

Electro-Magnetic Field Equation and Wave Function, Equation in Rindler spacetime

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ABSTRACT

In the general relativity theory, we find the electro-magnetic field transformation and the electro-magnetic field equation (Maxwell equation) in Rindler spacetime. We find the electro-magnetic wave equation and the electro-magnetic wave function in Rindler space-time. We treat Lorentz gauge condition in Rindler spacetime. Specially, this article says the uniqueness of the accelerated frame because the accelerated frame can treat electro-magnetic field equation.

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1. Introduction

In the general relativity theory, our article's aim is that we find the electro-magnetic field equation in Rindler space-time.

The Rindler coordinate is

$$ct = \left(\frac{c^2}{a_0} + \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$x = \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) - \frac{c^2}{a_0}, y = \xi^2, z = \xi^3 \quad (1)$$

In this time, the tetrad $e^a{}_\mu$ is

$$d\tau^2 = dt^2 - \frac{1}{c^2} [dx^2 + dy^2 + dz^2]$$

$$= -\frac{1}{c^2} \eta_{ab} \frac{\partial x^a}{\partial \xi^\mu} \frac{\partial x^b}{\partial \xi^\nu} d\xi^\mu d\xi^\nu$$

$$= -\frac{1}{c^2} \eta_{ab} e^a{}_\mu e^b{}_\nu d\xi^\mu d\xi^\nu = -\frac{1}{c^2} g_{\mu\nu} d\xi^\mu d\xi^\nu, e^a{}_\mu = \frac{\partial x^a}{\partial \xi^\mu} \quad (2)$$

$$e^{\alpha}{}_0(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^0} = \left(\left(1 + \frac{a_0}{c^2} \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right), \left(1 + \frac{a_0}{c^2} \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right), 0, 0 \right) \quad (3)$$

About y -axis's and z -axis's orientation

$$e^{\alpha}{}_2(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^2} = (0, 0, 1, 0), \quad e^{\alpha}{}_3(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^3} = (0, 0, 0, 1) \quad (4)$$

The other unit vector $e^{\alpha}{}_1(\xi^0)$ is

$$e^{\alpha}{}_1(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^1} = \left(\sinh\left(\frac{a_0 \xi^0}{c}\right), \cosh\left(\frac{a_0 \xi^0}{c}\right), 0, 0 \right) \quad (5)$$

Therefore,

$$cdt = c \cosh\left(\frac{a_0 \xi^0}{c}\right) d\xi^0 \left(1 + \frac{a_0}{c^2} \xi^1\right) + \sinh\left(\frac{a_0 \xi^0}{c}\right) d\xi^1$$

$$= c \cosh\left(\frac{a_0 \xi^0}{c}\right) d\hat{\xi}^0 + \sinh\left(\frac{a_0 \xi^0}{c}\right) d\hat{\xi}^1$$

$$dx = c \sinh\left(\frac{a_0 \xi^0}{c}\right) d\xi^0 \left(1 + \frac{a_0}{c^2} \xi^1\right) + \cosh\left(\frac{a_0 \xi^0}{c}\right) d\xi^1$$

$$= c \sinh\left(\frac{a_0 \xi^0}{c}\right) d\hat{\xi}^0 + \cosh\left(\frac{a_0 \xi^0}{c}\right) d\hat{\xi}^1, dy = d\xi^2 = d\hat{\xi}^2, dz = d\xi^3 = d\hat{\xi}^3 \quad (6)$$

The vector transformation is

$$V'^{\mu} = \frac{\partial X'^{\mu}}{\partial X^{\alpha}} V^{\alpha}, \quad U'_{\mu} = \frac{\partial X^{\alpha}}{\partial X'^{\mu}} U_{\alpha} \quad (7)$$

Therefore, the transformation of the electro-magnetic 4-vector potential $(\phi, \vec{A}) = A^{\alpha}$ is

$$A^{\alpha} = \frac{\partial X^{\alpha}}{\partial X'^{\mu}} A'^{\mu} = \frac{\partial X^{\alpha}}{\partial \xi^{\mu}} A_{\xi}^{\mu} = e^{\alpha}_{\mu} A_{\xi}^{\mu}, \quad e^{\alpha}_{\mu} = \frac{\partial X^{\alpha}}{\partial \xi^{\mu}}$$

$$dx^{\alpha} = \frac{\partial X^{\alpha}}{\partial X'^{\mu}} dx'^{\mu} = \frac{\partial X^{\alpha}}{\partial \xi^{\mu}} d\xi^{\mu} = e^{\alpha}_{\mu} d\xi^{\mu}, \quad e^{\alpha}_{\mu} = \frac{\partial X^{\alpha}}{\partial \xi^{\mu}} \quad (8)$$

Hence, the transformation of the electro-magnetic 4-vector potential (ϕ, \vec{A}) in inertial frame and the

electro-magnetic 4-vector potential $(\phi_{\xi}, \vec{A}_{\xi})$ in uniformly accelerated frame is

