

The Electro-Magnetic Field Equation and the Electro-Magnetic Field Transformation in Rindler spacetime

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ABSTRACT

In the general relativity theory, we find the electro-magnetic field transformation and the electro-magnetic field equation (Maxwell equation) in Rindler spacetime. We find the electro-magnetic wave equation and the electro-magnetic wave function in Rindler space-time. Specially, this article say the uniqueness of the accelerated frame because the accelerated frame can treat electro-magnetic field equation.

PACS Number:04,04.90.+e,03.30, 41.20

Key words:The general relativity theory,

The Rindler spacetime,

The electro-magnetic field transformation,

The electro-magnetic field equation

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1. Introduction

In the general relativity theory, our article's aim is that we find the electro-magnetic field equation in Rindler space-time.

The expansion of Rindler coordinate[9] is

$$ct = \gamma \left(\frac{c^2}{a_0} + \xi^1 \right) \left\{ \sinh \left(\frac{a_0 \xi^0}{c} \right) + \frac{V_0}{c} \cosh \left(\frac{a_0 \xi^0}{c} \right) \right\} - \gamma \frac{V_0 c}{a_0},$$

$$x = \gamma \left(\frac{c^2}{a_0} + \xi^1 \right) \left\{ \cosh \left(\frac{a_0 \xi^0}{c} \right) + \frac{V_0}{c} \sinh \left(\frac{a_0 \xi^0}{c} \right) \right\} - \gamma \frac{c^2}{a_0},$$

$$y = \xi^2, z = \xi^3 \quad \gamma = \frac{1}{\sqrt{1 - \frac{V_0^2}{c^2}}} \quad (1)$$

In this time, the tetrad $e^a{}_\mu$ is

$$\begin{aligned} d\tau^2 &= dt^2 - \frac{1}{c^2} [dx^2 + dy^2 + dz^2] \\ &= -\frac{1}{c^2} \eta_{ab} \frac{\partial x^a}{\partial \xi^\mu} \frac{\partial x^b}{\partial \xi^\nu} d\xi^\mu d\xi^\nu \\ &= -\frac{1}{c^2} \eta_{ab} e^a{}_\mu e^b{}_\nu d\xi^\mu d\xi^\nu = -\frac{1}{c^2} g_{\mu\nu} d\xi^\mu d\xi^\nu, \quad e^a{}_\mu = \frac{\partial x^a}{\partial \xi^\mu} \quad (2) \end{aligned}$$

$$\begin{aligned} e^{\alpha_0}(\xi^0) &= \frac{\partial x^\alpha}{\partial \xi^0} \\ &= \left(\left(1 + \frac{a_0}{c^2} \xi^1\right) \left(\gamma \cosh \left(\frac{a_0}{c} \xi^0 \right) + \frac{V_0}{c} \gamma \sinh \left(\frac{a_0}{c} \xi^0 \right) \right), \right. \\ &\quad \left. \left(1 + \frac{a_0}{c^2} \xi^1\right) \left(\gamma \sinh \left(\frac{a_0}{c} \xi^0 \right) + \frac{V_0}{c} \gamma \cosh \left(\frac{a_0}{c} \xi^0 \right) \right), 0, 0 \right), \quad \gamma = \frac{1}{\sqrt{1 - \frac{V_0^2}{c^2}}} \quad (3) \end{aligned}$$

About y -axis's and z -axis's orientation

$$e^{\alpha_2}(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^2} = (0, 0, 1, 0) \quad , \quad e^{\alpha_3}(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^3} = (0, 0, 0, 1) \quad (4)$$

The other unit vector $e^{\alpha_1}(\xi^0)$ is

$$e^{\alpha}_1(\xi^0) = \frac{\partial X^{\alpha}}{\partial \xi^1} = (\gamma \sinh(\frac{a_0}{c} \xi^0) + \frac{V_0}{c} \gamma \cosh(\frac{a_0}{c} \xi^0),$$

$$\gamma \cosh(\frac{a_0}{c} \xi^0) + \frac{V_0}{c} \gamma \sinh(\frac{a_0}{c} \xi^0), 0, 0), \gamma = \frac{1}{\sqrt{1 - \frac{V_0^2}{c^2}}} \quad (5)$$

(5)

Therefore,

$$cdt = \gamma \left[\left(1 + \frac{a_0}{c^2} \xi^1\right) \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} cd\xi^0 \right.$$

$$\left. + \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} d\xi^1 \right]$$

$$dx = \gamma \left[\left(1 + \frac{a_0}{c^2} \xi^1\right) \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} cd\xi^0 \right.$$

$$\left. + \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} d\xi^1 \right], \gamma = \frac{1}{\sqrt{1 - \frac{V_0^2}{c^2}}}$$

$$, dy = d\xi^2, \quad dz = d\xi^3 \quad (6)$$

The vector transformation is

$$V'^{\mu} = \frac{\partial X'^{\mu}}{\partial X^{\alpha}} V^{\alpha}, \quad U'_{\mu} = \frac{\partial X^{\alpha}}{\partial X'^{\mu}} U_{\alpha} \quad (7)$$

