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Affiliation: None

Escape from a Black Hole

Abstract

The proposed flux model of the gravitational and inertial interactions posits the existence of a field that I label the temporal-inertial (TI) field. The TI field is subject to gravity and, in response to the acceleration of gravity, transmits its own acceleration to massive particles and objects comprising massive particles. The relation between the Higgs field or the Higgs mechanism and what I designate as the TI field is undefined. The flux model asserts that in a gravitational field, the velocity of the TI field combines with that of gravitons emitted by the gravitational body to increase the flux of gravitons relative to the TI field. Thus the response of the TI field to gravity adds to the force of gravity. The flux model describes a system in which particles of the TI field fall radially toward the central gravitational mass. Equations defining the acceleration and velocity profiles about a gravitational body are developed for the flux model. Near a massive gravitational body, such as a black hole, the infall velocity of the TI field reaches relativistic speeds. The relativistic mass increase of particles of the field at these speeds resists further acceleration in accord with the Lorentz factor. Accordingly, particles of the TI field approach light speed as they approach the singularity. The event horizon does not exist in this model.
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^ First Matters
To support reading this paper as a stand-alone document, substantial content has been taken from other papers [1] [2] [3] written by this author. The black hole considered in this paper is a non-rotating black hole without an electric charge.

^ Properties of The Temporal-Inertial (TI) Field
The relation of the Higgs field or the Higgs mechanism [4] and what I designate as the TI field is undefined. I may attribute properties to the TI field (such as the particles of the field being subject to gravity) that are not attributed to the Higgs field.

The characteristics of the TI field as they affect gravity are developed in reference [1]. Some of the conclusions of the referenced paper are summarized below.
1. When a matter particle or an object composed of matter particles is accelerated by an external force, its motion is resisted by its acceleration relative to the TI field. This reactive force of the TI field of space is the familiar inertial force.
2. Particles of the TI field are accelerated by gravity directly toward the center of each gravitational body just as a test particle would be and reach the escape velocity of such a particle at the distance of that particle from the center of mass of the gravitational body. (See Note 1.)
3. The flux model of gravity [2] is part of the TI field model of gravity and posits that in a gravitational field, the velocity of the TI field combines with that of the gravitons emitted by the gravitational body to increase the flux of gravitons relative to the TI field. Thus the response of the TI field to gravity adds to the force of gravity.
4. Inherent in the flux model is the assertion that the TI field supports the propagation of gravitons.
5. The gravitational acceleration of the TI field relative to a matter particle or an object composed of matter particles applies a force to that matter particle or object. This force is the familiar gravitational force applied indirectly through the intermediary of the acceleration of the TI field of space.
6. The TI field accelerates massive particles at the same rate as its own acceleration.
7. Acceleration of the TI field in its own response to gravity is the sole accelerator of massive particles in response to gravity. Accordingly, massive particles are not directly subject to the gravitational force.
8. The TI field supports the propagation of light.
9. The speed of light and gravitons is their speed relative to the TI field.
10. Acceleration of the TI field is moderated by a second field termed the static field which, itself, is not subject to gravity.

Note 1. The first iteration of calculating the infall velocity of particles of the TI field toward a gravitational body is made using a Newtonian dynamics formula for calculating the escape velocity of massive particles from a gravitational body. See Appendix B.
Considerations of Mass in the Temporal-Inertial Field Model of Gravity and Inertia [6]

I co-opt Wikipedia's definitions of mass [5], modify their meanings and apply them to the TI field model of gravity and inertia. In the TI field model, ordinary matter particles are not directly subject to gravity. The TI field mediates gravity. Its particles are directly subject to gravity and their acceleration in response to gravity forces matter particles and objects comprising matter particles to be accelerated at the same rate. Acceleration of an object relative to the TI field produces the familiar inertial reaction force, just as in the Newtonian model.

In sum, we have two sets of definitions for mass in the TI field model; one for ordinary matter particles and one for particles of the TI field itself.

