Abstract. In this paper I present three functions based on the digital sum of a number which might be interesting to study and ten conjectures. These functions are: (I) $F(x)$ defined as the digital sum of the number $2^x - x^2$; (II) $G(x)$ equal to $F(x) - x$ and (III) $H(x)$ defined as the digital sum of the number $2^x + x^2$.

Let $F(x)$ be the sum of the digits of the number $2^x - x^2$, where $x$ is an odd positive number. Then:

Conjecture 1:

There exist an infinity of primes $p$ such that $F(p) = p$. Such primes $p$ are 13, 61 (...). Note that, up to $x = 241$, there is no other odd number $x$ for which $F(x) = x$.

Conjecture 2:

There exist an infinity of pairs of twin primes $(p, q)$ such that $F(p) = F(q)$. Such pairs are $(59, 61)$, $(239, 241)$ (...) with corresponding $F(p) = F(q)$ equal to 61, 331 (...).

Conjecture 3:

There exist an infinity of pairs of primes $(p, q)$ such that $F(p) = q$. Such pairs are $(5, 7)$, $(11, 19)$, $(23, 43)$, $(29, 37)$, $(43, 61)$, $(59, 61)$, $(101, 109)$, $(157, 229)$, $(167, 241)$, $(239, 331)$, $(241, 331)$ (...).

Conjecture 4:

There exist an infinity of pairs of primes $(p, q)$ such that $F(p) = q^2$. Such pairs are $(31, 7)$, $(83, 11)$, $(103, 11)$, (...).

Conjecture 5:

There exist an infinity of pairs of primes $(p, q)$ such that $F(p^2) = q$. Such pairs are $(13, 223)$, $(19, 541)$, $(29, 1129)$, (...).
Conjecture 6:

There exist an infinity of pairs of primes \((p, F(p))\) such that \(F(p) - p = 2\) (in other words, \(p\) and \(F(p)\) are twin primes). Such pairs of twin primes are \((5, 7), (59, 61)\) (...).

(II)

Let \(G(x) = F(x) - x\), where \(x\) and \(F(x)\) are those defined above. Then:

Conjecture 7:

There exist an infinity of pairs of primes \((p, F(p))\) such that \(G(p)\) is a multiple of 9. Such pairs of primes are \((43, 61)\) (...) with corresponding \(G(p)\) equal to 18 (...).

Conjecture 8:

There exist an infinity of pairs of primes \((p, F(p))\) such that \(G(p)\) is a power of the number 2. Such pairs of primes are \((5, 7), (11, 19), (29, 37), (101, 109)\) (...) with corresponding exponents (powers of 2): 1, 3, 3, 3 (...).

Conjecture 9:

There exist an infinity of primes \(p\) such that \(G(p)\) is also prime. Such pairs of primes \((p, G(p))\) are \((17, 5), (41, 11), (47, 17), (53, 23), (71, 5), (113, 47), (173, 53)\) (...).

Problem 1:

Which is the longest possible sequence of ordered odd numbers \(n\) such that \(F(n)\) has the same value for all of them? The longest sequence I met is: 75, 81, 87, 93, 99, for all of them \(F(n)\) having the value 116.

Problem 2:

Which have in common the odd numbers \(n\) for that \(F(n)\) is equal to a power of two (such number is the prime 179 for which \(F(p) = 256)\)?

(III)

Let \(H(x)\) be the sum of the digits of the number \(2^x + x^2\), where \(x\) is an odd positive number. Then:

Conjecture 10:
There exist an infinity of pairs of twin primes \( (x = 11 + 18k, y = 13 + 18k) \) such that \( H(x) = H(y) \). Such pairs of twin primes are: \((11, 13), (29, 31), (101, 103), (191, 193), (227, 229), (569, 571)\) with corresponding \( H(x) = H(y) \) equal to: 18, 45, 117, 243, 315, 810 (...).