Therefore, the transformation of the electro-magnetic 4-vector potential $(\phi, \vec{A}) = A^{\alpha}$ is

$$A^{\alpha} = \frac{\partial X^{\alpha}}{\partial X'^{\mu}} A'^{\mu} = \frac{\partial X^{\alpha}}{\partial \xi^{\mu}} A_{\xi}^{\mu} = e^{\alpha}_{\mu} A_{\xi}^{\mu}, \quad e^{\alpha}_{\mu} = \frac{\partial X^{\alpha}}{\partial \xi^{\mu}}$$

$$dx^{\alpha} = \frac{\partial X^{\alpha}}{\partial X'^{\mu}} dx'^{\mu} = \frac{\partial X^{\alpha}}{\partial \xi^{\mu}} d\xi^{\mu} = e^{\alpha}_{\mu} d\xi^{\mu}, \quad e^{\alpha}_{\mu} = \frac{\partial X^{\alpha}}{\partial \xi^{\mu}} \quad (8)$$

Hence, the transformation of the electro-magnetic 4-vector potential (ϕ, \vec{A}) in inertial frame and the

electro-magnetic 4-vector potential $(\phi_{\xi}, \vec{A}_{\xi})$ in uniformly accelerated frame is

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi = 4\pi\rho$$

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right)\vec{A} = \frac{4\pi}{c} \vec{j}$$

$$\text{4-vector } (c\rho, \vec{j}) = \rho_0 \frac{dx^\alpha}{d\tau}$$

$$\phi = \gamma \left[\left(1 + \frac{a_0}{c^2} \xi^1\right) \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{v_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \phi_\xi \right. \\ \left. + \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{v_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} A_{\xi^1} \right]$$

$$A_x = \gamma \left[\left(1 + \frac{a_0}{c^2} \xi^1\right) \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{v_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \phi_{\xi^1} \right. \\ \left. + \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{v_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} A_{\xi^1} \right], \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}}$$

$$A_y = A_{\xi^2}, A_z = A_{\xi^3} \quad (9)$$

$$g = \begin{pmatrix} -\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$e^a{}_\mu e_b{}^\mu = \delta^a_b, \quad e^a{}_\mu e_a{}^\nu = \delta_\mu{}^\nu$$

$$e^a{}_\mu e^b{}_\nu \eta_{ab} = g_{\mu\nu} \rightarrow A^T \eta A = g$$

$$e_a{}^\mu e_b{}^\nu g_{\mu\nu} = \eta_{ab} \rightarrow (A^T)^{-1} g A^{-1} = (A^T)^{-1} A^T \eta A A^{-1} = \eta$$

$$e^a{}_\mu = \eta^{ab} g_{\mu\nu} e_b{}^\nu \rightarrow \eta^{-1} (A^T)^{-1} A^T \eta A = A = \eta^{-1} (A^T)^{-1} g \quad (10)$$

$$\begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix}$$

$$= \begin{pmatrix} \gamma(1 + \frac{a_0 \xi^1}{c^2}) \{ \cosh(\frac{a_0 \xi^0}{c}) + \frac{v_0}{c} \sinh(\frac{a_0 \xi^0}{c}) \} & \gamma \{ \sinh(\frac{a_0 \xi^0}{c}) + \frac{v_0}{c} \cosh(\frac{a_0 \xi^0}{c}) \} & 0 & 0 \\ \gamma(1 + \frac{a_0 \xi^1}{c^2}) \{ \sinh(\frac{a_0 \xi^0}{c}) + \frac{v_0}{c} \cosh(\frac{a_0 \xi^0}{c}) \} & \gamma \{ \cosh(\frac{a_0 \xi^0}{c}) + \frac{v_0}{c} \sinh(\frac{a_0 \xi^0}{c}) \} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ dz^3 \end{pmatrix}$$

$$= A \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ dz^3 \end{pmatrix} \quad (11)$$

$$e_{\mu}^{\alpha} = \frac{\partial \xi^{\alpha}}{\partial x^{\mu}} = A^{-1} = \begin{pmatrix} \frac{c\partial \xi^0}{c\partial t} & \frac{c\partial \xi^0}{\partial x} & \frac{c\partial \xi^0}{\partial y} & \frac{c\partial \xi^0}{\partial z} \\ \frac{\partial \xi^1}{\partial \xi^1} & \frac{\partial \xi^1}{\partial x} & \frac{\partial \xi^1}{\partial y} & \frac{\partial \xi^1}{\partial z} \\ \frac{\partial \xi^2}{\partial \xi^2} & \frac{\partial \xi^2}{\partial x} & \frac{\partial \xi^2}{\partial y} & \frac{\partial \xi^2}{\partial z} \\ \frac{\partial \xi^3}{\partial \xi^3} & \frac{\partial \xi^3}{\partial x} & \frac{\partial \xi^3}{\partial y} & \frac{\partial \xi^3}{\partial z} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\gamma \{ \cosh(\frac{a_0 \xi^0}{c}) + \frac{v_0}{c} \sinh(\frac{a_0 \xi^0}{c}) \}}{(1 + \frac{a_0 \xi^1}{c^2})} & - \frac{\gamma \{ \sinh(\frac{a_0 \xi^0}{c}) + \frac{v_0}{c} \cosh(\frac{a_0 \xi^0}{c}) \}}{(1 + \frac{a_0 \xi^1}{c^2})} & 0 & 0 \\ - \gamma \{ \sinh(\frac{a_0 \xi^0}{c}) + \frac{v_0}{c} \cosh(\frac{a_0 \xi^0}{c}) \} & \gamma \{ \cosh(\frac{a_0 \xi^0}{c}) + \frac{v_0}{c} \sinh(\frac{a_0 \xi^0}{c}) \} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(12)