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right) \phi = 4\pi\rho$$

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right) \vec{A} = \frac{4\pi}{c} \vec{j}$$

$$\text{4-vector } (c\rho, \vec{j}) = \rho_0 \frac{dx^{\alpha}}{d\tau}$$

$$\phi = \cosh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_{\xi} + \sinh\left(\frac{a_0 \xi^0}{c}\right) A_{\xi^1}$$

$$= \cosh\left(\frac{a_0 \xi^0}{c}\right) \hat{\phi}_{\xi} + \sinh\left(\frac{a_0 \xi^0}{c}\right) \hat{A}_{\xi^1}$$

$$A_x = \sinh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_{\xi} + \cosh\left(\frac{a_0 \xi^0}{c}\right) A_{\xi^1}$$

$$= \sinh\left(\frac{a_0 \xi^0}{c}\right) \hat{\phi}_{\xi} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \hat{A}_{\xi^1}$$

$$A_y = A_{\xi^2} = \hat{A}_{\xi^2}, A_z = A_{\xi^3} = \hat{A}_{\xi^3} \quad (9)$$

$$g = \begin{pmatrix} -(1 + \frac{a_0 \xi^1}{c^2})^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$e^a{}_\mu e_b{}^\mu = \delta^a{}_b, \quad e^a{}_\mu e_a{}^\nu = \delta_\mu{}^\nu$$

$$e^a{}_\mu e^b{}_\nu \eta_{ab} = g_{\mu\nu} \rightarrow A^T \eta A = g$$

$$e_a{}^\mu e_b{}^\nu g_{\mu\nu} = \eta_{ab} \rightarrow (A^T)^{-1} g A^{-1} = (A^T)^{-1} A^T \eta A A^{-1} = \eta$$

$$e^a{}_\mu = \eta^{ab} g_{\mu\nu} e_b{}^\nu \rightarrow \eta^{-1} (A^T)^{-1} A^T \eta A = A = \eta^{-1} (A^T)^{-1} g \quad (10)$$

$$\begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} \cosh(\frac{a_0 \xi^0}{c})(1 + \frac{a_0 \xi^1}{c^2}) & \sinh(\frac{a_0 \xi^0}{c}) & 0 & 0 \\ \sinh(\frac{a_0 \xi^0}{c})(1 + \frac{a_0 \xi^1}{c^2}) & \cosh(\frac{a_0 \xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ d\xi^3 \end{pmatrix}$$

$$= A \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ d\xi^3 \end{pmatrix}$$

$$= \begin{pmatrix} \cosh(\frac{a_0 \xi^0}{c}) & \sinh(\frac{a_0 \xi^0}{c}) & 0 & 0 \\ \sinh(\frac{a_0 \xi^0}{c}) & \cosh(\frac{a_0 \xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cd\hat{\xi}^0 \\ d\hat{\xi}^1 \\ d\hat{\xi}^2 \\ d\hat{\xi}^3 \end{pmatrix}$$

(11)

$$e_{\mu}^{\alpha} = \frac{\partial \xi^{\alpha}}{\partial x^{\mu}} = A^{-1} = \begin{pmatrix} \frac{c \partial \xi^0}{c \partial t} & \frac{c \partial \xi^0}{\partial x} & \frac{c \partial \xi^0}{\partial y} & \frac{c \partial \xi^0}{\partial z} \\ \frac{\partial \xi^1}{\partial \xi^1} & \frac{\partial \xi^1}{\partial x} & \frac{\partial \xi^1}{\partial y} & \frac{\partial \xi^1}{\partial z} \\ \frac{c \partial \xi^2}{\partial \xi^2} & \frac{c \partial \xi^2}{\partial x} & \frac{c \partial \xi^2}{\partial y} & \frac{c \partial \xi^2}{\partial z} \\ \frac{\partial \xi^3}{\partial \xi^3} & \frac{\partial \xi^3}{\partial x} & \frac{\partial \xi^3}{\partial y} & \frac{\partial \xi^3}{\partial z} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} & -\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} & 0 & 0 \\ -\frac{\sinh(\frac{a_0 \xi^0}{c})}{c} & \frac{\cosh(\frac{a_0 \xi^0}{c})}{c} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (12)$$

$$\begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = (A^{-1})^T \begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix} = (A^T)^{-1} \begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} & -\frac{\sinh(\frac{a_0 \xi^0}{c})}{c} & 0 & 0 \\ -\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} & \frac{\cosh(\frac{a_0 \xi^0}{c})}{c} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix}$$

$$= \begin{pmatrix} \cosh\left(\frac{a_0 \xi^0}{c}\right) & -\sinh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 \\ -\sinh\left(\frac{a_0 \xi^0}{c}\right) & \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial \hat{\xi}^0} \\ \frac{\partial}{\partial \hat{\xi}^1} \\ \frac{\partial}{\partial \hat{\xi}^2} \\ \frac{\partial}{\partial \hat{\xi}^3} \end{pmatrix} \quad (13)$$

$$\frac{1}{c} \frac{\partial}{\partial t} = \frac{\partial \xi^0}{\partial t} \frac{1}{c} \frac{\partial}{\partial \xi^0} + \frac{\partial \xi^1}{\partial t} \frac{\partial}{\partial \xi^1}$$

$$= \frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1}$$

$$= \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \hat{\xi}^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \hat{\xi}^1}$$

$$\frac{\partial}{\partial x} = \frac{\partial \xi^0}{\partial x} \frac{1}{c} \frac{\partial}{\partial \xi^0} + \frac{\partial \xi^1}{\partial x} \frac{\partial}{\partial \xi^1}$$

$$= -\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1}$$

$$= -\sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \hat{\xi}^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \hat{\xi}^1}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \xi^2} = \frac{\partial}{\partial \hat{\xi}^2}, \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial \xi^3} = \frac{\partial}{\partial \hat{\xi}^3}$$

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{1}{c^2 \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2 = \frac{1}{c^2} \left(\frac{\partial}{\partial \hat{\xi}^0}\right)^2 - \nabla_{\hat{\xi}}^2$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right), \quad \vec{\nabla}_{\xi} = \left(\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3}\right), \quad \vec{\nabla}_{\hat{\xi}} = \left(\frac{\partial}{\partial \hat{\xi}^1}, \frac{\partial}{\partial \hat{\xi}^2}, \frac{\partial}{\partial \hat{\xi}^3}\right)$$

