v}\right\rangle\right) \frac{v}{c^{2}} \times \frac{\boldsymbol{v} \delta q_{0}^{\prime}}{\gamma} \stackrel{0}{=}
\end{aligned}
$$

Or:

$$
p_{0}^{\prime} \stackrel{\stackrel{2}{=}}{=}\left(1+\frac{c^{4} \delta q_{0}^{\prime}}{\left\langle\delta \boldsymbol{q}^{\prime}, \boldsymbol{v}\right\rangle}\right)\left\langle\boldsymbol{p}^{\prime}, \boldsymbol{v}\right\rangle
$$

This does not seem to be generally valid for all values of $\delta q$ and $p$. The conclusion is that the invariance only holds for the real part of the generator $\delta g$.

## 18 Movement in an embedding field

We use the strategy that Denis Sciama applied in his "On the origin of inertia" ${ }^{2}$.
An elementary particle represents an artifact in its embedding continuum. The "strength" of the artifact can be characterized by the total energy $m$, which acts as a charge in a surrounding field. Here the field equations for charged object apply. This consideration also holds for composites.

The influence of the charge is determined by the Green's function. For a free moving elementary particle that shows a Gaussian density distribution of its location swarm this Green's function has a shape:

$$
\frac{E r f(r)}{r}
$$

Already at a small distance from the center this can be approached by

$$
\frac{1}{r}
$$

Together with the strength $m$ this corresponds to a potential.

[^14]$$
\varphi=\iiint_{V} \frac{\rho}{\boldsymbol{r}} d V
$$

Here $\rho$ represents the "charge" density distribution of the contributing $N$ "charges" of (average) size $m / N$ to the potential $\varphi$.

$$
\begin{gathered}
\varphi \propto m \\
\varphi=g m
\end{gathered}
$$

$g$ is a proportionality factor.
A uniformly moving potential with speed $v$ corresponds to a vector potential $\boldsymbol{A}$.

$$
\begin{aligned}
\boldsymbol{A} & =\iiint_{V} \frac{\rho}{\boldsymbol{r}} \boldsymbol{v} d V=\varphi \boldsymbol{v} \\
A & =\phi+\varphi \boldsymbol{v}
\end{aligned}
$$

$\phi$ is the scalar part of the embedding field $\boldsymbol{A}$.
$\boldsymbol{A}$ is the imaginary part. We will use the quaternionic differential field equations.

If the scalar potential $\varphi$ accelerates, then this goes together with a new vector field $\boldsymbol{E}$.

$$
\frac{\partial A}{\partial \tau}=\frac{\partial \phi}{\partial \tau}+\frac{\partial \varphi}{\partial \tau} \boldsymbol{v}+\varphi \dot{\boldsymbol{v}}
$$

We separate the imaginary part of this equation:

$$
\frac{\partial A}{\partial \tau}=\frac{\partial \varphi}{\partial \tau} v+\varphi \dot{\boldsymbol{v}}
$$

Further we know:

$$
\begin{align*}
& E_{0}=\frac{\partial \phi}{\partial \tau}-\langle\boldsymbol{\nabla}, \boldsymbol{A}\rangle \\
& \boldsymbol{E}=\frac{\partial \boldsymbol{A}}{\partial \tau}+\boldsymbol{\nabla} \phi+\boldsymbol{\nabla} \times \boldsymbol{A}  \tag{6}\\
& \boldsymbol{E}=\frac{\partial \varphi}{\partial \tau} \boldsymbol{v}+\varphi \dot{\boldsymbol{v}}+\boldsymbol{\nabla} \phi+\boldsymbol{\nabla} \times \boldsymbol{A}
\end{align*}
$$

All terms on the right side, except the second term are very small.

$$
\boldsymbol{E} \approx \varphi \dot{\boldsymbol{v}}=g m \dot{\boldsymbol{v}}
$$

Thus the acceleration goes together with an extra vector field $\boldsymbol{E}$ that counteracts the acceleration of the moving potential. In physical terminology $\boldsymbol{E}$ is a force field. In fact $\boldsymbol{E}$ represents the action of inertia.

With other words the fact that elementary particles emit wave fronts, whose averaged effect is noticeable as a
gravitation potential, results in the existence of inertia. This effect represents itself as a force field.

## 19 Photons

### 19.1 Basic phenomena

It is not yet clear why the maximum speed of building blocks $c$ conforms to the maximum speed of information transfer. For that reason we must investigate the photons, which are the carriers of that information.

The rest mass of photons equals zero. They may progress in space for billions of years and after that they can still be detected by a suitable detector. They can be interpreted as one dimensional wave fronts. With other words they are governed by the one dimensional Huygens principle. Their sources may emit the photons in isotropic patterns, but each separate photon is a one dimensional wave front.
The photons are emitted in energy quanta that have a discrete energy, which is encoded in the frequency of the photon.

Atoms may emit photons. The electrons that belong to the atom travel in oscillation paths that are hidden by the geoditches, which are produced by these oscillating electrons. Due to this coverage the electromagnetic radiation that normally would be emitted, does not appear. The up and down hopping in the micro-path completely disturbs the generation of the EM radiation. However, the kinetic energy of the oscillation adds to the kinetic
energy of the hops and thus it adds to the total energy of the electron and as a consequence it adds to the mass of the atom.