$$\begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = (A^{-1})^T \begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix} = (A^T)^{-1} \begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\}}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} - \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} & 0 & 0 \\ -\frac{\gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} & \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix} \quad (13)$$

$$\frac{1}{c} \frac{\partial}{\partial t} = \frac{\partial \xi^0}{\partial t} \frac{1}{c} \frac{\partial}{\partial \xi^0} + \frac{\partial \xi^1}{\partial t} \frac{\partial}{\partial \xi^1}$$

$$= \frac{\gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\}}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial}{\partial \xi^1}$$

$$\frac{\partial}{\partial x} = \frac{\partial \xi^0}{\partial x} \frac{1}{c} \frac{\partial}{\partial \xi^0} + \frac{\partial \xi^1}{\partial x} \frac{\partial}{\partial \xi^1}$$

$$\begin{aligned}
&= -\frac{\gamma\{\sinh(\frac{a_0\xi^0}{c}) + \frac{V_0}{c}\cosh(\frac{a_0\xi^0}{c})\}}{(1 + \frac{a_0\xi^1}{c^2})} \frac{\partial}{\partial\xi^0} + \gamma\{\cosh(\frac{a_0\xi^0}{c}) + \frac{V_0}{c}\sinh(\frac{a_0\xi^0}{c})\} \frac{\partial}{\partial\xi^1} \\
&\quad \frac{\partial}{\partial y} = \frac{\partial}{\partial\xi^2}, \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial\xi^3} \\
&\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{1}{c^2(1 + \frac{a_0}{c^2}\xi^1)^2} \left(\frac{\partial}{\partial\xi^0}\right)^2 - \nabla_\xi^2 \\
&\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right), \quad \vec{\nabla}_\xi = \left(\frac{\partial}{\partial\xi^1}, \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial\xi^3}\right) \tag{14}
\end{aligned}$$

2. Electro-magnetic Field in the Rindler space-time

The electro-magnetic field (\vec{E}, \vec{B}) is in the inertial frame,

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial\vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla} \times \vec{A} \tag{15}$$

$$\begin{aligned}
E_x &= -\frac{\partial\phi}{\partial x} - \frac{\partial A_x}{\partial t} \\
&= -\left[-\frac{\gamma\{\sinh(\frac{a_0\xi^0}{c}) + \frac{V_0}{c}\cosh(\frac{a_0\xi^0}{c})\}}{(1 + \frac{a_0\xi^1}{c^2})} \frac{\partial}{\partial\xi^0} + \gamma\{\cosh(\frac{a_0\xi^0}{c}) + \frac{V_0}{c}\sinh(\frac{a_0\xi^0}{c})\} \frac{\partial}{\partial\xi^1}\right] \cdot \\
&\quad \cdot [\gamma\{\cosh(\frac{a_0\xi^0}{c}) + \frac{V_0}{c}\sinh(\frac{a_0\xi^0}{c})\}(1 + \frac{a_0\xi^1}{c^2})\phi_\xi + \gamma\{\sinh(\frac{a_0\xi^0}{c}) + \frac{V_0}{c}\cosh(\frac{a_0\xi^0}{c})\}A_{\xi^1}] \\
&\quad - \left[\frac{\gamma\{\cosh(\frac{a_0\xi^0}{c}) + \frac{V_0}{c}\sinh(\frac{a_0\xi^0}{c})\}}{(1 + \frac{a_0\xi^1}{c^2})} \frac{\partial}{\partial\xi^0} - \gamma\{\sinh(\frac{a_0\xi^0}{c}) + \frac{V_0}{c}\cosh(\frac{a_0\xi^0}{c})\} \frac{\partial}{\partial\xi^1}\right] \cdot \\
&\quad \cdot [\gamma\{\sinh(\frac{a_0\xi^0}{c}) + \frac{V_0}{c}\cosh(\frac{a_0\xi^0}{c})\}(1 + \frac{a_0\xi^1}{c^2})\phi_\xi + \gamma\{\cosh(\frac{a_0\xi^0}{c}) + \frac{V_0}{c}\sinh(\frac{a_0\xi^0}{c})\}A_{\xi^1}] \\
&= -\frac{1}{(1 + \frac{a_0\xi^1}{c^2})} \frac{\partial}{\partial\xi^1} \left[(1 + \frac{a_0}{c^2}\xi^1)^2 \phi_\xi\right] - \frac{1}{(1 + \frac{a_0\xi^1}{c^2})} \frac{\partial A_{\xi^1}}{\partial\xi^0} \tag{16} \\
E_y &= -\frac{\partial\phi}{\partial y} - \frac{\partial A_y}{\partial t}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\partial}{\partial \xi^2} \left[\gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_\xi + \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} A_{\xi^1} \right] \\
&\quad - \left[\frac{\gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\}}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial}{\partial \xi^1} \right] A_{\xi^2} \\
&= -\left(1 + \frac{a_0 \xi^1}{c^2}\right) \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial \phi_\xi}{\partial \xi^2} \\
&\quad - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial A_{\xi^2}}{\partial \xi^0} \\
&\quad + \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2} \right] \\
&= \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[-\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^2} \left[\phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right] - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial A_{\xi^2}}{\partial \xi^0} \right] \\
&\quad + \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2} \right]
\end{aligned}$$