(14)

2. Electro-magnetic Field in the Rindler space-time

The electro-magnetic field (\vec{E}, \vec{B}) is in the inertial frame,

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}, \vec{B} = \vec{\nabla} \times \vec{A} \quad (15)$$

$$E_x = -\frac{\partial \phi}{\partial x} - \frac{\partial A_x}{\partial t}$$

$$= -\left[-\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} + \cosh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1} \right] \cdot \left[\cosh(\frac{a_0 \xi^0}{c}) \left(1 + \frac{a_0 \xi^1}{c^2}\right) \phi_\xi + \sinh(\frac{a_0 \xi^0}{c}) A_{\xi^1} \right]$$

$$- \left[\frac{\cosh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} - \sinh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1} \right] \cdot \left[\sinh(\frac{a_0 \xi^0}{c}) \left(1 + \frac{a_0 \xi^1}{c^2}\right) \phi_\xi + \cosh(\frac{a_0 \xi^0}{c}) A_{\xi^1} \right]$$

$$= -\frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial A_{\xi^1}}{\partial \xi^0} - \left(1 + \frac{a_0 \xi^1}{c^2}\right) \frac{\partial \phi_\xi}{\partial \xi^1} - 2\phi_\xi \frac{a_0}{c^2}$$

$$= -\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^1} \left[\left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \phi_\xi \right] - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial A_{\xi^1}}{\partial \xi^0}$$

$$= -\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \hat{\xi}^1} \left[\left(1 + \frac{a_0}{c^2} \xi^1\right) \hat{\phi}_\xi \right] - \frac{\partial \hat{A}_{\xi^1}}{\partial \hat{\xi}^0} \quad (16)$$

$$E_y = -\frac{\partial \phi}{\partial y} - \frac{\partial A_y}{\partial t} = -\frac{\partial}{\partial \xi^2} \left[\cosh(\frac{a_0 \xi^0}{c}) \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_\xi + \sinh(\frac{a_0 \xi^0}{c}) A_{\xi^1} \right]$$

$$- \left[\frac{\cosh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} - \sinh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1} \right] A_{\xi^2}$$

$$= -\left(1 + \frac{a_0 \xi^1}{c^2}\right) \cosh(\frac{a_0 \xi^0}{c}) \frac{\partial \phi_\xi}{\partial \xi^2} - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \cosh(\frac{a_0 \xi^0}{c}) \frac{\partial A_{\xi^2}}{\partial \xi^0}$$

$$+ \sinh(\frac{a_0}{c} \xi^0) \left[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2} \right]$$

$$\begin{aligned}
&= \cosh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^2} \left[\phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right] - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial A_{\xi^2}}{\partial \xi^0} \right] \\
&+ \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2} \right] \\
&= \cosh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \hat{\xi}^2} \left[\hat{\phi}_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right) \right] - \frac{\partial \hat{A}_{\xi^2}}{\partial \hat{\xi}^0} \right] \\
&+ \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial \hat{A}_{\xi^2}}{\partial \hat{\xi}^1} - \frac{\partial \hat{A}_{\xi^1}}{\partial \hat{\xi}^2} \right] \tag{17}
\end{aligned}$$

$$\begin{aligned}
E_z &= -\frac{\partial \phi}{\partial z} - \frac{\partial A_z}{\partial t} = -\frac{\partial}{\partial \xi^3} \left[\cosh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_\xi + \sinh\left(\frac{a_0 \xi^0}{c}\right) A_{\xi^1} \right] \\
&\quad - \left[\frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] A_{\xi^3} \\
&= -\left(1 + \frac{a_0 \xi^1}{c^2}\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial \phi_\xi}{\partial \xi^3} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial A_{\xi^3}}{\partial \xi^0} \\
&\quad + \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial A_{\xi^3}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^3} \right] \\
&= \cosh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^3} \left[\phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right] - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial A_{\xi^3}}{\partial \xi^0} \right] \\
&+ \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial A_{\xi^3}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^3} \right] \\
&= \cosh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \hat{\xi}^3} \left[\hat{\phi}_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right) \right] - \frac{\partial \hat{A}_{\xi^3}}{\partial \hat{\xi}^0} \right]
\end{aligned}$$

$$+ \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial \hat{A}_{\xi^3}}{\partial \hat{\xi}^1} - \frac{\partial \hat{A}_{\xi^1}}{\partial \hat{\xi}^3} \right] \quad (18)$$

$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = \frac{\partial A_{\xi^3}}{\partial \xi^2} - \frac{\partial A_{\xi^2}}{\partial \xi^3} = \frac{\partial \hat{A}_{\xi^3}}{\partial \hat{\xi}^2} - \frac{\partial \hat{A}_{\xi^2}}{\partial \hat{\xi}^3} \quad (19)$$

$$\begin{aligned} B_y &= \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = \frac{\partial A_x}{\partial \xi^3} - \frac{\partial A_{\xi^3}}{\partial x} \\ &= \frac{\partial}{\partial \xi^3} \left[\sinh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_\xi + \cosh\left(\frac{a_0 \xi^0}{c}\right) A_{\xi^1} \right] \\ &\quad - \left[-\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] A_{\xi^3} \\ &= \cosh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial A_{\xi^1}}{\partial \xi^3} - \frac{\partial A_{\xi^3}}{\partial \xi^1} \right] \\ &\quad - \sinh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^3} \left[\phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right] - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial A_{\xi^3}}{\partial \xi^0} \right] \\ &= \cosh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial \hat{A}_{\xi^1}}{\partial \hat{\xi}^3} - \frac{\partial \hat{A}_{\xi^3}}{\partial \hat{\xi}^1} \right] \\ &\quad - \sinh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \hat{\xi}^3} \left[\hat{\phi}_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right) \right] - \frac{\partial \hat{A}_{\xi^3}}{\partial \hat{\xi}^0} \right] \quad (20) \end{aligned}$$