Definitions of Mass for Matter Particles in the TI Field Model of Gravity and Inertia

- **Inertial mass** is a measure of an object's resistance to being accelerated relative to the TI field by a force (represented by the relationship $F = ma$).
- **Active gravitational mass** is a measure of the gravitational force exerted by an object.
- **Passive gravitational mass** is a measure of the gravitational force experienced by matter particles in a gravitational field. Passive gravitational mass does not exist for matter particles in the TI field model of gravity.

Definitions of Mass for Particles of the TI Field in the TI Field Model of Gravity and Inertia

- **Inertial mass** is a measure of the resistance of a particle of the TI field to being accelerated relative to the static field by the force of gravity.
- **Passive gravitational mass** is a measure of the gravitational force experienced by a particle of the TI field in a known gravitational field.

These properties are summarized in Table 1. See the section Properties of the Temporal-Inertial (TI) Field for more information on the TI field.
Table 1. Distinctions Between the Newtonian and TI Field Models of Gravity

<table>
<thead>
<tr>
<th>Model</th>
<th>Active Gravitational Mass</th>
<th>Passive Gravitational Mass</th>
<th>Inertial Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Massive Object in Newtonian Model</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Massive Object in TI Field Model</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Particles of the TI Field</td>
<td>No</td>
<td>Yes</td>
<td>Yes*</td>
</tr>
</tbody>
</table>

* This property is a measure of the resistance of particles of the TI field to acceleration relative to the static field.

Gravitational Interaction of the TI Field Model [3]

The development in this section does not include the effects predicted by the flux model. Discussion of these effects is deferred until Appendix B.

The objectives of this section are:

- Show that the inertial mass of particles of the TI field increases relativistically with the increase in their velocity.
- Show that the passive gravitational mass of particles of the TI field does not increase with the increase in their velocity.
- Accordingly, the velocity of particles of the TI field approaching a black hole and the velocity of massive particles entrained in the TI field do not reach the speed of light until the singularity is reached. The consequence of this is the absence of an event horizon about a black hole.
- The absence of an event horizon about a black hole enables light and gravitons to escape from a black hole.

The gravitational force $F_{TI}$ on a particle of the TI field located at a distance $r$ from the gravitational body is:

$$F_{TI} = G_0 \frac{M_{pTI} M_{g1}}{r^2}$$  \hspace{1cm} (1)

where

- $F_{TI}$ is the gravitational force on a particle of the TI field.
- $G_0$ is the provisional gravitational constant.
- $M_{pTI}$ is the passive gravitational mass of a particle of the TI field.
- $M_{g1}$ is the active gravitational mass of the gravitational body.
r is the distance from the body where the flux is measured.

This force exists on all particles of the TI field and accelerates those particle as given in Eq (2).

\[ a_{TI} = \frac{F_{TI}}{M_{ITI}} = \frac{G_0 M_{pTI} M_{g1}}{M_{ITI} r^2} \]  

(2)

where

\( a_{TI} \) is the acceleration of a particle of the TI field.

\( M_{ITI} \) is the inertial mass of a particle of the TI field.

At this point we can start combining terms to get the equation in a form of our choosing. We sequester the ratio of \( M_{pTI} / M_{ITI} \) in the gravitational constant to give an equation containing only the inertial mass of a particle of the TI field and the active gravitational mass of the gravitational body. The ratio of \( M_{pTI} / M_{ITI} \) is constant as all particles of the TI field are the same.

The provision gravitational constant \( G_0 \) can now be combined with the ratio of \( M_{pTI} / M_{ITI} \) to yield a new gravitational constant.

\[ G = G_0 \frac{M_{pTI}}{M_{ITI}} \]  

(3)

This new gravitational constant \( G \) in the TI field model is the same constant of proportionality used to calculate forces between gravitational bodies that is accepted today.