(17)

$$\begin{aligned}
E_z &= -\frac{\partial \phi}{\partial z} - \frac{\partial A_z}{\partial t} \\
&= -\frac{\partial}{\partial \xi^3} \left[\gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_\xi + \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} A_{\xi^1} \right] \\
&\quad - \left[\frac{\gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\}}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial}{\partial \xi^1} \right] A_{\xi^3} \\
&= -\left(1 + \frac{a_0 \xi^1}{c^2}\right) \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial \phi_\xi}{\partial \xi^3}
\end{aligned}$$

$$\begin{aligned}
& - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial A_{\xi^3}}{c \partial \xi^0} \\
& + \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[\frac{\partial A_{\xi^3}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^3} \right] \\
& = \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[- \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^3} [\phi_\xi (1 + \frac{a_0 \xi^1}{c^2})^2] - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial A_{\xi^3}}{c \partial \xi^0} \right] \\
& + \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[\frac{\partial A_{\xi^3}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^3} \right]
\end{aligned}$$

(18)

$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = \frac{\partial A_{\xi^3}}{\partial \xi^2} - \frac{\partial A_{\xi^2}}{\partial \xi^3} \quad (19)$$

$$B_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = \frac{\partial A_x}{\partial \xi^3} - \frac{\partial A_{\xi^3}}{\partial x}$$

$$\begin{aligned}
& = - \frac{\partial}{\partial \xi^3} \left[\gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} (1 + \frac{a_0 \xi^1}{c^2}) \phi_\xi + \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} A_{\xi^1} \right] \\
& - \left[- \frac{\gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{c \partial \xi^0} + \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial}{\partial \xi^1} \right] A_{\xi^3} \\
& = \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[\frac{\partial A_{\xi^1}}{\partial \xi^3} - \frac{\partial A_{\xi^3}}{\partial \xi^1} \right] \\
& - \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[- \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^3} [\phi_\xi (1 + \frac{a_0 \xi^1}{c^2})^2] - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial A_{\xi^3}}{c \partial \xi^0} \right]
\end{aligned}$$

(20)

$$\begin{aligned}
B_z &= \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = \frac{\partial A_{\xi^2}}{\partial x} - \frac{\partial A_x}{\partial \xi^2} \\
&= \left[-\frac{\gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial}{\partial \xi^1} \right] A_{\xi^3} \\
&\quad - \frac{\partial}{\partial \xi^2} \left[\gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_\xi + \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} A_{\xi^1} \right] \\
&= \gamma \left\{ \cosh\left(\frac{a_0}{c} \xi^0\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2} \right] \\
&\quad + \gamma \left\{ \sinh\left(\frac{a_0}{c} \xi^0\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^2} \left[\phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right] - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial A_{\xi^2}}{\partial \xi^0} \right]
\end{aligned} \tag{21}$$

Hence, we can define the electro-magnetic field $(\vec{E}_\xi, \vec{B}_\xi)$ in Rindler spacetime.

$$\begin{aligned}
\vec{E}_\xi &= -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \vec{\nabla}_\xi \left\{ \phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right\} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial \vec{A}_\xi}{\partial \xi^0} \\
\vec{B}_\xi &= \vec{\nabla}_\xi \times \vec{A}_\xi
\end{aligned}$$

In this time, $\vec{\nabla}_\xi = \left(\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3} \right)$, $\vec{A}_\xi = (A_{\xi^1}, A_{\xi^2}, A_{\xi^3})$ (22)

We obtain the transformation of the electro-magnetic field.

$$\begin{aligned}
E_x &= -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^1} \left\{ \phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right\} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial A_{\xi^1}}{\partial \xi^0} = E_{\xi^1}, \\
E_y &= E_{\xi^2} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} + B_{\xi^3} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\},
\end{aligned}$$

$$E_z = E_{\xi^3} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} - B_{\xi^2} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

$$B_x = B_{\xi^1},$$

$$B_y = B_{\xi^2} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} - E_{\xi^3} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

$$B_z = B_{\xi^3} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} + E_{\xi^2} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

(23)