$$\begin{aligned} B_z &= \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = \frac{\partial A_{\xi^2}}{\partial x} - \frac{\partial A_x}{\partial \xi^2} \\ &= \left[-\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] A_{\xi^3} \end{aligned}$$

$$\begin{aligned}
& -\frac{\partial}{\partial \xi^2} \left[\sinh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_\xi + \cosh\left(\frac{a_0 \xi^0}{c}\right) A_{\xi^1} \right] \\
& = \cosh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2} \right] \\
& + \sinh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^2} \left[\phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right] - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial A_{\xi^2}}{\partial \xi^0} \right] \\
& = \cosh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial \hat{A}_{\xi^2}}{\partial \hat{\xi}^1} - \frac{\partial \hat{A}_{\xi^1}}{\partial \hat{\xi}^2} \right] \\
& + \sinh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \hat{\xi}^2} \left[\hat{\phi}_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right) \right] - \frac{\partial \hat{A}_{\xi^2}}{\partial \hat{\xi}^0} \right] \quad (21)
\end{aligned}$$

Hence, we can define the electro-magnetic field $(\vec{E}_\xi, \vec{B}_\xi)$ in Rindler spacetime.

$$\begin{aligned}
\vec{E}_\xi & = -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \vec{\nabla}_\xi \left\{ \phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right\} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial \vec{A}_\xi}{\partial \xi^0} \\
& = -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \vec{\nabla}_\xi \left\{ \hat{\phi}_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right) \right\} - \frac{\partial \vec{\hat{A}}_\xi}{\partial \hat{\xi}^0} \\
\vec{B}_\xi & = \vec{\nabla}_\xi \times \vec{A}_\xi = \vec{\nabla}_\xi \times \vec{\hat{A}}_\xi
\end{aligned}$$

In this time, $\vec{\nabla}_\xi = \left(\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3} \right), \vec{A}_\xi = (A_{\xi^1}, A_{\xi^2}, A_{\xi^3})$

$$\vec{\nabla}_\xi = \left(\frac{\partial}{\partial \hat{\xi}^1}, \frac{\partial}{\partial \hat{\xi}^2}, \frac{\partial}{\partial \hat{\xi}^3} \right), \vec{\hat{A}}_\xi = (\hat{A}_{\xi^1}, \hat{A}_{\xi^2}, \hat{A}_{\xi^3}) \quad (22)$$

Hence, Lorentz gauge condition is in Rindler spacetime,

$$\phi_\xi \rightarrow \phi_\xi - \frac{1}{c} \frac{\partial \Lambda}{\partial \xi^0} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2}, \quad \vec{A}_\xi \rightarrow \vec{A}_\xi + \vec{\nabla}_\xi \Lambda, \quad \Lambda \text{ is a scalar function.} \quad (22-1)$$

$$\begin{aligned}
\vec{E}_\xi &= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})} \vec{\nabla}_\xi \left\{ \phi_\xi \left(1 + \frac{a_0\xi^1}{c^2} \right)^2 \right\} + \frac{1}{(1+\frac{a_0\xi^1}{c^2})} \vec{\nabla}_\xi \frac{\partial \Lambda}{\partial \xi^0} \\
&\quad - \frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{\partial \vec{A}_\xi}{\partial \xi^0} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{\partial}{\partial \xi^0} \vec{\nabla}_\xi \Lambda \\
&= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})} \vec{\nabla}_\xi \left\{ \phi_\xi \left(1 + \frac{a_0\xi^1}{c^2} \right)^2 \right\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{\partial \vec{A}_\xi}{\partial \xi^0} \\
\vec{B}_\xi &= \vec{\nabla}_\xi \times \vec{A}_\xi + \vec{\nabla}_\xi \times \vec{\nabla}_\xi \times \Lambda = \vec{\nabla}_\xi \times \vec{A}_\xi
\end{aligned} \tag{22-2}$$

Lorentz gauge fix condition is in Rindler spacetime,

$$\begin{aligned}
0 &= \frac{1}{c} \frac{\partial \phi_\xi}{\partial \xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi \rightarrow \frac{1}{c} \frac{\partial \phi_\xi}{\partial \xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi - \left[\frac{1}{c^2} \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 - \nabla_\xi^2 \right] \Lambda = 0 \\
&\quad \left[\frac{1}{c^2} \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 - \nabla_\xi^2 \right] \Lambda = 0
\end{aligned} \tag{22-3}$$

We obtain the transformation of the electro-magnetic field.

$$\begin{aligned}
E_x &= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{\partial}{\partial \xi^1} \left\{ \phi_\xi \left(1 + \frac{a_0\xi^1}{c^2} \right)^2 \right\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{\partial A_{\xi^1}}{\partial \xi^0} \\
&= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{\partial}{\partial \hat{\xi}^1} \left\{ \hat{\phi}_\xi \left(1 + \frac{a_0\xi^1}{c^2} \right) \right\} - \frac{\partial \hat{A}_{\xi^1}}{\partial \hat{\xi}^0} = E_{\xi^1},
\end{aligned}$$

$$E_y = E_{\xi^2} \cosh\left(\frac{a_0\xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0\xi^0}{c}\right),$$

$$E_z = E_{\xi^3} \cosh\left(\frac{a_0\xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0\xi^0}{c}\right)$$

$$B_x = B_{\xi^1},$$

$$B_y = B_{\xi^2} \cosh\left(\frac{a_0\xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0\xi^0}{c}\right)$$

$$B_z = B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right) \quad (23)$$