### 19.2 Photon emission

If the electron switches to a lower energy oscillation mode, then a corresponding photon is emitted.

The HBM suggests that this emission takes the completion of a full micro-path, which has a fixed duration. That time is used to encode the energy jump into the photon.

The amplitudes of the regeneration wave fronts diminish with progression, but the photon wave front keeps its amplitude.

In order to produce the one-dimensional wave front or series of wave fronts, the electron must emit the photon from nearly the same location and in exactly the same direction.

### 19.2.1 Explanation

The situation can simply be explained, however that explanation has strong implications.

The elementary particle can obtain extra hops by participating in a harmonic oscillation. The number of extra hops corresponds to the energy of the oscillation. When a photon is emitted it takes a part of these extra hops as its bunch of energy. The extra hops are sent in a fixed progression interval. Thus the energy is encoded in the frequency of the photon.

During this special cycle of the micro path, the electron emits the photon in a preselected direction. After this procedure the oscillation restarts in the other oscillation mode.

The duration of the special micro-path cycle must not differ from the standard micro-path cycle. Also the harmonic oscillation must not influence the duration of the micro-path cycle. With other words, during the harmonic oscillation the hops are smaller.

## Harmonic oscillation energy is related to the number of hops in a micro-path cycle.

If this explanation fits, then this is a strong indication that the duration of the regeneration of elementary particles is fixed.

### 19.3 Red shift

Due to space expansion as a function of progression the photon becomes red shifted. The wave length of the photon changes, but the number of progression ticks that this train of wave fronts last increases as well. The train still fits the duration of the special micro-path. However it corresponds with a lower energy jump.

### 19.1 Photon absorption

Photon absorption is the reverse of photon emission. At the arrival of a fitting photon the current oscillation and the regeneration of the elementary particle enter a special regeneration cycle in which the hops of the photon are added to the hops of the elementary particle. During the special cycle of the micro-path the photon energy is absorbed. Next the oscillation is restarted in the new oscillation mode.

### 19.2 Annihilation of elementary particles

The annihilation of a closed path elementary particle involves the emission of all hops into two photons that have the same frequency and that when the particle is at rest will leave in opposite directions.

### 19.1 Creation of elementary particles

The creation of a closed path elementary particle involves the absorption of the hops of two photons that
have the same frequency and arrive from nearly opposite directions. The created particle is at rest with respect to the frame in which the photons have opposite directions.

## 20 Detection patterns

The wave nature of free elementary particles and photons can be visualized by their detection patterns. Both free elementary particles and photons can be characterized by the probability density distribution of their potential detection locations. This probability density distribution has a Fourier transform. As a consequence the probability density distribution is a wave package.

The shape of the actual probability density distribution is affected by space curvature and by boundary conditions. Since these conditions change with progression, the actual probability density distribution is also a function of progression. Thus the movement path of the considered object plays an important role.

Large sets of nearly similar objects that have such density distributions may show interference of the waves that are contained in the wave packages. As a consequence the detection patterns may take the form of interference patterns. This gives the impression that the separate objects show wave behavior. However, this behavior only appears when sets of similar objects interfere and they are the result of the interaction of the probability density distributions of the particles and the boundary conditions. The potentials of the objects may
interact and as such the potentials of other objects are part of the boundary conditions of the considered object.

If a trail of elementary particles that all are prepared in the same way passes a given potential configuration or an aperture configuration, then the geometric configuration of these boundary conditions acts as an imaging mechanism.

The configuration of this imaging mechanism has an Optical Transfer Function that affects the resulting probability density distribution that defines the detection probability of the objects that are being imaged. The OTF is the Fourier transform of the geometric potential or aperture configuration.

The Fourier transform of the probability density distribution of the considered object is the mapping quality characteristic of that object.

At every progression instant the product of this OTF and the mapping quality characteristic of a moving object determine the resulting detection density probability distribution of the moving object.

At the instant of detection, part of the influencing distributions is moving with the detected object, while the other part is rather static and represents the boundary
conditions. Their Fourier transforms multiply in order to form the Fourier transform of the effective probability density distribution. The object just obeys the restrictions that are set by this final distribution.

For a set of objects the angular distributions and the distribution of properties such as color (for photons), energy and charge play a role. A homogeneous set gives a different detection pattern than an inhomogeneous set.

## 21 Spin suggestion

### 21.1 The micro-path

The micro-path can be walked in two directions
The micro-path may implement a quasi rotation If not closed the path defines a swarm movement step. A closed micro-path can still be stretched along an oscillation path.