Hence,

$$E_x = E_{\xi^1}, B_x = B_{\xi^1},$$

$$\begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix} = H \begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix}$$

$$H = \begin{pmatrix} \alpha & 0 & 0 & \beta \\ 0 & \alpha & -\beta & 0 \\ 0 & -\beta & \alpha & 0 \\ \beta & 0 & 0 & \alpha \end{pmatrix}$$

$$\alpha = \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\}, \beta = \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

(24)

The inverse-transformation of the electro-magnetic field is

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$\begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix} = H^{-1} \begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix}$$

$$H^{-1} = \begin{pmatrix} \alpha & 0 & 0 & -\beta \\ 0 & \alpha & \beta & 0 \\ 0 & \beta & \alpha & 0 \\ -\beta & 0 & 0 & \alpha \end{pmatrix}$$

$$\alpha = \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\}, \beta = \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

(25)

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$E_{\xi^2} = E_y \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} - B_z \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\},$$

$$B_{\xi^2} = B_y \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} + E_z \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

$$E_{\xi^3} = E_z \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} + B_y \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

$$B_{\xi^3} = B_z \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} - E_y \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

(26)

3. Electro-magnetic Field Equation(Maxwell Equation) in the Rindler space-time

Maxwell equation is

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \quad (27-i)$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j} \quad (27-ii)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (27-iii)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (27-iv)$$

$$1. \vec{\nabla} \cdot \vec{E} = 4\pi\rho$$

$$E_x = E_{\xi^1},$$

$$E_y = E_{\xi^2} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} + B_{\xi^3} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\},$$

$$\begin{aligned}
E_z &= E_{\xi^3} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} - B_{\xi^2} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \\
4\pi\rho &= \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \\
&= \left[-\frac{\gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial}{\partial \xi^1} \right] E_{\xi^1} \\
&+ \frac{\partial}{\partial \xi^2} \left[E_{\xi^2} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} + B_{\xi^3} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \right] \\
&+ \frac{\partial}{\partial \xi^3} \left[E_{\xi^3} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} - B_{\xi^2} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \right] \\
&= \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} (\vec{\nabla}_\xi \cdot \vec{E}_\xi) \\
&+ \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial E_{\xi^1}}{\partial \xi^0} \right] \quad (28)
\end{aligned}$$

$$2. \vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j}$$

$$B_x = B_{\xi^1}$$

$$B_y = B_{\xi^2} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} - E_{\xi^3} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

$$B_z = B_{\xi^3} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} + E_{\xi^2} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

$$\text{X-component) } \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}$$

$$= \frac{\partial}{\partial \xi^2} \left[B_{\xi^3} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} + E_{\xi^2} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \right]$$

$$- \frac{\partial}{\partial \xi^3} \left[B_{\xi^2} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} - E_{\xi^3} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \right]$$

$$\begin{aligned}
&= \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} \right] \\
&+ \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[\frac{\partial E_{\xi^2}}{\partial \xi^2} + \frac{\partial E_{\xi^3}}{\partial \xi^3} \right] \\
&= \frac{\partial E_x}{\partial t} + \frac{4\pi}{c} j_x \\
&= \left[\frac{\gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\}}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial}{\partial \xi^1} \right] E_{\xi^1} + \frac{4\pi}{c} j_x
\end{aligned}$$

Hence,

$$\begin{aligned}
&\frac{4\pi}{c} j_x \\
&= \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} (\vec{\nabla}_{\xi} \cdot \vec{E}_{\xi}) \\
&+ \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial E_{\xi^1}}{\partial \xi^0} \right] \quad (29)
\end{aligned}$$

$$\text{Y-component) } \frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}$$

$$\begin{aligned}
&= \frac{\partial B_{\xi^1}}{\partial \xi^3} \\
&- \left[- \frac{\gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial}{\partial \xi^1} \right] \\
&\cdot \left[B_{\xi^3} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} + E_{\xi^2} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \right] \\
&= \frac{\partial E_y}{\partial t} + \frac{4\pi}{c} j_y
\end{aligned}$$

$$\begin{aligned}
&= \left[\frac{\gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\}}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial}{\partial \xi^1} \right] \\
&\cdot \left[E_{\xi^2} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} + B_{\xi^3} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \right] \\
&\quad + \frac{4\pi}{c} j_y \\
\frac{4\pi}{c} j_y &= \frac{\partial B_{\xi^1}}{\partial \xi^3} - \frac{\partial B_{\xi^3}}{\partial \xi^1} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{a_0}{c^2} B_{\xi^3} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial E_{\xi^2}}{\partial \xi^0} \\
&= \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^3} \left\{ B_{\xi^1} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^1} \left\{ B_{\xi^3} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial E_{\xi^2}}{\partial \xi^0} \\
&\hspace{20em} (30)
\end{aligned}$$