Hence,

$$E_x = E_{\xi^1}, B_x = B_{\xi^1},$$

$$\begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix} = H \begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix}$$

$$H = \begin{pmatrix} \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 & \sinh\left(\frac{a_0 \xi^0}{c}\right) \\ 0 & \cosh\left(\frac{a_0 \xi^0}{c}\right) & -\sinh\left(\frac{a_0 \xi^0}{c}\right) & 0 \\ 0 & -\sinh\left(\frac{a_0 \xi^0}{c}\right) & \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 \\ \sinh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 & \cosh\left(\frac{a_0 \xi^0}{c}\right) \end{pmatrix} \quad (24)$$

The inverse-transformation of the electro-magnetic field is

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$\begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix} = H^{-1} \begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix}$$

$$H^{-1} = \begin{pmatrix} \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 & -\sinh\left(\frac{a_0 \xi^0}{c}\right) \\ 0 & \cosh\left(\frac{a_0 \xi^0}{c}\right) & \sinh\left(\frac{a_0 \xi^0}{c}\right) & 0 \\ 0 & \sinh\left(\frac{a_0 \xi^0}{c}\right) & \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 \\ -\sinh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 & \cosh\left(\frac{a_0 \xi^0}{c}\right) \end{pmatrix} \quad (25)$$

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$\begin{aligned}
E_{\xi^2} &= E_y \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_z \sinh\left(\frac{a_0 \xi^0}{c}\right), \\
B_{\xi^2} &= B_y \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_z \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
E_{\xi^3} &= E_z \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_y \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
B_{\xi^3} &= B_z \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_y \sinh\left(\frac{a_0 \xi^0}{c}\right)
\end{aligned} \tag{26}$$

3. Electro-magnetic Field Equation(Maxwell Equation) in the Rindler space-time

Maxwell equation is

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \tag{27-i}$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{c \partial t} + 4\pi \vec{j} \tag{27-ii}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{27-iii}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{c \partial t} \tag{27-iv}$$

$$1. \vec{\nabla} \cdot \vec{E} = 4\pi\rho$$

$$E_x = E_{\xi^1},$$

$$E_y = E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right),$$

$$E_z = E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$\begin{aligned}
4\pi\rho &= \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \\
&= \left[-\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] E_{\xi^1}
\end{aligned}$$

$$+ \frac{\partial}{\partial \xi^2} \left[E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right]$$

$$+ \frac{\partial}{\partial \xi^3} \left[E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right]$$

$$= \cosh\left(\frac{a_0}{c} \xi^0\right) (\vec{\nabla}_\xi \cdot \vec{E}_\xi) + \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial E_{\xi^1}}{\partial \xi^0} \right]$$

$$= \cosh\left(\frac{a_0}{c} \xi^0\right) (\vec{\nabla}_\xi \cdot \vec{E}_\xi) + \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial B_{\xi^3}}{\partial \hat{\xi}^2} - \frac{\partial B_{\xi^2}}{\partial \hat{\xi}^3} - \frac{\partial E_{\xi^1}}{\partial \hat{\xi}^0} \right] \quad (28)$$

$$2. \vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j}$$

$$B_x = B_{\xi^1}$$

$$B_y = B_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$B_z = B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$\text{X-component) } \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}$$

$$= \frac{\partial}{\partial \xi^2} [B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)]$$

$$- \frac{\partial}{\partial \xi^3} [B_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right)]$$

$$= \cosh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} \right] + \sinh\left(\frac{a_0 \xi^0}{c}\right) \left[\frac{\partial E_{\xi^2}}{\partial \xi^2} + \frac{\partial E_{\xi^3}}{\partial \xi^3} \right]$$

$$= \frac{\partial E_x}{\partial t} + \frac{4\pi}{c} j_x$$

$$= \left[\frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] E_{\xi^1} + \frac{4\pi}{c} j_x$$

Hence,

$$\frac{4\pi}{c} j_x$$

$$\begin{aligned}
&= \sinh\left(\frac{a_0 \xi^0}{c}\right) (\vec{\nabla}_{\xi} \cdot \vec{E}_{\xi}) + \cosh\left(\frac{a_0 \xi^0}{c}\right) \left[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial E_{\xi^1}}{\partial \xi^0} \right] \\
&= \sinh\left(\frac{a_0 \xi^0}{c}\right) (\vec{\nabla}_{\hat{\xi}} \cdot \vec{E}_{\xi}) + \cosh\left(\frac{a_0 \xi^0}{c}\right) \left[\frac{\partial B_{\xi^3}}{\partial \hat{\xi}^2} - \frac{\partial B_{\xi^2}}{\partial \hat{\xi}^3} - \frac{\partial E_{\xi^1}}{\partial \hat{\xi}^0} \right] \quad (29)
\end{aligned}$$

$$\text{Y-component) } \frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}$$

$$= \frac{\partial B_{\xi^1}}{\partial \xi^3}$$

$$- \left[-\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] \cdot [B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)]$$

$$= \frac{\partial E_y}{\partial t} + \frac{4\pi}{c} j_y$$

$$= \left[\frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] \cdot [E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right)]$$

$$+ \frac{4\pi}{c} j_y$$

$$\frac{4\pi}{c} j_y = \frac{\partial B_{\xi^1}}{\partial \xi^3} - \frac{\partial B_{\xi^3}}{\partial \xi^1} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{a_0}{c^2} B_{\xi^3} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial E_{\xi^2}}{\partial \xi^0}$$

$$= \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^3} \{B_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^1} \{B_{\xi^3} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial E_{\xi^2}}{\partial \xi^0}$$

$$= \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^3} \{B_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \hat{\xi}^1} \{B_{\xi^3} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{\partial E_{\xi^2}}{\partial \hat{\xi}^0}$$