### 21.2 The generation of the swarm

The generation of a planned swarm is started by a Poisson process that delivers a germ. This germ is used by the subsequent binomial process that is implemented by a binomial process. The spread function uses a spherical coordinate system, which can be characterized by three parameters. They are used in the sequence:

1. The zenith vector $r$
a. The germ selects the value of $r$.
2. The polar angle $\varphi$.
a. The polar angle axis is perpendicular on the zenith vector
b. The polar angle has range 0 to $2 \pi$
3. The azimuth angle axis ( $\theta$ range $\pi$ )
a. The azimuth angle axis is perpendicular on
i. the zenith vector
ii. polar angle axis
b. The azimuth angle has range 0 to $\pi$ In this case the spin vector is along the azimuth angle.
Or the parameters are used in the sequence:
4. The zenith vector $r$
a. The germ selects the value of $r$.
5. The azimuth angle axis ( $\theta$ range $\pi$ )
a. The azimuth angle axis is perpendicular on
i. the zenith vector
ii. polar angle axis
b. The azimuth angle has range 0 to $\pi$
6. The polar angle $\varphi$.
a. The polar angle axis is perpendicular on the zenith vector
b. The polar angle has range 0 to $2 \pi$

In this case the spin vector is along the polar angle.

### 21.2.1 The spin axis

The spin axis can be along the $\varphi$ axis, which leads to half integer spin or along the $\theta$ axis which leads to integer spin.

## 22 The unit speed path

A difference exists between the description of the planned micro-path and the Frenet Serret description of the unit speed path of the swarm.

At any point in the Palestra and in any direction a path can be started
Also Qpatches that represent particles follow such paths In the Palestra the "length" of the quaternionic path is the coordinate time duration

$$
s(d)=\int_{0}^{d}\|d \wp\|=\int_{0}^{d}\left\|\frac{d \wp}{d \tau}\right\| d \tau
$$

$\boldsymbol{d}$ is the duration in proper time ticks.
$\boldsymbol{\tau}$ is the progression parameter. It equals proper time.
$\boldsymbol{s}$ is the coordinate time.
We investigate constant speed curves in the imaginary Palestra.
$\mathcal{R}$ is the imaginary part of $\wp$.

$$
\boldsymbol{T}=\frac{\operatorname{Im}\left(\frac{d \wp}{d s}\right)}{\left\|\operatorname{Im}\left(\frac{d \wp}{d s}\right)\right\|}=\frac{\frac{d \mathcal{R}}{d s}}{\left\|\frac{d \mathfrak{R}}{d s}\right\|}
$$

$T$ is the tangent unit vector.

$$
\boldsymbol{N}=\frac{\frac{d \boldsymbol{T}}{d s}}{\left\|\frac{d \boldsymbol{T}}{d s}\right\|}
$$

$N$ is the principle normal unit vector.
Since $\|\boldsymbol{T}\|=1$ are $\boldsymbol{N}$ and $\boldsymbol{T}$ perpendicular.

$$
B=T \times N
$$

$\boldsymbol{B}$ is the binormal unit vector
The sign of $\boldsymbol{T}, \boldsymbol{N}$, and $\boldsymbol{B}$ depends on the discrete symmetry set of the embedding field.

### 22.1 Path characteristics

$$
\frac{d \boldsymbol{T}}{d s}=\kappa \boldsymbol{N}
$$

$\kappa$ is the curvature.

$$
\frac{d \boldsymbol{N}}{d s}=-\kappa \boldsymbol{T}+\tau \boldsymbol{B}
$$

$\tau$ is the torque.

$$
\frac{d \boldsymbol{B}}{d s}=-\tau \boldsymbol{N}
$$

## 23 Elementary particle types

The coupling equation for the electron and the positron as it is depicted by Paul Dirac, gives us a lead how to interpret this equation for other types of elementary particles.

The HBM suggests that elementary particles are the result of couplings between the members of the symmetry flavor bundle that constitutes the Palestra.

One of these members is the reference member and has the same symmetry flavor as the parameter space of the bundle has.

One of the member fields plays the role of the quantum state function of the particle. The other field plays the role of the embedding field.

The operating system maps the quantum state function on the embedding field. Only the partial mirror of the quantum state function in this embedding field is used in the coupling.

Thus the coupling equation for elementary particles will be:

## 24 Generations

## 25 Superposition

In a composite superposition of elementary particles can be considered in configuration space and it can be considered in Fourier space. At each progression instant these superposition coefficients may differ. With other words they are functions of progression. The Fourier transform $\mathcal{F}$ is a linear operator.

$$
\begin{align*}
& \mathcal{F}(g(q))=\tilde{g}(p)  \tag{1}\\
& \mathcal{F}(a g(q)+b h(q))=a \tilde{g}(p)+b \tilde{h}(p) \tag{2}
\end{align*}
$$

With a superposition in Fourier space the superposition coefficients can be interpreted as displacement generators.

$$
\begin{align*}
\mathrm{g}(q)=\nabla f(q) & =\nabla_{0} f_{0}(q) \mp\langle\boldsymbol{\nabla}, \boldsymbol{f}(q)\rangle  \tag{3}\\
& \pm \nabla_{0} \boldsymbol{f}(q)+\boldsymbol{\nabla} f_{0}(q) \\
& \pm( \pm \boldsymbol{\nabla} \times \boldsymbol{f}(q)) \\
\tilde{\mathrm{g}}(k)=\mathrm{k} \tilde{f}(k) & =\mathrm{k}_{0} \widetilde{f}_{0}(k) \mp\langle\mathbf{k}, \tilde{\boldsymbol{f}}(k)\rangle  \tag{4}\\
& \pm \mathrm{k}_{0} \tilde{\boldsymbol{f}}(k)+\mathbf{k} \tilde{f}_{0}(k) \\
& \pm( \pm \mathbf{k} \times \tilde{\boldsymbol{f}}(k))
\end{align*}
$$

Thus superposition coefficients that are a function of progression can implement internal oscillations of the constituents.
The internal oscillation paths become internal geoditches. Together these geoditches implement the binding of the elementary particles into the composite. On the other hand the geoditches obscure the locations of the constituents. The hopping inside the micro-paths also disturbs the generation of EM radiation. With other words the identity of the constituents is hidden from the environment of the composite.

