The Concept of Gravitorotational Acceleration and its Consequences for Compact Stellar Objects and Sun

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Abstract: In a previous series of papers relating to the Combined Gravitational Action (CGA), we have exclusively studied orbital motion without spin. In the present paper we apply CGA to any self-rotating material body, *i.e.*, an axially spinning massive object, which itself may be locally seen as a gravitorotational source because it is capable of generating the gravitorotational (field) acceleration, which seems to be unknown to previously existing theories of gravity. The consequences of such an acceleration are very interesting, particularly for Compact Stellar Objects and Sun.

Keywords: CGA; gravitorotational acceleration; gravitorotational energy; neutron stars; pulsars; sun.

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1. Introduction

1.1. A brief summary of the CGA

We feel that we are obliged to give a careful physical justification to the creation of the Combined Gravitational Action (CGA) as a refinement and a generalization of the Newton's gravity theory. The key *idea* in the CGA-formalism is the physical fact of taking into account the relative motion of the test(secondary)body which is under the gravitational influence of the primary one. Historically, the *idea* itself is not new since Laplace [1], Lorentz [2], Poincaré [3,4] and Oppenheim [5] have already thought of adjusting the Newton's law of gravitation by adding a certain velocity-dependent-term, but unfortunately their effort could not explain, *e.g.*, the remaining secular perihelion advance rate of Mercury discovered by Le Verrier in 1859. We have previously shown in a series of articles [6,7,8,9,10] that the CGA as an alternative gravity theory is very capable of investigating, explaining and predicting, in its proper framework, some *old* and *new* gravitational phenomena. Conceptually, the CGA is basically founded on the concept of the combined gravitational potential energy (CGPE) which is actually a new form of velocity-dependent-GPE defined by the expression

$$U \equiv U(r,v) = -\frac{k}{r}\left(1 + \frac{v^2}{w^2}\right), \tag{1}$$

where $k = GMm$; G being Newton's gravitational constant; M and m are the masses of the gravitational source A and the moving test-body B; $r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$ is the relative distance between A and B; $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ is the velocity of the test-body B relative

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to the inertial reference frame of source A; and w is a specific kinematical parameter having the physical dimensions of a constant velocity defined by

$$w = \begin{cases} c, \text{ if } B \text{ is in relative motion inside the vicinity of } A \\ v_{\text{esc}} = \sqrt{2GM/R}, \text{ if } B \text{ is in relative motion outside the vicinity of } A \end{cases} \tag{2}$$

where c is the light speed in local vacuum and v_{esc} is the escape velocity at the surface of the gravitational source *A*.

In the CGA-context, the velocity-dependent-GPE (1) is simply called CGPE because it is, in fact, a combination of the static-GPE $V(r) = k\,r^{-1}$ and the dynamic-GPE $W(r,v) = k\,r^{-1}(v/w)^2$. The main difference between the CGPE (1) as a generalization of classical GPE and the previously well-known velocity-dependent-GPEs is clearly situated in the originality and simplicity of Eq.(1), which may be rewritten in the form $U \equiv U(r,\dot{r}) = -\ k\,r^{-1}\left(1 + (\dot{r}/w)^2\right)$, with $v \equiv \dot{r} = dr/dt$. The originality of CGPE is reflected by the fact that the CGPE is explicitly depending on r and v but also is implicitly depending on w since the latter is, by definition, – a specific kinematical parameter having the physical dimensions of a constant velocity – .The implicit dependence of CGPE on w is expressed in terms of '*inside the vicinity of A*' and '*outside the vicinity of A*' in (2). Furthermore, the CGPE may be reduced to the static-GPE when $v << w$ or $v = 0$. Thus the main physical reason for the choice of the expression (1) for the CGPE lies in its consequence as a generalization of the static-GPE.

Now, let us show when and how we could apply the CGA. As we know, the Newton's gravitational theory is a very good depiction of gravity for many situations of practical, astronomical and cosmological interest. However, it is currently well established that the Newton's theory is only an approximate description of the law of gravity. As early as the middle of the nineteenth century, observations of the Mercury's orbit revealed a discrepancy with the prediction of Newton's gravity theory. In fact, this famous discrepancy was historically the first evidence of the limit of validity of Newtonian gravity theory. This disagreement between theory and observations was resolved by taking into account the CGA-effects inside and outside the solar system [6,7,8,9], which are known as the crucial tests support the general relativity theory (GRT).