(30)

$$\text{Z-component) } \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}$$

$$\begin{aligned}
&= \left[-\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] \cdot [B_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right)] \\
&\quad - \frac{\partial B_{\xi^1}}{\partial \xi^2} \\
&= \frac{\partial E_z}{\partial t} + \frac{4\pi}{c} j_z \\
&= \left[\frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] \cdot [E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)] \\
&\quad + \frac{4\pi}{c} j_z \\
\frac{4\pi}{c} j_z &= \frac{\partial B_{\xi^2}}{\partial \xi^1} - \frac{\partial B_{\xi^1}}{\partial \xi^2} + \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{a_0}{c^2} B_{\xi^2} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial E_{\xi^3}}{\partial \xi^0} \\
&= \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^1} \{B_{\xi^2} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^2} \{B_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial E_{\xi^3}}{\partial \xi^0} \\
&= \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^1} \{B_{\xi^2} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^2} \{B_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{\partial E_{\xi^3}}{\partial \xi^0}
\end{aligned}$$

(31)

$$3. \vec{\nabla} \cdot \vec{B} = 0$$

$$\begin{aligned}
\vec{\nabla} \cdot \vec{B} &= \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \\
&= \left[-\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] B_{\xi^1} \\
&\quad + \frac{\partial}{\partial \xi^2} [B_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right)]
\end{aligned}$$

$$\begin{aligned}
& + \frac{\partial}{\partial \xi^3} [B_{\xi^3} \cosh(\frac{a_0 \xi^0}{c}) + E_{\xi^2} \sinh(\frac{a_0 \xi^0}{c})] \\
& = \cosh(\frac{a_0 \xi^0}{c}) (\vec{\nabla}_{\xi} \cdot \vec{B}_{\xi}) + \sinh(\frac{a_0 \xi^0}{c}) [-(-\frac{\partial E_{\xi^2}}{\partial \xi^3} + \frac{\partial E_{\xi^3}}{\partial \xi^2}) - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial B_{\xi^1}}{\partial \xi^0}] = 0 \\
& = \cosh(\frac{a_0 \xi^0}{c}) (\vec{\nabla}_{\xi} \cdot \vec{B}_{\xi}) + \sinh(\frac{a_0 \xi^0}{c}) [-(-\frac{\partial E_{\xi^2}}{\partial \xi^3} + \frac{\partial E_{\xi^3}}{\partial \xi^2}) - \frac{\partial B_{\xi^1}}{\partial \xi^0}] = 0
\end{aligned}$$

(32)

$$4. \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$E_x = E_{\xi^1},$$

$$E_y = E_{\xi^2} \cosh(\frac{a_0 \xi^0}{c}) + B_{\xi^3} \sinh(\frac{a_0 \xi^0}{c}),$$

$$E_z = E_{\xi^3} \cosh(\frac{a_0 \xi^0}{c}) - B_{\xi^2} \sinh(\frac{a_0 \xi^0}{c})$$

$$\text{X-component) } \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}$$

$$= \frac{\partial}{\partial \xi^2} [E_{\xi^3} \cosh(\frac{a_0 \xi^0}{c}) - B_{\xi^2} \sinh(\frac{a_0 \xi^0}{c})]$$

$$- \frac{\partial}{\partial \xi^3} [E_{\xi^2} \cosh(\frac{a_0 \xi^0}{c}) + B_{\xi^3} \sinh(\frac{a_0 \xi^0}{c})]$$

$$= \cosh(\frac{a_0}{c} \xi^0) [\frac{\partial E_{\xi^3}}{\partial \xi^2} - \frac{\partial E_{\xi^2}}{\partial \xi^3}] - \sinh(\frac{a_0 \xi^0}{c}) [\frac{\partial B_{\xi^2}}{\partial \xi^2} + \frac{\partial B_{\xi^3}}{\partial \xi^3}]$$

$$= -\frac{\partial B_x}{\partial t}$$

$$= -[\frac{\cosh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} - \sinh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1}] B_{\xi^1}$$

$$\text{Hence, } -\sinh\left(\frac{a_0\xi^0}{c}\right)(\vec{\nabla}_{\xi} \cdot \vec{B}_{\xi}) + \cosh\left(\frac{a_0\xi^0}{c}\right) \left[\frac{\partial E_{\xi^3}}{\partial \xi^2} - \frac{\partial E_{\xi^2}}{\partial \xi^3} \right] + \frac{1}{\left(1 + \frac{a_0\xi^1}{c^2}\right)} \frac{\partial B_{\xi^1}}{\partial \xi^0}$$

$$= -\sinh\left(\frac{a_0\xi^0}{c}\right)(\vec{\nabla}_{\xi} \cdot \vec{B}_{\xi}) + \cosh\left(\frac{a_0\xi^0}{c}\right) \left[\left(\frac{\partial E_{\xi^3}}{\partial \xi^2} - \frac{\partial E_{\xi^2}}{\partial \xi^3} \right) + \frac{\partial B_{\xi^1}}{\partial \xi^0} \right] = 0 \quad (33)$$

$$\text{Y-component) } \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}$$

$$= \frac{\partial E_{\xi^1}}{\partial \xi^3}$$

$$- \left[-\frac{\sinh\left(\frac{a_0\xi^0}{c}\right)}{\left(1 + \frac{a_0\xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0\xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] \cdot [E_{\xi^3} \cosh\left(\frac{a_0\xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0\xi^0}{c}\right)]$$