### 25.1 Entanglement

The fact that the quantum state function of the composite equals the superposition of the quantum state functions of the constituents is usually interpreted as the entanglement of the composite.

### 25.2 Pauli principle

If two components of an entangled (sub)system that have the same quantum state function are exchanged, then we can take the system location at the center of the location of the two components. Now the exchange means for bosons that the (sub)system quantum state function is not affected:

$$
\begin{aligned}
\forall_{\alpha, \beta}\{\alpha \varphi(-x) & +\beta \varphi(x)=\alpha \varphi(x)+\beta \varphi(-x)\} \\
\Rightarrow \varphi(-x) & =\varphi(x)
\end{aligned}
$$

And for fermions that the corresponding part of the (sub)system quantum state function changes sign.

$$
\begin{aligned}
& \forall_{\alpha, \beta}\{\alpha \varphi(-x)+\beta \varphi(x)=-\alpha \varphi(x)-\beta \varphi(-x)\} \\
& \Rightarrow \varphi(-x)=-\varphi(x)
\end{aligned}
$$

This conforms to the Pauli principle. It also indicates that the operating system, which controls the entanglement, takes care of the fact that if one of these two twin components exposes any of its properties (e.g. its spin) that it has IMMEDIATE effect on the properties of the other component.

### 25.3 Gauge transformations

In quaternionic quantum mechanics the superposition of elementary particles achieve what in complex quantum mechanics gauge transformations will do. When the change of the quaternionic superposition coefficients restricts to phase shifts, then the change represents a complex gauge transformation.

## 26 Non-locality

### 26.1 Within a particle

In the Hilbert Book Model, the notion of non-locality exists. This is due to the fact that nature's building blocks have a set of discrete properties that can be prepared or that can be observed via indirect means that does not touch their state, while the swarm that represents the building block may extend over relative large distances.

So measuring the same property at nearly the same instant at quite different locations will give the same result.

If shortly before that these measurements were performed the property is changed, then it might give the impression that an "instant action at a distance" occurred, because neither light nor the wave fronts that constitute the potentials could bridge these locations in the period between the two measurements.

The explanation is that the building block at each progression instant hops to a different step stone while at the same time these step stones may lay far apart.

Apart from the property measurements, in this process no information transfer needs to take place.

At each arrival at a step stone the building block transmits contributions to its potentials. If the measurement uses these potentials, then the building block is not affected by the measurement ${ }^{21}$.

According to this explanation, at least one progression step must separate the two measurements.

### 26.2 Between particles

Non-locality between particles means that the reach of control of the operating system covers multiple elementary particles. This can be caused by the fact that the particles are considered to form a an entangled system. In that system the Pauli principle will take its role. It means that if the constituting particles are fermions, then at all progression instances they must al take different states. This sounds familiar in composites and atoms, but it can also happen in other entangled systems.

The coupling of entangled particles and the support of the Pauli principle are supported by special capabilities of the operating system.

[^15]
## 27 Modular construction

### 27.1 Modularization

A very powerful influencer is modularization. Together with the corresponding encapsulation it has a very healthy influence on the relational complexity of the ensemble of objects on which modularization works.
The encapsulation takes care of the fact that most relations are kept internal to the module.

When relations between modules are reduced to a few types, then the module becomes reusable.

The most influential kind of modularization is achieved when modules can be configured from lower order modules.

Elementary particles can be considered as the lowest level of modules. All composites are higher level modules.

When sufficient resources in the form of reusable modules are present, then modularization can reach enormous heights.
On earth it was capable to generate intelligent species.

### 27.1.1 Complexity

Potential complexity of a set of objects is a measure that is defined by the number of potential relations that exist between the members of that set.

If there are $n$ elements in the set, then there exist $n^{*}(n-1)$ potential relations.

Actual complexity of a set of objects is a measure that is defined by the number of relevant relations that exist between the members of the set.

In human affairs and with intelligent design it takes time and other resources to determine whether a relation is relevant or not. Only an expert has the knowledge that a given relation is relevant. Thus it is advantageous to have as little irrelevant potential relations as is possible, such that mainly relevant and preferably usable relations result.

Physics is based on relations. Quantum logic is a set of axioms that restrict the relations that exist between quantum logical elements.
In applications in physics the quantum logical elements are not propositions, but instead they are construction elements.

Via its isomorphism with Hilbert spaces quantum logic forms a fundament for quantum physics.

Classical logic is a similar set of restrictions that define how we can communicate logically. Like classical logic, quantum logic only describes static relations.