$$\begin{aligned}
&\text{Z-component) } \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \\
&= \left[- \frac{\gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial}{\partial \xi^1} \right] \\
&\cdot \left[B_{\xi^2} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} - E_{\xi^3} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \right] \\
&\quad - \frac{\partial B_{\xi^1}}{\partial \xi^2} \\
&= \frac{\partial E_z}{\partial t} + \frac{4\pi}{c} j_z \\
&= \left[\frac{\gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\}}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial}{\partial \xi^1} \right] \\
&\cdot \left[E_{\xi^3} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} - B_{\xi^2} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{4\pi}{c} j_z \\
\frac{4\pi}{c} j_z &= \frac{\partial B_{\xi^2}}{\partial \xi^1} - \frac{\partial B_{\xi^1}}{\partial \xi^2} + \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{a_0}{c^2} B_{\xi^2} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial E_{\xi^3}}{c \partial \xi^0} \\
&= \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^1} \{B_{\xi^2} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^2} \{B_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial E_{\xi^3}}{c \partial \xi^0}
\end{aligned} \tag{31}$$

$$3. \vec{\nabla} \cdot \vec{B} = 0$$

$$\begin{aligned}
\vec{\nabla} \cdot \vec{B} &= \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \\
&= \left[-\frac{\gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{c \partial \xi^0} + \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial}{\partial \xi^1} \right] B_{\xi^1} \\
&+ \frac{\partial}{\partial \xi^2} \left[B_{\xi^2} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} - E_{\xi^3} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \right] \\
&+ \frac{\partial}{\partial \xi^3} \left[B_{\xi^3} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} + E_{\xi^2} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \right] \\
&= \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} (\vec{\nabla}_\xi \cdot \vec{B}_\xi) \\
&+ \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[-\left(-\frac{\partial E_{\xi^2}}{\partial \xi^3} + \frac{\partial E_{\xi^3}}{\partial \xi^2} \right) - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial B_{\xi^1}}{c \partial \xi^0} \right] = 0
\end{aligned} \tag{32}$$

$$4. \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{c \partial t}$$

$$E_x = E_{\xi^1},$$

$$E_y = E_{\xi^2} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} + B_{\xi^3} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\},$$

$$E_z = E_{\xi^3} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} - B_{\xi^2} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

$$\begin{aligned}
& \text{X-component) } \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \\
&= -\frac{\partial}{\partial \xi^2} \left[E_{\xi^3} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} - B_{\xi^2} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \right] \\
&\quad - \frac{\partial}{\partial \xi^3} \left[E_{\xi^2} \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} + B_{\xi^3} \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \right] \\
&= \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[\frac{\partial E_{\xi^3}}{\partial \xi^2} - \frac{\partial E_{\xi^2}}{\partial \xi^3} \right] \\
&\quad - \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[\frac{\partial B_{\xi^2}}{\partial \xi^2} + \frac{\partial B_{\xi^3}}{\partial \xi^3} \right] \\
&= -\frac{\partial B_x}{\partial t} \\
&= -\left[\frac{\gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\}}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial}{\partial \xi^1} \right] B_{\xi^1}
\end{aligned}$$

Hence,

$$\begin{aligned}
& -\gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} \nabla_{\xi} \cdot \vec{B}_{\xi} \\
&+ \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \left[\left(\frac{\partial E_{\xi^3}}{\partial \xi^2} - \frac{\partial E_{\xi^2}}{\partial \xi^3} \right) + \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial B_{\xi^1}}{\partial \xi^0} \right] = 0 \quad (33)
\end{aligned}$$

$$\text{Y-component) } \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}$$

$$= \frac{\partial E_{\xi^1}}{\partial \xi^3}$$

$$-\left[-\frac{\gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \frac{\partial}{\partial \xi^1} \right]$$

$$\begin{aligned}
& \cdot [E_{\xi^3} \gamma \{ \cosh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \sinh(\frac{a_0 \xi^0}{c}) \} - B_{\xi^2} \gamma \{ \sinh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \cosh(\frac{a_0 \xi^0}{c}) \}] \\
&= -\frac{\partial B_y}{\partial t} \\
&= -\left[\frac{\gamma \{ \cosh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \sinh(\frac{a_0 \xi^0}{c}) \}}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} - \gamma \{ \sinh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \cosh(\frac{a_0 \xi^0}{c}) \} \frac{\partial}{\partial \xi^1} \right] \\
& \cdot [B_{\xi^2} \gamma \{ \cosh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \sinh(\frac{a_0 \xi^0}{c}) \} - E_{\xi^3} \gamma \{ \sinh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \cosh(\frac{a_0 \xi^0}{c}) \}] \\
& \frac{\partial E_{\xi^1}}{\partial \xi^3} - \frac{\partial E_{\xi^3}}{\partial \xi^1} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{a_0}{c^2} E_{\xi^3} + \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial B_{\xi^2}}{\partial \xi^0} \\
&= \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^3} \{ E_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1) \} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^1} \{ E_{\xi^3} (1 + \frac{a_0}{c^2} \xi^1) \} + \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial B_{\xi^2}}{\partial \xi^0} \\
&= 0 \tag{34}
\end{aligned}$$