On the whole, the criterion that we should use to decide whether to employ Newton's gravity or CGA is the magnitude of a dimensionless physical quantity called the "CGA-correction factor" :

$$\zeta = \frac{GM}{rc^2} \cong \frac{v^2}{c^2}\ , \tag{3}$$

which is actually derived from (1) and (2) for the case when the test-body *B* is evolving inside the vicinity of the gravitational source *A*. The same dimensionless physical quantity (3) exists in GRT and for this reason we have already shown in Ref.[10] the existence of an important similarity between the CGA-equations of motion and those of GRT. Moreover, it is worthwhile to note that the smaller this factor (3), the bitter is Newtonian gravity theory as an approximation. As an illustration, we have *e.g.*, for the system {Earth, Moon}: $\zeta \cong 10^{-11}$ and for the system {Sun, Earth}: $\zeta \cong 10^{-8}$.

Hence, starting from the CGPE and using only the very familiar tools of classical gravitomechanics and the Euler-Lagrange equations, we have established the CGA-formalism [6,7,8,9,10] .The main consequence of CGA is the dynamic gravitational field (DGF), $\mathbf{\Lambda}$, which is phenomenologically an induced field that is more precisely a sort of gravitational induction due to the relative motion of material body inside the vicinity of the gravitational source [6,7,8,9,10]. In general, the magnitude of DGF is of the form

$$\Lambda = \pm \frac{GM}{r^2}\left(\frac{v}{w}\right)^2 . \tag{4}$$

Eq.(4) means that DGF may play a double role, that is to say, when perceived/interpreted as an extra-gravitational acceleration, $\Lambda > 0$, or an extra-gravitational deceleration, $\Lambda < 0$, (see Ref. [8] for a detailed discussion).

In the papers [6,7,8,9,10] we have exclusively focused our interest on the orbital motion and gravitational two-body problem. In the present paper, we shall apply CGA to any self-rotating (spinning) material body, *i.e.*, axially rotating massive object that itself may be locally seen as a gravitorotational source since it is capable of generating the gravitorotational (field) acceleration $\boldsymbol{\lambda}$, which seems to be unknown to previous theories of gravity.

2. Concept of the Gravitorotational Acceleration

Phenomenologically speaking, the concept of the gravitorotational acceleration (GRA), $\boldsymbol{\lambda}$, is very similar to DGF, that is if $\mathbf{\Lambda}$ is mainly induced by the relative motion of the massive test body in the vicinity of the principal gravitational source, the GRA is intrinsically generated by any massive body in a state of rotational motion, independently of the principal gravitational source, which itself may be characterized by its proper GRA during its axial-rotation, and therefore the GRA is, in fact, a combination of *gravity* and *rotation*.

3. Expression of GRA's magnitude

In order to derive an explicit expression for GRA's magnitude, let us first rewrite Eq.(4) for the case when $\Lambda > 0$, that is

$$\Lambda = \frac{GM}{r^2}\left(\frac{v}{w}\right)^2 , \tag{5}$$

and consider a massive body of mass M and radius R, which is intrinsically in a state of axial-rotation in its proper reference frame at rotational velocity of magnitude $v_{\mathrm{rot}} = \Omega R$ independently of the presence of any other gravitational source. Therefore, according to the concept of GRA, in such a case, the rotating/spinning massive body should be *locally* seen as a gravitorotational source when $\|\mathbf{\Lambda}\| \to \|\boldsymbol{\lambda}\| \equiv \lambda$ as $r \to R$, $v \to v_{\mathrm{rot}}$ and $w \to c$, thus (5) becomes after substitution

$$\lambda = GM\left(\frac{\Omega}{c}\right)^2 . \tag{6}$$

Since $\Omega = 2\pi P^{-1}$, where P is the rotational period, hence we get after substitution into (6), the expected expression of GRA's magnitude