Traditional quantum logic does not treat physical fields and it does not touch dynamics. However, the model that is based on traditional quantum logic can be extended such that physical fields are included as well and by assuming that dynamics is the travel along subsequent versions of extended quantum logics, also dynamics will be treated.

The set of elements of traditional quantum logic is isomorphic with the set of closed subspaces of a Hilbert space. The Hilbert space is a mathematical construct in which quantum physicists do their investigations and calculations.
In this way fundamental physics can be constructed. Here holds very strongly that only relevant relations have significance.

### 27.1.2 Relational complexity

We define relational complexity as the ratio of the number of actual relations divided by the number of potential relations.

### 27.1.3 Interfaces

Modules connect via interfaces. Interfaces are used by interactions.
Interactions run via (relevant) relations.

Relations that act within modules are lost to the outside world of the module.

Thus interfaces are collections of relations that are used by interactions.
Inbound interactions come from the past.
Outbound interactions go to the future.
Two-sided interactions are cyclic. They are either oscillations or rotations of the inter-actor.

In physics interactions are implemented by potentials. The solutions in the Huygens principle cover both outgoing as well as incoming waves.
The outbound waves implement outbound interfaces of elementary particles.
The inbound waves implement inbound interfaces of elementary particles.

### 27.1.4 Interface types

Apart from the fact that they are inbound, outbound or cyclic the interfaces can be categorized with respect to the type of relations that they represent.

Each category corresponds to an interface type.
An interface that possesses a type and that installs the possibility to couple the corresponding module to other modules is called a standard interface.

### 27.1.5 Modular subsystems

Modular subsystems consist of connected modules.
They need not be modules. They become modules when they are encapsulated and offer standard interfaces that makes the encapsulated system a reusable object.

The cyclic interactions bind the corresponding modules together.

Like the coupling factor of elementary particles characterizes the binding of the pair of Qpatterns will a similar characteristic characterize the binding of modules. This binding characteristic directly relates to the total energy of the constituted sub-system.

Let $\psi$ represent the renormalized superposition of the involved distributions. We treat the sources and drains separately.

$$
\begin{equation*}
\nabla \psi=\phi=m \varphi \tag{1}
\end{equation*}
$$

$$
\begin{align*}
& \int_{V}|\psi|^{2} d V=\int_{V}|\varphi|^{2} d V=1  \tag{2}\\
& \int_{V}|\phi|^{2} d V=m^{2} \tag{3}
\end{align*}
$$

Here $\psi$ represents a superposition of local sources, while $\varphi$ represents a superposition of drains that themselves might reside at distant locations.

And for the anti-particles (that act as drains):

$$
\begin{equation*}
\nabla^{*} \psi_{a}^{*}=m \varphi_{a}^{*} \tag{4}
\end{equation*}
$$

Here $\psi_{a}^{*}$ represents a superposition of local drains, while $\varphi_{a}^{*}$ represents a superposition of sources that themselves might reside at distant locations.

The whole composite contains both local sources and local drains that are neutralized by local and distant counterparts.
The corresponding integral equations must define a closed system.

The binding factor is the total energy of the sub-system minus the sum of the total energies of the separate constituents.

### 27.1.6 Quantum oscillations

An interaction that runs via information transfer always runs from a previous instant to a later instant. Bidirectional interactions must be cyclic. Thus, bidirectional interfaces between system components are formed by exchanging messages into two directions or by actual oscillations. In case of an interface consisting of oscillating elementary particles the micro-path of the particle is stretched along the oscillation path.

For an elementary particle at rest, the singularities that are caused by the step stones dig a potential well. In this way a particle creates its own inertia. In case of an oscillation, the singularities that accompany the step stones dig a potential ditch that stretches along the oscillation path. This ditch forms a geodesic path in which the particle can travel freely. These oscillations can be coupled to other potential wells or ditches. In this way the nucleus and the electrons are coupled in atoms.

### 27.1.7 Relational complexity indicators

The inner product of two Hilbert vectors is a measure of the relational complexity of the combination.
A Hilbert vector represents a linear combination of atomic Hilbert elements.

When all coefficients are equal, then the vector represents an assembly of atoms.
When the coefficients are not equal, then the vector represents a weighted assembly of atoms.

For two normalized vectors $|a\rangle$ and $|b\rangle$ :

$$
\begin{align*}
& \langle a \mid a\rangle=1  \tag{1}\\
& \langle b \mid b\rangle=1  \tag{2}\\
& \langle a \mid b\rangle=0 \text { means }|a\rangle \text { and }|b\rangle \text { are not related. }  \tag{3}\\
& \langle a \mid b\rangle \neq 0 \text { means }|a\rangle \text { and }|b\rangle \text { are related. }  \tag{4}\\
& |\langle a \mid b\rangle|=1 \text { means }|a\rangle \text { and }|b\rangle \text { are optimally related. } \tag{5}
\end{align*}
$$

### 27.1.8 Modular actions

Subsystems that have the ability to choose their activity can choose to organize their actions in a modular way.

As with static relational modularization the modular actions reduce complexity and for the decision maker it eases control.

### 27.1.9 Random design versus intelligent design

At lower levels of modularization nature designs modular structures in a stochastic way. This renders the modularization process rather slow. This way of modularization is called random design.