The acceleration of a particle of the TI field at a distance of \( r \) from the gravitational body is now given by Eq (4) using the new valuation of the gravitational constant of Eq (3).

\[ a_{TI} = \frac{F_{TI}}{M_{ITI}} = \frac{G M_{g1}}{r^2} \]  

(4)

Any massive object in the gravitational field accelerates at the same rate as particles of the TI field and so its acceleration is the same as expressed by Eq (4).

\[ a_2 = \frac{G M_{g1}}{r^2} \]  

(5)

where

\( a_2 \) is the acceleration of a massive particle or object comprising massive particles at the radius \( r \) from the gravitational body.

\( G \) is the gravitational constant.

\( M_{g1} \) is the active gravitational mass of the gravitational body.

\( r \) is the radius from the center of mass of the gravitational body.
The familiar form of Eq (5) expresses the acceleration profile about a gravitational body solely as a function of the active gravitational mass of the body and the distance of the point of measurement from the center of mass of the body. The contributions to the valuation of the profile made by the properties of mass of the TI field are hidden in their inclusion in the gravitational constant G.

^ On the Separability of Passive Gravitational Mass and Inertial Mass in the TI Field Model [3]

I attribute two properties of mass to particles of the TI field, namely passive gravitational mass and inertial mass as stated in the section Definitions of Mass for Particles of the TI Field in the TI Field Model of Gravity and Inertia. The question here is whether these two properties are separate and distinct. The argument for separability is based on the behavior of these properties as the particles of the TI field move at relativistic speed. If the passive gravitational mass and inertial mass of particles of the TI field were one and the same then the acceleration and the velocity of the field toward a black hole would be unbounded because the increase in relativistic mass of the particles would be matched by an increase in the passive gravitational mass of the particles. Massive particles within the field would be accelerated at the same rate as the field and could exceed the speed of light which would violate Special Relativity. The conclusion is that the two properties of mass of the TI field are separate and distinct.

There is no hint in the classic expression for acceleration of Eq (5) that the inertial mass of particles of the TI field plays any role in the expression. That's because we sequestered the ratio of $M_{\text{pTI}} / M_{\text{ITI}}$ in the valuation of the gravitational constant itself. We have to revert to Eq (2) to see the contribution of the inertial mass $M_{\text{ITI}}$ of particles of the TI field to the acceleration of the TI field in response to gravity.

At relativistic speed the inertial mass $M_{\text{ITI}}$ of particles of the TI field increases in accord with the Lorentz factor, $1 / \sqrt{1 - v^2 / c^2}$. So the effect on the acceleration expressed in Eq (5) is to increase the value of the gravitational ‘constant’ G. This is not how the calculation of the effect of relativistic mass increase will be handled. Instead, we'll accommodate the Lorentz factor as a multiplier of the right side of Eq (5). Much simpler.
Results of the Numerical Integration

Descent Into A Black Hole
To determine the acceleration and velocity profiles of an object descending into a black hole, a numerical integration was made for a 30 solar mass black hole. Results of this calculation are shown in Figures 1, 2 and 3. Figure 1 portrays the infall velocity of the TI field vs the radius from the singularity. As seen in the figure, the infall velocity approaches light speed as the singularity is approached.

The so-called Schwarzschild radius for a 30 solar mass black hole is 88.5 km. Figure 1 shows a range of the radius from the singularity of 73 to 99 km, thus bracketing the Schwarzschild radius. Note that according to the flux model the infall velocity of the TI field at the Schwarzschild radius is about 2.9E5 km/sec or about 97% of the speed of light, insufficient to qualify the Schwarzschild radius as the event horizon. In fact the flux model asserts that there is no event horizon about a black hole. There is no event horizon, but the infall velocity approaches the velocity of light c asymptotically as the factor \((1 - v^2 / c^2)^{1/2}\) approaches zero. This near luminal velocity occurs as the radius from the singularity approaches zero. The escape velocity is the negative of the infall velocity of the TI field.