$$\lambda = \kappa \frac{M}{P^2}, \quad \kappa = 4\pi^2 G/c^2 . \tag{7}$$

It is clear from Eq.(7), λ depends exclusively on the mass and rotational period, therefore, mathematically may be treated as a function of the form $\lambda \equiv \lambda(M, P)$. Moreover, the structure of Eq.(7) allows us to affirm that for any astrophysical massive object, the magnitude of $\boldsymbol{\lambda}$ should be infinitesimally small for slowly rotating massive stellar objects and enormous for rapidly rotating ones. Furthermore, in order to confirm our assertion numerically, we have selected seven well-known (binary) pulsars and calculated their GRAs' magnitudes, and compared them with the magnitude of the Sun's GRA. The values are listed in Table 1.

OBJECT **Sun + PRS**	P (s)	M (M_Θ)	λ $(\mathrm{m\,s^{-2}})$	REF.
Sun	2.164320×10^{6}	1	1.244823×10^{-8}	
B 1913+16	5.903000×10^{-2}	1.4410	2.409380×10^{7}	a
B 1534+12	3.790000×10^{-2}	1.3400	5.435171×10^{7}	b,c
B 2127+11C	3.053000×10^{-2}	1.3600	8.501044×10^{7}	d
B 1257+12	6.200000×10^{-3}	1.4000	2.121932×10^{9}	e
J 0737-3039	2.280000×10^{-2}	1.3381	1.500000×10^{8}	f
B 1937+21	1.557800×10^{-3}	1.4000	3.364000×10^{10}	g
J 1748-2446ad	1.395000×10^{-3}	1.4000	4.194982×10^{10}	h

Table 1: The values of GRA's magnitude for seven well-known (binary) pulsars compared with that of the Sun.

Ref.: a) Taylor and Weisberg [11]; b) Arzoumanian [12]; c) Wolszcan [13]; d) Deich and Kulkarni [14]; e) Konacki and Wolszcan [15]; f) Kramer and Wex [16]; g) Takahashi *et al.* [17]; h) Hessels *et al.* [18].

Note: To calculate these values, we have used $G = 6.67384 \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$, $c = 299792458\ \mathrm{m\,s^{-1}}$, $M_\Theta = 1.9891 \times 10^{30}\ \mathrm{kg}$ and sidereal rotation period at equator $P_\Theta = 25.05\,\mathrm{d}$.

Analysis of Table1 gives us the following results: 1) The magnitude of the Sun's GRA, $\lambda_\Theta = 1.244823 \times 10^{-8}\,\mathrm{m\,s^{-2}}$, is extremely weak that's why its effect on the solar system is unobservable, but perhaps it is only the Sun's immediate vicinity that should be affected by it. Since GRA is explicitly independent of the radius of the rotating massive object thus the extreme weakness of the magnitude of the Sun's GRA is mainly due to the huge value of the rotational period, $P_\Theta = 2.164320 \times 10^6\,\mathrm{s}$, compared with those of the pulsars. 2) In spite of the fact that the pulsars' masses are nearly equal, the pulsars' rotational periods show a neat inequality between them. Also, the different values of GRA's magnitude for each celestial object show us how sensitive GRA is to variation in rotational period.

4. Mutual dependence between the Mass and the Rotational Period

Since GRA's magnitude may be treated as a function of the form $\lambda \equiv \lambda(M, P)$ hence we can show more clearly the existence of the mutual dependence between the mass and rotational period of the same rotating body *via* GRA's magnitude. For this purpose, we deduce from Eq.(7) the following expression

$$\frac{M}{P^2} = \kappa^{-1}\lambda\,. \tag{8}$$

Obviously, Eq.(8) shows us the expected mutual dependence between the mass and rotational period *via* GRA's magnitude. Moreover, because the rotational period is an intrinsic physical quantity, here, according to Eq.(8), the spin of any massive celestial body should vary with mass independently of cosmic time.