It takes a huge amount of progression steps in order to achieve a relatively complicated structure. Still the complexity of that structure can be orders of magnitude less than the complexity of an equivalent monolith.

As soon as more intelligent subsystems arrive, then these systems can design and construct modular systems in a more intelligent way. They use resources efficiently.
This speeds the modularization process in an enormous way.

### 27.1.10 Probability distributions

Much in quantum physics has to do with the fact that the wave function has a direct relation to a probability density distribution and that the Fourier transform of this probability density distribution describes a probability distribution of momenta that describe the motion of the considered object.

The HBM relates the wave function to a coherent discrete distribution of step stones that form a stochastic micro-path. During movements or quantum oscillations the micro-path stretches along the oscillation or movement path. This is done such that the above relation between locations and momenta is kept. With other words the mechanism that controls this, keeps Heisenberg's uncertainty principle intact.

The result of these measures is that under certain conditions the step stones can form interference patterns. This leads to the particle-wave duality of quantum scale objects.

## 28 Cosmology

Where quantum physics is ruled by the coupling equation, which is a special form of the differential continuity equation is cosmology ruled by integral continuity equation.
At large scale also the sharp allocation function and its differentials plays a role. The differential can be interpreted as a metric.

The terms in the integral continuity equation

$$
\begin{equation*}
\Phi=\int_{V} \nabla \psi d V=\int_{V} \phi d V \tag{1}
\end{equation*}
$$

can be interpreted as representing the influence of a local object onto the rest of the universe or as the influence of the rest of the universe onto a local object. In the second case the influence diminishes with distance and the number of influencers increases such that the most distant contributors together poses the largest influence. These influencers sit at the information horizon. In the history of the model they are part of the birth state of the current episode of the universe. This was a state of densest packaging.

### 28.1 The differential and integral continuity equations

Let us approach the balance equation from the integral variety of the balance equation. Balance equation is another name for continuity equation.

We replace $\psi$ by $\rho, \psi_{0}$ by $\rho_{0}$ and $\boldsymbol{\psi}$ by $\boldsymbol{\rho}=\rho_{0} \boldsymbol{v} / c$.

$$
\begin{equation*}
\rho \stackrel{\text { def }}{=} \rho_{0}+\boldsymbol{\rho} \tag{1}
\end{equation*}
$$

When $\rho_{0}$ is interpreted as a charge density distribution, then the conservation of the corresponding charge ${ }^{22}$ is given by the continuity equation:

[^16]Total change within $V=$ flow into $V+$ production inside $V$

In formula this means:

$$
\begin{align*}
& \frac{d}{d \tau} \int_{V} \rho_{0} d V=\oint_{S} \widehat{\boldsymbol{n}} \rho_{0} \frac{\boldsymbol{v}}{c} d S+\int_{V} s_{0} d V  \tag{3}\\
& \int_{V} \nabla_{0} \rho_{0} d V=\int_{V}\langle\nabla, \boldsymbol{\rho}\rangle d V+\int_{V} s_{0} d V \tag{4}
\end{align*}
$$

The conversion from formula (2) to formula (3) uses the Gauss theorem ${ }^{23}$.
Here $\widehat{\boldsymbol{n}}$ is the normal vector pointing outward the surrounding surface $S$,
$\boldsymbol{v}(\tau, \boldsymbol{q})$ is the velocity at which the charge density
$\rho_{0}(\tau, \boldsymbol{q})$ enters volume $V$ and $s_{0}$ is the source density inside $V$.

In the above formula $\boldsymbol{\rho}$ stands for

$$
\begin{equation*}
\boldsymbol{\rho}=\rho_{0} \boldsymbol{v} / c \tag{5}
\end{equation*}
$$

It is the flux (flow per unit area and unit time) of $\rho_{0}$.
${ }^{23} \mathrm{http}$ ://en.wikipedia.org/wiki/Divergence theorem

The combination of $\rho_{0}(q)$ and $\boldsymbol{\rho}(q)$ is a quaternionic skew field $\rho(q)$ and can be seen as a probability density distribution (QPDD).
$\rho$ is a function of $q$.

$$
\begin{equation*}
q \stackrel{\text { def }}{=} q_{0}+\boldsymbol{q} ; q_{0}=\tau \tag{6}
\end{equation*}
$$

$\rho(q) \rho^{*}(q)$ can be seen as an overall probability density distribution of the presence of the carrier of the charge.
$\rho_{0}(q)$ is a charge density distribution. $\boldsymbol{\rho}(q)$ is the current density distribution.
This results in the law of charge conservation:

$$
\begin{gathered}
s_{0}(q)=\nabla_{0} \rho_{0}(q) \\
\mp\left\langle\boldsymbol{\nabla},\left(\rho_{0}(q) \boldsymbol{v}(q)+\boldsymbol{\nabla} \times \boldsymbol{a}(q)\right)\right\rangle \\
=\nabla_{0} \rho_{0}(q) \mp\langle\boldsymbol{\nabla}, \boldsymbol{\rho}(q)+\boldsymbol{A}(q)\rangle \\
=\nabla_{0} \rho_{0}(q) \mp \begin{array}{c}
\left.\mp \boldsymbol{v}(q), \boldsymbol{\nabla} \rho_{0}(q)\right\rangle \\
\mp\langle\boldsymbol{\nabla}, \boldsymbol{v}(q)\rangle \rho_{0}(q) \\
\mp\langle\boldsymbol{\nabla}, \boldsymbol{A}(q)\rangle
\end{array}
\end{gathered}
$$

The blue colored $\pm$ indicates quaternionic sign selection through conjugation of the field $\rho(q)$.