Escape From A Black Hole
We are told that nothing can escape from inside the event horizon of a black hole. How then do gravitons escape from a black hole? If the infall velocity of the TI field reaches light speed at the event horizon, both light and gravitons cannot emerge from inside the event horizon. However, according to the flux model there is no event horizon, so light and gravitons can indeed escape from a black hole.

The flux model accommodates the emission (escape) of gravitons from a black hole. The flux model also accommodates the emission of light from a black hole, even from deep inside the black hole ‘close’ to the singularity.
For a 30 solar mass black hole, the change in infall velocity of particles of the TI field vs distance from the center of the black hole is shown in Figure 1.

Operative Equation: \( v = \left( \frac{GM}{r^2} \right) \left( 1 + \frac{v}{c} \right) \left( 1 - \left(\frac{v}{c}\right)^2 \right)^{1/2} \)
As seen in Fig. 2 and Fig. 3 the infall velocity of the TI field approaches light speed at the singularity.

Escape from a Black Hole

Distance r from Singularity, km (30 Solar Mass Black Hole)

Figure 2. Infall Velocity / Light Speed vs Radius from the Singularity

Operative Equation: \( \frac{v}{c} = \left( \frac{GM}{r^2} \right) \times \left( 1 + \frac{v}{c} \right) \times \left( 1 - \left(\frac{v}{c}\right)^2 \right)^{1/2} \)
Escape from a Black Hole

Distance $r$ from Singularity, km (30 Solar Mass Black Hole)

Figure 3. Infall Velocity / $c$ Inside the Schwarzschild Radius

Operative Equation: $v / c = (GM / r^2 c) \times (1 + v/c) \times (1 - (v/c)^2)^{1/2}$
\textbf{Conclusions}

1. As particles of the TI field fall toward a black hole their velocity remains less than the speed of light as their relativistic mass increases in accord with the Lorentz factor.

2. The velocity of particles of the TI field reach light speed only at the singularity of the black hole.

3. According to the flux model, a black hole has no event horizon.

4. The flux model accommodates the emission (escape) of gravitons from a black hole.

5. The flux model accommodates the emission of light from a black hole, even from deep inside the black hole ‘close’ to the singularity. Expect that this light will be strongly redshifted.
Appendix A

Graviton Flux is Increased by the Motion of the TI Field Toward a Gravitational Body [2]

As given in the section Properties of The Temporal-Inertial (TI) Field:

- Particles of the TI field are accelerated by gravity directly toward the center of each gravitational body just as a test particle would be and reach the escape velocity of such a particle at the distance of that particle from the center of mass of the gravitational body.

- The flux model of gravity [2] posits that in a gravitational field, the velocity of the TI field combines with that of the gravitons emitted by the gravitational body to increase the flux of gravitons relative to the TI field. Thus the response of the TI field to gravity adds to the force of gravity.

Before consideration of the flux model the acceleration profile about a gravitational body is given by Eq (A-1).

\[ a = \frac{GM}{r^2} \]  

(A-1)

The value of acceleration in Eq (A-1) is only a first cut at establishing the acceleration profile about a gravitational body. We must account for the effect of graviton flux on the particles of the TI field as those particles fall toward the gravitational body. The increase in graviton flux over the flux at a stationary point at a given distance from the gravitational body is proportional to \( v / c \).

\[ \frac{\text{FluxMoving}}{\text{FluxStationary}} = 1 + \frac{v}{c} \]  

(A-2)

The expression for the acceleration ‘a’ of particles of the TI field at a distance \( r \) from a gravitational mass \( M \), given in Eq (A-1), must now be augmented by the increase in graviton flux caused by the infall velocity of the TI field.

\[ a = \left( \frac{GM}{r^2} \right) \ast \left( 1 + \frac{v}{c} \right) \]  

(A-3)

At this point we move on to Appendix B to see the iterative process by which we determine the effects of the infall velocity on the acceleration profile about a gravitational body.
Appendix B