5. Link between GRA and Rotational Acceleration

Now, returning to Eq.(7) and showing that GRA's magnitude and the rotational acceleration

$$a_{\mathrm{rot}} = \Omega^2 R\,, \tag{9}$$

are in fact proportional, $\lambda \propto a_{\mathrm{rot}}$, and the constant of proportionality is precisely the compactness factor $\varepsilon = GM/c^2R$ that characterizes any massive celestial body. To this end, it suffices to multiply and divide by the radius R the right hand side of Eq.(7) to get the expected expression

$$\lambda = \varepsilon\, a_{\mathrm{rot}}\,. \tag{10}$$

According to the expression (10), GRA is at the same time an *old* and a *new* natural physical quantity that should play a crucial role, especially for compact stellar objects, *e.g.*, the rotating neutron stars and pulsars for which the compactness ε has a large value compared to that of normal stellar objects. By way of illustration, the Sun's compactness has the value $\varepsilon_\Theta = 2.123679 \times 10^{-6}$.

6. Consequences of GRA

In what follows we will show that, in the CGA-context , the transitional state, dynamical stability and instability of a uniformly rotating neutron star (NS) depend on the *'antagonism'* between centrifugal force and gravitational force, or in energetic terms, between rotational kinetic energy (RKE) and gravitational binding energy (GBE).

Usually the physics of NS considers that the source of the emitted energy is essentially the RKE, however, such a consideration should immediately imply that, at least in the medium term, the GBE should absolutely dominate the RKE and as a result the NS should be prematurely in a state of gravitational collapse. Hence, as we will see, the main source of the emitted energy is not the RKE but the gravitorotational energy (GRE), a sort of *new* physical quantity which is a direct consequence of GRA.

Let us now determine the conditions of transitional state, dynamical stability and instability that may be characterized any NS at least in the medium term. With this aim , we assume a uniformly rotating NS as a homogeneous rigid spherical body of mass M, radius R and rotational velocity $\Omega = 2\pi P^{-1}$, where P is the rotational period. The NS's RKE and GBE are, respectively, defined by the well-known formulae:

$$E_{\mathrm{rot}} = I\,\Omega^2/2, \tag{11}$$

and

$$E_{\mathrm{g}} = -\frac{3}{5}\frac{GM^2}{R}, \tag{12}$$

where $I = 2MR^2/5$ is the moment of inertia of NS under consideration. Hence, the total energy is

$$W = E_{\mathrm{rot}} + E_{\mathrm{g}}, \tag{13}$$

which presents the following conditions:

a) $W < 0$, NS is in a state of dynamical stability,
b) $W = 0$, NS is in a state of transition,
c) $W > 0$, NS is in a state of dynamical instability.

It is worth noting that the three suggested conditions a), b) and c) are taken in the medium term because NS may be suddenly in a state of dynamical perturbation or in a state of transition from stability to instability and vice versa.

7. Critical Rotational Period

Knowing the critical rotational period (CRP) of NS is very important because CRP should be treated as a parameter of reference on which the temporal evolution of NS depends. Furthermore, since the change from stability to instability and vice versa should pass obligatorily *via* the transitional state, therefore, an expression for the CRP may be deduced from the transitional state (b), so after performing a simple algebraic calculation, we get the following expected expression

$$P_{\mathrm{c}} = 2\pi R\sqrt{\frac{R}{3GM}} \ . \tag{14}$$

We can numerically evaluate the CRP by taking, through this paper, the standard NS mass and radius, namely, $M = 1.4 M_{\Theta}$ and $R = 10\,\mathrm{km}$, thus by substituting these values into (14), we find

$$P_{\mathrm{c}} = 2.660963\times 10^{-4} \cong 0.2661 \text{ ms}\,, \tag{15}$$

which is a tiny fraction of the smallest yet observed rotational period, $P = 1.3950\,\mathrm{ms}$, of PRS J1748-2446ab [13]. Further, according to (15), the critical value of GRA's magnitude for a standard NS should be

$$\lambda_{\mathrm{c}} = 1.153\times 10^{12}\,\mathrm{ms}^{-2}\,. \tag{16}$$

8. Gravitorotational Energy

Now we approach the most important consequence of GRA, that is, the gravitorotational energy (GRE), which should qualitatively and quantitatively characterize any massive rotating body. As we will see, GRE is quantitatively very comparable to the amount of RKE, particularly for NS and pulsars. Since GRE is a direct consequence of GRA, hence GRE should be proportional to GRA's magnitude, *i.e.*, $\mathscr{E} \propto \lambda$ or equivalently