The field $\boldsymbol{a}(q)$ is an arbitrary differentiable vector function.

$$
\begin{equation*}
\langle\boldsymbol{\nabla}, \boldsymbol{\nabla} \times \boldsymbol{a}(q)\rangle=0 \tag{8}
\end{equation*}
$$

$\boldsymbol{A}(q) \stackrel{\text { def }}{=} \boldsymbol{\nabla} \times \boldsymbol{a}(q)$ is always divergence free. In the following we will neglect $\boldsymbol{A}(q)$.

Equation (6) represents a balance equation for charge density. What this charge actually is, will be left in the middle. It can be one of the properties of the carrier or it can represent the full ensemble of the properties of the carrier.

Up to this point the investigation only treats the real part of the full equation. The full continuity equation runs:

$$
\begin{align*}
s(q)= & \nabla \rho(q)=s_{0}(q)+\boldsymbol{s}(q)  \tag{9}\\
=\nabla_{0} \rho_{0}(q) \mp & \langle\boldsymbol{\nabla}, \boldsymbol{\rho}(q)\rangle \pm \nabla_{0} \boldsymbol{\rho}(\tau, \boldsymbol{q}) \\
& +\boldsymbol{\nabla} \rho_{0}(\tau, \boldsymbol{q}) \\
& \pm( \pm \boldsymbol{\nabla} \times \boldsymbol{\rho}(\tau, \boldsymbol{q})) \\
= & \nabla_{0} \rho_{0}(\tau, \boldsymbol{q}) \\
\mp & \left\langle\boldsymbol{v}(q), \boldsymbol{\nabla} \rho_{0}(q)\right\rangle \\
& \mp\langle\boldsymbol{\nabla}, \boldsymbol{v} \boldsymbol{q}\rangle \rho_{0}(q)
\end{align*}
$$

$$
\begin{aligned}
\pm \nabla_{0} \boldsymbol{v}(q)+\nabla_{0} \rho_{0}(q) & +\nabla \rho_{0}(q) \\
\pm\left( \pm\left(\rho_{0}(q) \nabla\right.\right. & \times \boldsymbol{v}(q)-\boldsymbol{v}(q) \\
& \left.\times \nabla \rho_{0}(q)\right)
\end{aligned}
$$

After splitting into real and imaginary equations, this leads to:

$$
\begin{align*}
s_{0}(q)= & 2 \nabla_{0} \rho_{0}(q) \mp\left\langle\boldsymbol{v}(q), \boldsymbol{\nabla} \rho_{0}(q)\right\rangle  \tag{10}\\
& \mp\langle\boldsymbol{\nabla}, \boldsymbol{v}(q)\rangle \rho_{0}(q)  \tag{11}\\
\boldsymbol{s}(q)= & \pm \nabla_{0} \boldsymbol{v}(q) \pm \boldsymbol{\nabla} \rho_{0}(q) \\
\pm\left( \pm\left(\rho_{0}(q) \nabla\right.\right. & \times \boldsymbol{v}(q)-\boldsymbol{v}(q) \\
& \left.\left.\times \nabla \rho_{0}(q)\right)\right)
\end{align*}
$$

The red sign selection indicates a change of handedness by changing the sign of one of the imaginary base vectors. Conjugation also causes a switch of handedness. It changes the sign of all three imaginary base vectors.

In its simplest form the full continuity equation runs:

$$
\begin{equation*}
s(q)=\nabla \rho(q) \tag{12}
\end{equation*}
$$

Thus the full continuity equation specifies a quaternionic distribution $s$ as a flat differential $\nabla \rho$.

When we go back to the integral balance equation, then holds for the imaginary parts:

$$
\begin{align*}
\frac{d}{d \tau} \int_{V} \boldsymbol{\rho} d V= & -\oint_{S} \widehat{\boldsymbol{n}} \rho_{0} d S-\oint_{S} \widehat{\boldsymbol{n}} \times \boldsymbol{\rho} d S  \tag{13}\\
& +\int_{V} \boldsymbol{s} d V \\
\int_{V} \nabla_{0} \boldsymbol{\rho} d V= & -\int_{V} \nabla_{0} d V-\int_{V} \boldsymbol{\nabla} \times \boldsymbol{\rho} d V  \tag{1}\\
& +\int_{V} \boldsymbol{s} d V
\end{align*}
$$

For the full integral equation holds:

$$
\begin{align*}
& \frac{d}{d \tau} \int_{V} \rho d V+\oint_{S} \widehat{\boldsymbol{n}} \rho d S=\int_{V} s d V  \tag{15}\\
& \int_{V} \nabla \rho d V=\int_{V} s d V \tag{16}
\end{align*}
$$

Here $\widehat{\boldsymbol{n}}$ is the normal vector pointing outward the surrounding surface $S, \boldsymbol{v}(q)$ is the velocity at which the
charge density $\rho_{0}(q)$ enters volume $V$ and $s_{0}$ is the source density inside $V$. In the above formula $\rho$ stands for

$$
\begin{equation*}
\rho=\rho_{0}+\boldsymbol{\rho}=\rho_{0}+\frac{\rho_{0} \boldsymbol{v}}{c} \tag{17}
\end{equation*}
$$

It is the flux (flow per unit of area and per unit of progression) of $\rho_{0}$.
$\tau$ stands for progression (not observer's time).