$$\begin{aligned}
& \text{Z-component) } \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \\
&= \left[-\frac{\gamma \{ \sinh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \cosh(\frac{a_0 \xi^0}{c}) \}}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} + \gamma \{ \cosh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \sinh(\frac{a_0 \xi^0}{c}) \} \frac{\partial}{\partial \xi^1} \right] \\
& \cdot [E_{\xi^2} \gamma \{ \cosh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \sinh(\frac{a_0 \xi^0}{c}) \} + B_{\xi^3} \gamma \{ \sinh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \cosh(\frac{a_0 \xi^0}{c}) \}] \\
& - \frac{\partial E_{\xi^1}}{\partial \xi^2} \\
&= -\frac{\partial B_z}{\partial t} \\
&= -\left[\frac{\gamma \{ \cosh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \sinh(\frac{a_0 \xi^0}{c}) \}}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} - \gamma \{ \sinh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \cosh(\frac{a_0 \xi^0}{c}) \} \frac{\partial}{\partial \xi^1} \right]
\end{aligned}$$

$$\begin{aligned}
& \cdot [B_{\xi^3} \gamma \{ \cosh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \sinh(\frac{a_0 \xi^0}{c}) \} + E_{\xi^2} \gamma \{ \sinh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \cosh(\frac{a_0 \xi^0}{c}) \}] \\
& \frac{\partial E_{\xi^2}}{\partial \xi^1} - \frac{\partial E_{\xi^1}}{\partial \xi^2} + \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{a_0}{c^2} E_{\xi^2} + \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial B_{\xi^3}}{c \partial \xi^0} \\
& = \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^1} \{ E_{\xi^2} (1 + \frac{a_0}{c^2} \xi^1) \} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^2} \{ E_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1) \} + \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial B_{\xi^3}}{c \partial \xi^0} \\
& = 0 \tag{35}
\end{aligned}$$

Therefore, we obtain the electro-magnetic field equation by Eq (28)-Eq(35) in Rindler spacetime .

$$\vec{\nabla}_\xi \cdot \vec{E}_\xi = 4\pi\rho_\xi (1 + \frac{a_0 \xi^1}{c^2}) \tag{36-i}$$

$$\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \vec{\nabla}_\xi \times \{ \vec{B}_\xi (1 + \frac{a_0 \xi^1}{c^2}) \} = \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial \vec{E}_\xi}{c \partial \xi^0} + \frac{4\pi}{c} \vec{j}_\xi \tag{36-ii}$$

$$\vec{\nabla}_\xi \cdot \vec{B}_\xi = 0 \tag{36-iii}$$

$$\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \vec{\nabla}_\xi \times \{ \vec{E}_\xi (1 + \frac{a_0 \xi^1}{c^2}) \} = - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial \vec{B}_\xi}{c \partial \xi^0} \tag{36-iv}$$

$$\vec{E}_\xi = (E_{\xi^1}, E_{\xi^2}, E_{\xi^3}), \vec{B}_\xi = (B_{\xi^1}, B_{\xi^2}, B_{\xi^3}), \vec{\nabla}_\xi = (\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3})$$

Hence, the transformation of 4-vector $(c\rho, \vec{j}) = \rho_0 \frac{dx^\alpha}{d\tau}$ is

$$\begin{aligned}
\rho &= \rho_\xi (1 + \frac{a_0 \xi^1}{c^2}) \gamma \{ \cosh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \sinh(\frac{a_0 \xi^0}{c}) \} \\
&+ \frac{j_{\xi^1}}{c} \gamma \{ \sinh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \cosh(\frac{a_0 \xi^0}{c}) \} \\
j_x &= j_{\xi^1} \gamma \{ \cosh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \sinh(\frac{a_0 \xi^0}{c}) \} \\
&+ c\rho_\xi (1 + \frac{a_0}{c^2} \xi^1) \gamma \{ \sinh(\frac{a_0 \xi^0}{c}) + \frac{V_0}{c} \cosh(\frac{a_0 \xi^0}{c}) \}, \quad j_y = j_{\xi^2}, j_z = j_{\xi^3}
\end{aligned}$$

In this time, 4-vector $(c\rho_\xi, \vec{j}_\xi) = \rho_0 \frac{d\xi^\alpha}{d\tau}$ (37)

4. Electro-magnetic wave equation in Rindler space-time

The electro-magnetic wave function is

$$E_{\xi^1} = E_x = E_{x0} \sin \Phi,$$

$$B_{\xi^1} = B_x = B_{x0} \sin \Phi$$

$$E_{\xi^2} = E_y \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} - B_z \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\},$$

$$= E_{y0} \sin \Phi \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

$$- B_{z0} \sin \Phi \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

$$B_{\xi^2} = B_y \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} + E_z \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

$$= B_{y0} \sin \Phi \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

$$+ E_{z0} \sin \Phi \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

$$E_{\xi^3} = E_z \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} + B_y \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

$$= E_{z0} \sin \Phi \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

$$+ B_{y0} \sin \Phi \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

$$B_{\xi^3} = B_z \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} - E_y \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

$$= B_{z0} \sin \Phi \gamma \left\{ \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

$$- E_{y0} \sin \Phi \gamma \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) + \frac{V_0}{c} \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\}$$