$$= -\frac{\partial B_y}{\partial t}$$

$$= - \left[\frac{\cosh\left(\frac{a_0\xi^0}{c}\right)}{\left(1 + \frac{a_0\xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0\xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] \cdot [B_{\xi^2} \cosh\left(\frac{a_0\xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0\xi^0}{c}\right)]$$

$$\frac{\partial E_{\xi^1}}{\partial \xi^3} - \frac{\partial E_{\xi^3}}{\partial \xi^1} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{a_0}{c^2} E_{\xi^3} + \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial B_{\xi^2}}{\partial \xi^0}$$

$$= \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^3} \left\{ E_{\xi^1} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^1} \left\{ E_{\xi^3} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} + \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial B_{\xi^2}}{\partial \xi^0}$$

$$= \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^3} \left\{ E_{\xi^1} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^1} \left\{ E_{\xi^3} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} + \frac{\partial B_{\xi^2}}{\partial \xi^0} = 0$$

(34)

$$\text{Z-component) } \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}$$

$$\begin{aligned}
&= \left[-\frac{\sinh\left(\frac{a_0\xi^0}{c}\right)}{\left(1+\frac{a_0\xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0\xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] \cdot \left[E_{\xi^2} \cosh\left(\frac{a_0\xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0\xi^0}{c}\right) \right] \\
&\quad - \frac{\partial E_{\xi^1}}{\partial \xi^2} \\
&= -\frac{\partial B_z}{\partial t} \\
&= \left[-\frac{\cosh\left(\frac{a_0\xi^0}{c}\right)}{\left(1+\frac{a_0\xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0\xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] \cdot \left[B_{\xi^3} \cosh\left(\frac{a_0\xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0\xi^0}{c}\right) \right] \\
&\quad - \frac{\partial E_{\xi^2}}{\partial \xi^1} - \frac{\partial E_{\xi^1}}{\partial \xi^2} + \frac{1}{\left(1+\frac{a_0}{c^2}\xi^1\right)} \frac{a_0}{c^2} E_{\xi^2} + \frac{1}{\left(1+\frac{a_0}{c^2}\xi^1\right)} \frac{\partial B_{\xi^3}}{\partial \xi^0} \\
&= \frac{1}{\left(1+\frac{a_0}{c^2}\xi^1\right)} \frac{\partial}{\partial \xi^1} \left\{ E_{\xi^2} \left(1+\frac{a_0}{c^2}\xi^1\right) \right\} - \frac{1}{\left(1+\frac{a_0}{c^2}\xi^1\right)} \frac{\partial}{\partial \xi^2} \left\{ E_{\xi^1} \left(1+\frac{a_0}{c^2}\xi^1\right) \right\} + \frac{1}{\left(1+\frac{a_0}{c^2}\xi^1\right)} \frac{\partial B_{\xi^3}}{\partial \xi^0} \\
&= \frac{1}{\left(1+\frac{a_0}{c^2}\xi^1\right)} \frac{\partial}{\partial \xi^1} \left\{ E_{\xi^2} \left(1+\frac{a_0}{c^2}\xi^1\right) \right\} - \frac{1}{\left(1+\frac{a_0}{c^2}\xi^1\right)} \frac{\partial}{\partial \xi^2} \left\{ E_{\xi^1} \left(1+\frac{a_0}{c^2}\xi^1\right) \right\} + \frac{\partial B_{\xi^3}}{\partial \xi^0} = 0
\end{aligned}$$

(35)

Therefore, we obtain the electro-magnetic field equation by Eq (28)-Eq(35) in Rindler spacetime .

$$\vec{\nabla}_{\xi} \cdot \vec{E}_{\xi} = \vec{\nabla}_{\xi} \cdot \vec{E}_{\xi} = 4\pi\rho_{\xi} \left(1 + \frac{a_0\xi^1}{c^2}\right) \quad (36-i)$$

$$\frac{1}{\left(1+\frac{a_0\xi^1}{c^2}\right)} \vec{\nabla}_{\xi} \times \left\{ \vec{B}_{\xi} \left(1+\frac{a_0\xi^1}{c^2}\right) \right\} = \frac{1}{\left(1+\frac{a_0\xi^1}{c^2}\right)} \frac{\partial \vec{E}_{\xi}}{\partial \xi^0} + \frac{4\pi}{c} \vec{j}_{\xi}$$

$$\frac{1}{\left(1+\frac{a_0\xi^1}{c^2}\right)} \vec{\nabla}_{\xi} \times \left\{ \vec{B}_{\xi} \left(1+\frac{a_0\xi^1}{c^2}\right) \right\} = \frac{\partial \vec{E}_{\xi}}{\partial \xi^0} + \frac{4\pi}{c} \vec{j}_{\xi} \quad (36-ii)$$

$$\vec{\nabla}_{\xi} \cdot \vec{B}_{\xi} = \vec{\nabla}_{\xi} \cdot \vec{B}_{\xi} = 0 \quad (36-iii)$$

$$\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \vec{\nabla}_\xi \times \{ \vec{E}_\xi (1 + \frac{a_0 \xi^1}{c^2}) \} = - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial \vec{B}_\xi}{\partial \xi^0}$$

$$\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \vec{\nabla}_\xi \times \{ \vec{E}_\xi (1 + \frac{a_0 \xi^1}{c^2}) \} = - \frac{\partial \vec{B}_\xi}{\partial \xi^0} \quad (36-iv)$$

$$\vec{E}_\xi = (E_{\xi^1}, E_{\xi^2}, E_{\xi^3}), \vec{B}_\xi = (B_{\xi^1}, B_{\xi^2}, B_{\xi^3}),$$

$$\vec{\nabla}_\xi = (\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3}), \vec{\nabla}_\xi = (\frac{\partial}{\partial \hat{\xi}^1}, \frac{\partial}{\partial \hat{\xi}^2}, \frac{\partial}{\partial \hat{\xi}^3})$$