Development of the Acceleration and Velocity Profiles About a Weak Gravitational Body [2]

Review the Properties of the Flux Model of Gravity
The flux model of gravity posits the following:
1. The TI field is subject to gravity.
2. The infall velocity of particles of the TI field fall toward a gravitational body is the same magnitude (but opposite in sign) as the escape velocity at any given distance from the body.
3. The acceleration of particles of the TI field as they fall toward the gravitational body applies a force to massive particles and objects comprising massive particles. This force is the gravitational force as mediated by the TI field.
4. The velocity of the particles of the TI field toward a gravitational body increases the flux of gravitons ‘seen’ by these particles and thus increases the gravitational force acting on the particles. The infall velocity of the TI field is increased beyond the value expressed in the classic formula: \( v_{\text{Infall}} = - v_{\text{Escape}} = - \left(\frac{2GM}{r}\right)^{1/2} \).
5. We must evaluate the effect of this change. In other words, the increase of the infall velocity increases the gravitational force which then increases the infall velocity and so on.
6. The process to determine the outcome of this feedback problem is to step through the following:
   a. Calculate the acceleration at a given distance from a gravitational body for the current value of the infall velocity.
   b. Integrate the acceleration to yield a new expression for the infall velocity.
   c. Continue back to step ‘a’ until the contribution of the last iteration is negligible.
4. As the speed of particles falling toward the gravitational body increases, the relativistic mass of the particles increases by the Lorentz factor: \( \frac{1}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}} \). Relativistic mass is a measure of the resistance of particles of the TI field to acceleration. This increased resistance to acceleration must be accounted for.

Calculate the acceleration at a given distance from a gravitational body for the current value of the infall velocity (first iteration)
The acceleration ‘a’ of gravity by a central mass \( M \) at a distance \( r \) is given by Newtonian mechanics as
\[
a = \frac{GM}{r^2}
\]
If gravity is mediated by gravitons, then the acceleration in Eq (B-1) is proportional to the flux of gravitons at a distance \( r \) from a gravitational body of active gravitational mass \( M \). The graviton flux seen by particles of the TI field is augmented by the velocity of particles of the TI field at a given radius from the gravitational body. As described in Appendix A the augmentation in graviton flux relative to the value given implicitly in Eq (B-1) is proportional to \( v / c \), where \( v \) is the velocity of particles of the TI field toward the gravitational body and \( c \) is the velocity of light and of gravitons relative to the TI field. Particles of the TI field are thus accelerated as shown in Eq (B-2).

\[
a = \left( \frac{GM}{r^2} \right) \left( 1 + \frac{v}{c} \right) \tag{B-2}
\]

The escape velocity at a distance \( r \) from a gravitational body of mass \( M \) is given \[^6\] by

\[
v_{\text{Escape}} = \left( \frac{2GM}{r} \right)^{1/2} \tag{B-3}
\]

The magnitude of the infall velocity of particles of the TI field at a distance \( r \) is the same as the escape velocity of a particle falling from infinity radially toward the gravitational mass at that radius from the center of mass of the gravitational body as expressed in the conventional Newtonian model. The infall velocity of the TI field is the negative of the escape velocity of a particle; the magnitudes of the velocities are the same.

\[
v_{\text{Infall}} = v_{\text{Escape}} = \left( \frac{2GM}{r} \right)^{1/2} \tag{B-4}
\]

The value of \( v \) in Eq (B-2) is the value given for \( v_{\text{Infall}} \) in Eq (B-4). The expression for the acceleration of particles of the TI field in Eq (B-2) becomes Eq (B-5).

\[
a_{\text{Total}} = \left( \frac{GM}{r^2} \right) \left( \frac{v + c}{c} \right) = \left( \frac{GM}{r^2} \right) \left( \left( \frac{2GM}{r} \right)^{1/2} + c \right) / c
a_{\text{Total}} = \left( \frac{GM}{r^2} \right) \left( 1 + \frac{(2GM/c^2)^{1/2}}{c} \right) \tag{B-5}
\]

An object in free fall at a distance \( r \) from the gravitational mass would experience this same acceleration \[^2\].