$$\mathscr{E} = \eta\lambda \ . \tag{17}$$

Let us determine the constant of the proportionality η by using dimensional analysis as follows:

$$[\,\eta\,] = \frac{[\mathscr{E}]}{[\lambda]} = \frac{\mathrm{M\,L^2T^{-2}}}{\mathrm{LT^{-2}}} = \mathrm{M\,L}.$$

we can remark that the dimensional quantity $\mathrm{M\,L}$ has the physical dimensions of the product of mass and length, therefore, for our case η should take the form $\eta = MR = 5I/2R$ and by substituting into (17), we find the required expression for GRE

$$\mathscr{E} = \frac{5}{2}\frac{\lambda I}{R} \ . \tag{18}$$

In order to show that the amount of GRE $\mathscr{E}$ is quantitatively very comparable to that of RKE, particularly for the compact stellar objects, we can use Table 1. The numerical values of E_{rot} and $\mathscr{E}$ are listed in Table 2.

OBJECT Sun + PRS	E_{rot} (J)	$\mathcal{E}$ (J)
Sun	2.3900×10^{35}	2.5401×10^{30}
B 1913+16	6.4948×10^{41}	6.9180×10^{41}
B 1534+12	1.4651×10^{42}	1.4500×10^{42}
B 2127+11C	2.2915×10^{42}	2.3010×10^{42}
B 1257+12	5.7200×10^{43}	5.9140×10^{43}
J 0737-3039	4.0426×10^{42}	4.0000×10^{42}
B 1937+21	9.0604×10^{44}	9.3680×10^{44}
J 1748-2446ad	1.1300×10^{45}	1.1680×10^{45}

Table 2: Comparison of the numerical values of E_{rot} and $\mathcal{E}$ for the Sun and seven well known (binary) pulsars.

Note: To calculate the Sun's E_{rot} and $\mathcal{E}$, we have used the relation $I/MR^2 = 0.059$.

Analysis of Table 2: The numerical values listed in Table 2 show us, excepting the Sun's values, that all the values of E_{rot} and $\mathcal{E}$ are very comparable for the seven (binary) pulsars. This fact is mainly due, at the same time, to the rotational period and the compactness ε. To illustrate this fact, let us return to the expression (18) which may be written as follows:

$$\mathcal{E} = 5\varepsilon\, E_{rot} \ . \tag{19}$$

And as $5\varepsilon \cong 1$ for the NSs hence that's why $\mathcal{E} \cong E_{\text{rot}}$ as it is well illustrated in Table 2. From all this we arrive at the following result: In the CGA-context, the RKE cannot be considered as the main source of the emitted energy for rotating neutron stars and pulsars because –in energetic terms– its own role is to balance, approximately, the GBE, at least in the medium term. Therefore, the veritable principal source of the emitted energy should undoubtedly be GRE, as illustrated by the GRE numerical values listed in Table 2, which are quantitatively very comparable to those of RKE for pulsars. Moreover, if we take into account the critical value of GRA's magnitude (16), we get the following critical value for GRE

$$\mathcal{E}_{\text{c}} = 3.210\times10^{46}\,\text{J} = 3.210\times10^{53}\,\text{erg}\ . \tag{20}$$

9. Rotating Magnetars

Rotating magnetized neutron stars (magnetars) are also important compact stellar objects. That's why it is possible, in the CGA-context, to exploit GRE as an energetic reservoir for rotating magnetars by assuming that there is a certain physical mechanism that can convert all or at least a significant part of GRE into an extreme internal magnetic energy:

$$E_B = B^2 R^3 = \mathscr{E}\,(\text{erg}), \tag{21}$$

which could, of course, produce an extreme *internal* magnetic field strength

$$B = \sqrt{\mathscr{E} R^{-3}} \quad (\text{G}). \tag{22}$$

The observed surface (external) dipole magnetic field strength B_0 would be lower than the internal field strength B defined by (22). Besides the internal magnetic field there is the *critical* internal magnetic field whose strength may be evaluated according to (20) and (22) as follows:

$$B_c = \sqrt{\mathscr{E}_c R^{-3}} \quad (\text{G}). \tag{23}$$

As an illustration, let us evaluate the strength of internal magnetic field of radio pulsar B 1931+24. We have according to Ref.[19] the following parameters: $P = 0.813\,\text{s}$ and $B_0 \cong 3\times10^{12}\,\text{G}$. By taking, as usual, $M = 1.4 M_\Theta$ and $R = 10\,\text{km}$, we find for GRE $\mathscr{E} = 3.440\times10^{46}\,\text{erg}$, and after substitution into (22), we obtain

$$B = 1.855\times10^{14}\,\text{G}. \tag{24}$$

Now, let us evaluate the critical strength of internal magnetic field of pulsar B 1931+24. We have according to (23), the following value

$$B_c = 5.666\times10^{17}\,\text{G}. \tag{25}$$

10. The Sun

Contrary to the compact stellar objects like, *e.g.,* white dwarfs, neutron stars and pulsars, the Sun is a main-sequence star, and thus generates its energy by nuclear fusion of hydrogen nuclei into helium, and it is characterized by the following evident proprieties:

-The Sun's interior is in hydrostatic equilibrium.
- Nuclear fusion (reactions) is the main energetic source of the Sun.
-Energy is carried away from the Sun's core by radiative diffusion and convection.
-The Sun's interior can be probed by helioseismology.
-The Sun's magnetic field is, at the same time, the engine and energy source driving all phenomena collectively defining solar activity.

Since the nuclear fusion is the principal source of the Sun's energy, so how does the Sun strike a perfect balance between the explosive forces of fusion and the implosive forces of self gravity?

The perfect balance be between fusion and gravity –or in energetic terms, between fusion energy and gravitational binding energy– is mainly due to the fact that the Sun possesses a property which serves as an auto-regulation mechanism for the fusion reaction. When the force due to pressure exactly balances the force due to gravity, a system is in hydrostatic equilibrium. The Sun's hydrostatic equilibrium is stable. The balance is achieved by auto-regulation: a slight decrease in fusion energy would result in contraction that would heat up the core and increase fusion rate, and vice versa.

Thus, phenomenologically, the reason that the Sun neither expands nor collapses is that the two forces keep the balance. In the distant future, when this balance is disturbed because most of the hydrogen is used up, the Sun will expand. This will be the end of the solar system as we know it.

10.1. Central Magnetic Field

Since half-century, the stellar magnetism of massive stars was and is modeled according to the fossil-field theory and core dynamo theory. However, these two conceptual explanations are not the last word. Why? Because: the principal weakness of the fossil-field theory has been the absence of filed configurations stable enough to survive in a star over its lifetime. In other words, the fossil magnetic field, as a primordial magnetic field should disintegrate, decrease and vanish rapidly due, among other thing, to Ohmic decay and magnetic energy dissipation. The dynamo theory has difficulty explaining high field strengths and the lack of a correlation with rotation. However, if we assume that the Sun's (global) internal dynamic stability is essentially ensured by the internal differential rotation, and the fossil (magnetic) theory and the core dynamo theory are not the unique source of the Sun's internal/external magnetic fields, thus there is another source which is the *central magnetic field* (CMF). The mechanism of the partial or total transformation of GRE into magnetic energy generating the CMF which subsequently diffused throughout the Sun. This CMF is supposed to be sufficiently large-scale such that its decay time is at least comparable with the Sun's age [19] but adequately concentrated to the center that it should not distort the observed (almost spherical) shape of the Sun [20] and also it should not violate the virial theorem [21].

Thus CMF should be the principal source of the Sun's internal magnetic fields (IMFs). But the natural question is: How can CMF generate IMFs? It is due to the contribution of (partial and progressive) radial magnetic diffusion and the local mechanism of regeneration and amplification that lead to local IMFs. Accordingly, the regeneration and amplification of IMFs is purely local occurrence. For that reason, the observed (10% of) massive stars with very strong surface fields should be characterized by a very high amount of the GRE than the others (90% with no detectable fields). The same considerations should be theoretically applicable equally to Ap stars, magnetic white dwarfs and some highly magnetized neutron stars known as magnetars. This establishes the CMF as the natural, unifying explanation for the stellar magnetism. Historically speaking, the hypothesis of CMF in the Sun had been proposed in 1973 by Chitre, Ezer and Stothers [22] to explain the low solar neutrino flux measured by Davis [23].