In this time, $\Phi = \omega\left(t - l\frac{X}{c} - m\frac{Y}{c} - n\frac{Z}{c}\right)$, $l^2 + m^2 + n^2 = 1$ (38)

The electro-magnetic wave equation is in vacuum

$$\begin{aligned}
& \vec{\nabla}_\xi \times \left(1 + \frac{a_0}{c^2} \xi^1\right) \vec{\nabla}_\xi \times \left\{ \vec{E}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} \\
&= \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{\nabla}_\xi \times \left\{ \vec{E}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} + \left(1 + \frac{a_0}{c^2} \xi^1\right) \vec{\nabla}_\xi \times \vec{\nabla}_\xi \times \left\{ \vec{E}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} \\
&= \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{E}_\xi \\
&+ \left(1 + \frac{a_0}{c^2} \xi^1\right) \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{\nabla}_\xi \times \vec{E}_\xi \\
&+ \left(1 + \frac{a_0}{c^2} \xi^1\right) \vec{\nabla}_\xi \times \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{E}_\xi \\
&+ \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \vec{\nabla}_\xi \times \vec{\nabla}_\xi \times \vec{E}_\xi \\
&= \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{E}_\xi + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \vec{\nabla}_\xi \times \vec{\nabla}_\xi \times \vec{E}_\xi \\
&= [\vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \cdot \vec{E}_\xi] \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) - [\vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \cdot \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right)] \vec{E}_\xi \\
&\quad + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 [\vec{\nabla}_\xi (\vec{\nabla}_\xi \cdot \vec{E}_\xi) - \nabla_\xi^2 \vec{E}_\xi] \\
&= -\frac{1}{c} \frac{\partial}{\partial \xi^0} [\vec{\nabla}_\xi \times \{ \vec{B}_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right) \}] = -\frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \vec{E}_\xi,
\end{aligned}$$

In this time, $\vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) = \left(\frac{a_0}{c^2}, 0, 0\right)$ (39)

Hence,

$$\begin{aligned}
& \vec{\nabla}_\xi \times \left(1 + \frac{a_0}{c^2} \xi^1\right) \vec{\nabla}_\xi \times \left\{ \vec{E}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} + \frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \vec{E}_\xi \\
&= [\vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \cdot \vec{E}_\xi] \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) - [\vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \cdot \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right)] \vec{E}_\xi \\
&\quad + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 [\vec{\nabla}_\xi (\vec{\nabla}_\xi \cdot \vec{E}_\xi) - \nabla_\xi^2 \vec{E}_\xi] + \frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \vec{E}_\xi \\
&= \frac{a_0^2}{c^4} (E_{\xi^1}, 0, 0) - \frac{a_0^2}{c^4} (E_{\xi^1}, E_{\xi^2}, E_{\xi^3}) + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_\xi^2 \right] \vec{E}_\xi \\
&= \frac{a_0^2}{c^4} (0, -E_{\xi^2}, -E_{\xi^3}) + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_\xi^2 \right] \vec{E}_\xi
\end{aligned}$$

$$= \vec{0} \quad (40)$$

Hence, the magnetic wave equation is in vacuum

$$\begin{aligned} & \vec{\nabla}_\xi \times \left(1 + \frac{a_0}{c^2} \xi^1\right) \vec{\nabla}_\xi \times \left\{ \vec{B}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} + \frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \vec{B}_\xi \\ &= \left[\vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \cdot \vec{B}_\xi \right] \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) - \left[\vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \cdot \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \right] \vec{B}_\xi \\ & \quad + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \left[\vec{\nabla}_\xi (\vec{\nabla}_\xi \cdot \vec{B}_\xi) - \nabla_\xi^2 \vec{B}_\xi \right] + \frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \vec{B}_\xi \\ &= \frac{a_0^2}{c^4} (0, -B_{\xi^2}, -B_{\xi^3}) + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_\xi^2 \right] \vec{B}_\xi \\ &= \vec{0} \quad (41) \end{aligned}$$

The electromagnetic wave function, Eq(39) satisfy the electromagnetic wave equation, Eq(40), Eq(41)

5. Conclusion

We find the electro-magnetic field transformation and the electro-magnetic equation in uniformly accelerated frame.

Generally, the coordinate transformation of accelerated frame is

$$\begin{aligned} \text{(I)} \quad ct &= \left(\frac{c^2}{a_0} + \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \\ x &= \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) - \frac{c^2}{a_0}, \quad y = \xi^2, \quad z = \xi^3 \end{aligned} \quad (42)$$

$$\begin{aligned} \text{(II)} \quad ct &= \frac{c^2}{a_0} \exp\left(\frac{a_0}{c^2} \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \\ x &= \frac{c^2}{a_0} \exp\left(\frac{a_0}{c^2} \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) - \frac{c^2}{a_0}, \quad y = \xi^2, \quad z = \xi^3 \end{aligned} \quad (43)$$

Hence, this article say the accelerated frame is Rindler coordinate (I) that can treat electro-magnetic field equation.

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