Substitute the expression for the Schwarzschild radius \( r_S \) into Eq (B-5) and drop the subscript Total.

\[
r_S = \frac{2GM}{c^2} \tag{B-6}
\]

\[
a = \left( \frac{GM}{r^2} \right) \left( 1 + \frac{(r_S/r)^{1/2}}{} \right) \tag{B-7}
\]
Integrate the acceleration to yield a new expression for the infall velocity (first iteration continued)

We now integrate Eq (B-7) to derive the escape velocity. The integration procedure is adapted from reference [6] and results in Eq (B-8).

\[ v = \left( \frac{2GM}{r} \right)^{1/2} \times \left[ 1 + \frac{1}{3} \left( \frac{r_s}{r} \right)^{1/2} \right] \]  

(B-8)

We step through the sequence below to calculate expressions for the infall velocity and acceleration at a given radius from the gravitational body.

a. Calculate the acceleration at a given distance from a gravitational body for the current value of the infall velocity.

b. Integrate the acceleration to yield a new expression for the infall velocity.

c. Continue back to step ‘a’ until the contribution of the last iteration is negligible.

This procedure is done in reference [2]. The general form of the series and the results for the first four iterations are shown in Table B-1. The series converges rapidly for the modest gravitational fields of the Solar System. The stronger gravitational fields of neutron stars and black holes require consideration of relativistic effects that are described in Appendix C.
Table B-1. Summary of the Iterative Evaluation of the Acceleration and Infall Velocity Profiles About a Weak Gravitational Body

<table>
<thead>
<tr>
<th>Step</th>
<th>Expression **</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determine escape velocity v.</td>
<td>[ v = \left( \frac{2GM}{r} \right)^{1/2} ]</td>
</tr>
<tr>
<td>Determine acceleration. ( a = (GM/r^2)(1+v/c) )</td>
<td>[ a = \left( \frac{GM}{r^2} \right) \left( 1 + \left( \frac{rS}{r} \right)^{1/2} \right) ]</td>
</tr>
<tr>
<td>Integrate acceleration to get next iteration of escape velocity v.</td>
<td>[ v = \left( \frac{2GM}{r} \right)^{1/2} \left[ 1 + \left( \frac{1}{3} \right) \left( \frac{rS}{r} \right)^{1/2} \right] ]</td>
</tr>
<tr>
<td>Determine acceleration. ( a = (GM/r^2)(1+v/c) )</td>
<td>[ a = \left( \frac{GM}{r^2} \right) \left[ 1 + \left( \frac{rS}{r} \right)^{1/2} + \left( \frac{1}{3} \right) \left( \frac{rS}{r} \right) \right] ]</td>
</tr>
<tr>
<td>Integrate acceleration to get next iteration of escape velocity v.</td>
<td>[ v = \left( \frac{2GM}{r} \right)^{1/2} \left[ 1 + \left( \frac{1}{3} \right) \left( \frac{rS}{r} \right)^{1/2} + \left( \frac{1}{12} \right) \left( \frac{rS}{r} \right) \right] ]</td>
</tr>
<tr>
<td>Determine acceleration. ( a = (GM/r^2)(1+v/c) )</td>
<td>[ a = \left( \frac{GM}{r^2} \right) \left[ 1 + \left( \frac{rS}{r} \right)^{1/2} + \left( \frac{1}{3} \right) \left( \frac{rS}{r} \right) + \left( \frac{1}{60} \right) \left( \frac{rS}{r} \right)^{3/2} \right] ]</td>
</tr>
<tr>
<td>Integrate acceleration to get next iteration of escape velocity v.</td>
<td>[ v = \left( \frac{2GM}{r} \right)^{1/2} \left[ 1 + \left( \frac{1}{3} \right) \left( \frac{rS}{r} \right)^{1/2} + \left( \frac{1}{12} \right) \left( \frac{rS}{r} \right) + \left( \frac{1}{60} \right) \left( \frac{rS}{r} \right)^{3/2} \right] ]</td>
</tr>
<tr>
<td>Determine acceleration. ( a = (GM/r^2)(1+v/c) )</td>
<td>[ a = \left( \frac{GM}{r^2} \right) \left[ 1 + \left( \frac{rS}{r} \right)^{1/2} + \left( \frac{1}{3} \right) \left( \frac{rS}{r} \right)^{3/2} + \left( \frac{1}{60} \right) \left( \frac{rS}{r} \right)^2 \right] ]</td>
</tr>
<tr>
<td>The general form of the series for the infall velocity can now be written.</td>
<td>[ v = \left( \frac{2GM}{r} \right)^{1/2} \left[ 1 + \sum_{n=1}^{\infty} \left( \frac{2}{(n+2)!} \right) \left( \frac{rS}{r} \right)^{n/2} \right] ]</td>
</tr>
<tr>
<td>The general form of the series for the infall acceleration can now be written.</td>
<td>[ a = \left( \frac{GM}{r^2} \right) \left[ 1 + \sum_{n=1}^{\infty} \left( \frac{2}{(n+1)!} \right) \left( \frac{rS}{r} \right)^{n/2} \right] ]</td>
</tr>
</tbody>
</table>