Concerning the Sun's IMF, in 1989, Dziembowski and Good had already developed an important helioseismological model from which they determined the quadrupole toroidal magnetic field near the base of the Sun's convection zone [24]. Basing exclusively on the oscillation data of Libbrech [25] which yield information about the IMF strength between 0.6

and 0.8 of the Sun's radius; they found that the strength of this IMF is 2×10^6 G. However, In their seminal article entitled “Solar Neutrino and a Central Magnetic Field in the Sun” Chitre, Ezer and Stothers found the value of 10^9 G for the strength of the Sun's CMF –under certain considerations proper to their hypothesis [22]. Again, the two results reinforce in some way our initial claim, *i.e.*, the CMF should be the principal source of the Sun's IMFs.

10.2. Theoretical average strengths of the Sun's CMF

Finally, let us focus our attention on the formula (22). At first glance, the verification of this formula seems to be experimentally and/or observationally out of any expectation. Luckily, the situation is not practically so complicated since we have the Sun that may serve as a veritable celestial laboratory enabling us to understand physical processes that take place inside the Sun as well as in other similar stellar bodies and also to test some new gravity theories. Now, let us evaluate the strengths of the Sun's CMF by applying the formula (22). However, in order to be sufficiently close to the *reality*, we are naturally obliged to suppose that the theoretical average strengths of the CMF should be a function of radius. Consequently, the formula (22) may be rewritten as follows

$$B \equiv B(R) = R^{-2}\sqrt{\tfrac{5}{2}\lambda I} \quad . \tag{26}$$

Furthermore, the well-known solar internal structure allows us to investigate numerically the behavior/features (rapidly increasing and/or slowly decreasing) of the Sun's CMF by evaluating, on average, the strengths of CMF in different layers inside the *core*. Explicitly, we have to evaluate the theoretical average strengths of CMF in the radial region $r_{\mathrm{g}} \le R < 0.2R_\Theta$ (*i.e.*, the core, where $r_{\mathrm{g}} = GM_\Theta c^{-2}$ is the Sun's gravitational radius) and its theoretical maximum strength should be evaluated at the Sun's gravitational radius.

–*Theoretical maximum strength of the Sun's CMF*: Formula (26) permits us to evaluate the theoretical maximum strength of the Sun's CMF at the Sun's gravitational radius, $r_{\mathrm{g}} = 1.477\times10^5$ cm , and we get after a direct numerical application

$$B_{\max} \equiv B(r_{\mathrm{g}}) = r_{\mathrm{g}}^{-2}\sqrt{\tfrac{5}{2}\lambda_\Theta I_\Theta} = 6.1\times10^{13}\,\mathrm{G}. \tag{27}$$

Quantitatively and qualitatively, the theoretical maximum value, $B_{\max} = 6.1\times10^{13}$ G , of the Sun's CMF strength gives us an idea about the origin, localization and distribution of the Sun's IMF. That is, as it was already mentioned explicitly, the IMF is just a *local* manifestation of a small portion of the CMF via the progressive radial magnetic diffusion and the local mechanism of regeneration and amplification.

– *Numerical investigation of the Sun's CMF strengths*: In order to illustrate numerically the behavior/features of the Sun's CMF strengths as a function of radius, it is heuristically judged important to split the interval $r_{\mathrm{g}} \le R < 0.2R_\Theta$ in two subintervals, namely, $1 \le R/r_{\mathrm{g}} \le 100$ and

$0.01 \le R/R_\Theta \le 0.20$. In the context of the present work, we call the first subinterval '*critical core region*' in which there is a strong centrally concentrated MF. The values of the Sun's CMF strengths are listed in Tables 3,4,5 and 6 for the first and second subinterval, respectively.