### 28.2 The cosmological equations

The integral equations that describe cosmology are:

$$
\begin{align*}
& \frac{d}{d \tau} \int_{V} \rho d V+\oint_{S} \widehat{\boldsymbol{n}} \rho d S=\int_{V} s d V  \tag{1}\\
& \int_{V} \nabla \rho d V=\int_{V} s d V \tag{2}
\end{align*}
$$

Here $\widehat{\boldsymbol{n}}$ is the normal vector pointing outward the surrounding surface $S, \boldsymbol{v}(\tau, \boldsymbol{q})$ is the velocity at which the charge density $\rho_{0}(\tau, \boldsymbol{q})$ enters volume $V$ and $s_{0}$ is the source density inside $V$. In the above formula $\rho$ stands for

$$
\begin{equation*}
\rho=\rho_{0}+\boldsymbol{\rho}=\rho_{0}+\frac{\rho_{0} \boldsymbol{v}}{c} \tag{3}
\end{equation*}
$$

It is the flux (flow per unit of area and per unit of progression) of $\rho_{0} . t$ stands for progression (not observer's time).

### 28.1 Space cavities

A space cavity is characterized by:

$$
\frac{d}{d \tau} \int_{V} \rho d V=0
$$

All properties of this object depend on the surrounding surface.
These objects are known as black holes.

### 28.2 Inversion surfaces

An inversion surface $S$ is characterized by:

$$
\begin{equation*}
\oint_{S} \widehat{\boldsymbol{n}} \rho d S=0 \tag{1}
\end{equation*}
$$

Potentials and their constituting wave fronts can still pass this inversion surface.

### 28.3 Compartments

The inversion surfaces define compartments. Palestra is an affine space that is divided into compartments.


### 28.4 Cosmological history

The inversion surfaces divide universe into compartments. Think that these universe pockets contain matter that is on its way back to its natal state.

If there is enough matter in the pocket this state forms a black region. The rest of the pocket is cleared from its mass content.

Still the size of the pocket may increase. This corresponds to the expansion of the universe.

Inside the pocket the holographic principle governs. The black region represents the densest packaging mode of entropy.

The pockets may merge. Thus finally a very large part of the universe may return to its birth state, which is a state of densest packaging of entropy.

Then the resulting mass which is positioned at a huge distance will enforce a uniform attraction. This uniform attraction will install an isotropic extension of the central package.

This will disturb the densest packaging quality of that package.

The motor behind this is formed by the combination of the attraction through distant massive particles, which installs an isotropic expansion and the influence of the small scale random localization which is present even in the state of densest packaging.

This describes an eternal process that takes place in and between the pockets of an affine-like space.

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[^0]:    ${ }^{1}$ Superposition of particles is an indispensable ingredient in the comprehension of the formation of composites.

[^1]:    ${ }^{2}$ http://en.wikipedia.org/wiki/Logical_conjunction

[^2]:    ${ }^{3}$ http://en.wikipedia.org/wiki/Logical_connective

[^3]:    ${ }^{4}$ The restriction to a division ring is taken from the fact that also Hilbert space restricts its numbers to elements of a division ring.

[^4]:    ${ }^{5}$ A Gelfand triple is NOT a Hilbert space.

[^5]:    ${ }^{6}$ This refers to fermions.
    ${ }^{7}$ This refers to bosons.

[^6]:    ${ }^{8}$ The size of the e-charge is one third of the electron charge.

[^7]:    ${ }^{9}$ A quaternionic number system with a selected symmetry flavor.

[^8]:    ${ }^{11}$ http://cds.cern.ch/record/514621/files/0108083.pdf

[^9]:    ${ }^{12}$ See color confinement.

[^10]:    ${ }^{13}$ http://en.wikipedia.org/wiki/Yukawa_potential

[^11]:    ${ }^{14} \mathrm{http}: / / e n . w i k i p e d i a . o r g /$ wiki/Bertrand's_theorem.

[^12]:    ${ }^{15}$ Later we will see that this interpretation only holds for closed micro-paths. $N$ depends on the oscillation mode of the closed path particle.

[^13]:    ${ }^{17}$ See sections on photon emission and photon absorption.
    ${ }^{18}$ See section on relativistic energy and momentum.

[^14]:    ${ }^{20}$ http://www.adsabs.harvard.edu/abs/1953MNRAS.113...34S

[^15]:    ${ }^{21}$ It is also possible that the first measurement is used in order to prepare the state of the particle.

[^16]:    ${ }^{22}$ Also see Noether's laws: http://en.wikipedia.org/wiki/Noether\%27s_theorem