** I use the abbreviation \( r_S = \left( \frac{2GM}{c^2} \right) \) even though the development here is not valid ‘near’ a black hole. The term \( r_S \) is the Schwarzschild radius [7].
Appendix C

The Effect of Relativity on the Acceleration of Particles of the TI Field

As the speed of particles falling toward the gravitational body increases, the relativistic mass of the particles increases by the factor of Eq (C-1). Relativistic mass is a measure of the resistance of particles of the TI field to acceleration at very high velocity [9].

\[
m_{\text{Rel}} / m_{\text{Rest}} = 1 / (1 - v^2 / c^2)^{1/2}
\]

where

- \(m_{\text{Rel}}\) is the relativistic mass.
- \(m_{\text{Rest}}\) is the rest mass.
- \(v\) is the velocity of the particle mass.
- \(c\) is the velocity of light.

The infall acceleration of particles of the TI field is expressed in Eq (C-2). This expression is derived from the general form shown in Table B-1.

\[
a = \left(\frac{GM}{r^2}\right) \left[ (1 + (r_S / r)^{1/2} + 1/3 (r_S / r) + 1/12 (r_S / r)^{3/2} + 1/60 (r_S / r)^2 + 1/360 (r_S / r)^{5/2} \right]
\]

The acceleration expressed in Eq (C-2) must be reduced by the factor of Eq (C-1).

\[
a = \left(\frac{GM}{r^2}\right) \left[ (1 + (r_S / r)^{1/2} + 1/3 (r_S / r) + 1/12 (r_S / r)^{3/2} + 1/60 (r_S / r)^2 + 1/360 (r_S / r)^{5/2} \right] \times (1 - v^2 / c^2)^{1/2}
\]

Integration of this equation to obtain the infall velocity is intractable. Equation (C-2), not Eq (C-3), can be used to initialize the numerical integration for a given 'distant' value of \(r\), where the contribution of the increase in relativistic mass is negligible. The infall velocity can then be calculated by numerical integration. This process is described in Appendix D.
Appendix D

Development of the Acceleration and Velocity Profiles About a Black Hole

Introduction to the Effect of Relativity on the Acceleration of Particles of the TI Field

As shown in Appendix C the relativistic increase in inertial mass of particles of the TI field reduces the acceleration of those particles by the factor given in Eq (D-1).