Table 3

R/r_g	$B(R)$ (G)
1	6.100×10^{13}
5	2.438×10^{12}
15	2.710×10^{11}
20	1.524×10^{11}
25	9.754×10^{10}
30	6.774×10^{10}
35	4.976×10^{10}
40	3.810×10^{10}
45	3.010×10^{10}
50	2.438×10^{10}

Table 4

R/r_g	$B(R)$ (G)
55	2.015×10^{10}
60	1.693×10^{10}
65	1.443×10^{10}
70	1.244×10^{10}
75	1.083×10^{10}
80	9.526×10^{9}
85	8.438×10^{9}
90	7.526×10^{9}
95	6.755×10^{9}
100	6.100×10^{9}

Numerical values of the Sun's CMF strengths are listen for $1 \le R/r_g \le 100$, where $r_g = 1.477\times10^5$ cm is the Sun's gravitational radius.

Note: To calculate $B \equiv B(R)$, we have used the formula (26), and the Sun's λ_Θ and I_Θ.

Tables 5

R/R_Θ	$B(R)$ (G)
0.01	2.750×10^{6}
0.02	6.873×10^{5}
0.03	3.055×10^{5}
0.04	1.718×10^{5}
0.05	1.100×10^{5}
0.06	7.637×10^{4}

Tables 6

R/R_Θ	$B(R)$ (G)
0.11	2.272×10^{4}
0.12	1.910×10^{4}
0.13	1.626×10^{4}
0.14	1.402×10^{4}
0.15	1.222×10^{4}
0.16	1.074×10^{4}

0.07	5.611×10^4	0.17	9.513×10^3
0.08	4.296×10^4	0.18	8.486×10^3
0.09	3.394×10^4	0.19	7.616×10^3
0.10	2.750×10^4	0.20	6.873×10^3

Numerical values of the Sun's CMF strengths are listen for $0.01 \le R/R_\Theta \le 0.20$, where $R_\Theta = 6.95508 \times 10^{10}\,\text{cm}$ is the Sun's radius.

– *Analysis of Tables 3,4,5 and 6*: As the reader can easily remark it, the Sun's CMF strengths decreasing very slowly on the subintervals $1 \le R/r_\text{g} \le 100$ and $0.01 \le R/R_\Theta \le 0.20$, this means the radial magnetic diffusion from inside to outside the core should be, as it was already mentioned, at the same time partial and progressive. Statistically, the value of $10^{10}\,\text{G}$ seems to be the most predominant one since it is displayed uniformly. Hence, this feature allows us to suggest that, theoretically, the average strength of the Sun's CMF should be of the order of $10^{10}\,\text{G}$, which is only ten times the value found by Chitre, Ezer and Stothers [22].

11. Concept of the Stellar Magnetic Permeability

The concept of the stellar magnetic permeability is mainly inspired by the fact that since in Nature, any material is physically characterized by its proper magnetic permeability, thus it is quite logical to postulate the existence of the magnetic permeability for the stellar material. In this sense, we defined the stellar magnetic permeability as follows:

The stellar magnetic permeability is the measure of the ability of a stellar material to support the formation of a magnetic field within itself.

Therefore, the stellar magnetic permeability is the degree of *magnetization* that a stellar material obtains in response to an applied magnetic field. Since, in general, the stellar objects are characterized by an internal magnetic field and an external magnetic field, hence, in the context of the present work, the stellar magnetic permeability μ is physico-mathematically defined by the expression

$$\mu = \mu_0 \frac{B_\text{int}}{B_\text{ext}}, \tag{28}$$

where μ_0 is the magnetic permeability of free space; B_int and B_ext are, respectively, the strengths of internal and external magnetic field.– Conceptually, the stellar magnetic permeability should play an important role in stellar evolution.

12. Conclusion

Basing on our gravity model, Combined Gravitational Action, we have derived an explicit expression for the concept of gravitorotational acceleration (GRA), which is unknown to previously established gravity theories. The most significant result of GRA is the gravitorotational energy (GRE), which should qualitatively and quantitatively characterize any massive rotating body. Furthermore, GRE is exploited as an energetic reservoir, particularly for neutron stars and pulsars. Also, the hypothesis of the Sun's central magnetic field is revisited, investigated and exploited.

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