\[
\text{Relativistic reduction factor} = \left( 1 - \frac{v^2}{c^2} \right)^{1/2} \tag{D-1}
\]

We revert to the acceleration expressed in Eq (B-2) and reduce it by the factor of Eq (D-1) to begin the numerical integration.

\[
a_{\text{Total}} = \left( \frac{2GM}{r^2} \right) \times (1 + \frac{v}{c}) \times \left( 1 - \frac{v^2}{c^2} \right)^{1/2} \tag{D-2}
\]

The two terms on the right side of Eq (D-2) form an expression that multiplies the value of acceleration of particles of the TI field. This multiplier is a function of the ratio of the infall velocity of the TI field to the speed of light. This function is graphed in Figure D-1. When the velocity \( v \) of the TI field toward the black hole is zero the multiplier value is 1.0. The multiplier peaks at a value of \( v/c \) of 0.5, but as \( v/c \) approaches 1.0, the multiplier approaches zero. Equation (D-3) shows that when the velocity approaches the speed of light, the acceleration of particles of the TI field approaches zero.

\[
\text{Acceleration Multiplier} = \left( 1 + \frac{v}{c} \right) \times \left( 1 - \frac{v^2}{c^2} \right)^{1/2} \tag{D-3}
\]

The algorithm for the numerical integration is described in Appendix E.
Figure D-1. Acceleration Multiplier as a Function of $v / c$

Governing equation: Multiplier = $(1 + v / c) \times (1 - v^2 / c^2)^{1/2}$
Appendix E

Numerical Integration of the Flux Model of Gravity

Algorithm for the Numerical Integration

The numerical integration is initialized using the first six terms in the expressions for acceleration and velocity derived from the general form of the series equations in Table B-1. The initialization and computational steps of the integration are listed in Table E-1.

Table E-1. Algorithm for the Numerical Integration

<table>
<thead>
<tr>
<th>Step</th>
<th>Initialization **</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Initialize r(0)</td>
</tr>
</tbody>
</table>
| 2    | \( a = \frac{GM}{r^2} \left[ (1 + \left( \frac{r_s}{r} \right)^{1/2} + \frac{1}{3} \left( \frac{r_s}{r} \right) + \frac{1}{12} \left( \frac{r_s}{r} \right)^{3/2} \right. \)  
\( + \frac{1}{60} \left( \frac{r_s}{r} \right)^2 + \left( \frac{1}{360} \left( \frac{r_s}{r} \right)^5 \right] \)  
| 3    | \( v = \frac{2GM}{r^{1/2}} \left[ 1 + \frac{1}{3} \left( \frac{r_s}{r} \right)^{1/2} + \frac{1}{12} \left( \frac{r_s}{r} \right) + \frac{1}{60} \left( \frac{r_s}{r} \right)^{3/2} + \left( \frac{1}{360} \left( \frac{r_s}{r} \right)^2 \right] \)  
| 4    | deltaT = 0       |

<table>
<thead>
<tr>
<th>Step</th>
<th>Computation</th>
</tr>
</thead>
</table>
| 1    | \( a(i+1) = \frac{GM}{r(i)^2} \left( 1 + \frac{v(i)}{c} \right) \left( 1 - \frac{v(i)^2}{c^2} \right)^{1/2} \)  
| 2    | Enter a value for deltaT  
| 3    | deltaV(i+1) = a(i) * deltaT  
| 4    | \( v(i+1) = v(i) + \text{deltaV}(i+1) \)  
| 5    | \( \text{deltaR}(i+1) = v(i) \cdot \text{deltaT} + 0.5 \cdot a(i) \cdot \text{deltaT}^2 \)  
| 6    | \( r(i+1) = r(i) - \text{deltaR}(i+1) \)  
| 7    | Continue until the singularity is as close as intended for the analysis.  
| 8    | Go To Computation Step 1

** I use the notation \( r_s (r_s = 2GM/c^2) \) for brevity even though this paper asserts that a black hole has no event horizon.
References