Hence, the transformation of 4-vector $(c\rho, \vec{j}) = \rho_0 \frac{dx^\alpha}{d\tau}$ is

$$\rho = \rho_\xi (1 + \frac{a_0 \xi^1}{c^2}) \cosh(\frac{a_0 \xi^0}{c}) + \frac{j_{\xi^1}}{c} \sinh(\frac{a_0 \xi^0}{c})$$

$$j_x = j_{\xi^1} \cosh(\frac{a_0 \xi^0}{c}) + c\rho_\xi (1 + \frac{a_0}{c^2} \xi^1) \sinh(\frac{a_0 \xi^0}{c}), \quad j_y = j_{\xi^2}, j_z = j_{\xi^3}$$

$$\text{In this time, 4-vector } (c\rho_\xi, \vec{j}_\xi) = \rho_0 \frac{d\xi^\alpha}{d\tau} \quad (37)$$

4. Electro-magnetic wave equation in Rindler space-time

The electro-magnetic wave function is

$$E_x = E_{x0} \sin \Phi, E_y = E_{y0} \sin \Phi, E_z = E_{z0} \sin \Phi$$

$$B_x = B_{x0} \sin \Phi, B_y = B_{y0} \sin \Phi, B_z = B_{z0} \sin \Phi$$

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$E_{\xi^1} = E_{x0} \sin \Phi', B_{\xi^1} = B_{x0} \sin \Phi'$$

$$E_{\xi^2} = E_y \cosh(\frac{a_0 \xi^0}{c}) - B_z \sinh(\frac{a_0 \xi^0}{c}),$$

$$= (E_{y0} \sin \Phi') \cosh(\frac{a_0 \xi^0}{c}) - (B_{z0} \sin \Phi') \sinh(\frac{a_0 \xi^0}{c})$$

$$B_{\xi^2} = B_y \cosh(\frac{a_0 \xi^0}{c}) + E_z \sinh(\frac{a_0 \xi^0}{c})$$

$$\begin{aligned}
&= (B_{y0} \sin \Phi') \cosh\left(\frac{a_0 \xi^0}{c}\right) + (E_{z0} \sin \Phi') \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
E_{\xi^3} &= E_z \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_y \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
&= (E_{z0} \sin \Phi') \cosh\left(\frac{a_0 \xi^0}{c}\right) + (B_{y0} \sin \Phi') \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
B_{\xi^3} &= B_z \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_y \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
&= (B_{z0} \sin \Phi') \cosh\left(\frac{a_0 \xi^0}{c}\right) - (E_{y0} \sin \Phi') \sinh\left(\frac{a_0 \xi^0}{c}\right) \quad (38)
\end{aligned}$$

$$\begin{aligned}
\Phi &= \omega \left(t - l \frac{X}{c} - m \frac{Y}{c} - n \frac{Z}{c} \right), \\
\Phi' &= \omega' \left(\hat{\xi}^0 - l' \frac{\hat{\xi}^1}{c} - m' \frac{\hat{\xi}^2}{c} - n' \frac{\hat{\xi}^3}{c} \right)
\end{aligned}$$

In this time,

$$ct = \left(\frac{c^2}{a_0} + \xi^1 \right) \sinh\left(\frac{a_0 \xi^0}{c}\right), \quad x = \left(\frac{c^2}{a_0} + \xi^1 \right) \cosh\left(\frac{a_0 \xi^0}{c}\right) - \frac{c^2}{a_0}$$

$$y = \xi^2, \quad z = \xi^3$$

$$\xi^0 = \frac{c}{a_0} \tanh^{-1}\left(\frac{ct}{x + \frac{c^2}{a_0}}\right), \quad \xi^1 = \sqrt{\left(x + \frac{c^2}{a_0}\right)^2 - c^2 t^2} - \frac{c^2}{a_0}$$

$$\lim_{a_0 \rightarrow 0} \xi^0 = \lim_{a_0 \rightarrow 0} c \tanh^{-1}\left(\frac{cta_0}{a_0 x + c^2}\right) / a_0 = \lim_{a_0 \rightarrow 0} c \tanh^{-1}\left(\frac{cta_0}{c^2}\right) / a_0 = \lim_{a_0 \rightarrow 0} c \frac{1}{1 - \left(\frac{a_0 t}{c}\right)^2} \frac{t}{c} = t$$

$$\begin{aligned}
\lim_{a_0 \rightarrow 0} \xi^1 &= \lim_{a_0 \rightarrow 0} c^2 \left(\sqrt{\left(1 + \frac{a_0}{c^2} x\right)^2 - \frac{a_0^2 t^2}{c^2}} - 1 \right) / a_0 = \lim_{a_0 \rightarrow 0} c^2 \left(\sqrt{\left(1 + \frac{a_0}{c^2} x\right)^2} - 1 \right) / a_0 \\
&= \lim_{a_0 \rightarrow 0} c^2 \left(\frac{a_0}{c^2} x \right) / a_0 = x
\end{aligned}$$

Hence,

$$\lim_{a_0 \rightarrow 0} \hat{\xi}^0 = \lim_{a_0 \rightarrow 0} \int d\hat{\xi}^0 = \lim_{a_0 \rightarrow 0} \int \left(1 + \frac{a_0}{c^2} \xi^1\right) d\xi^0 = \lim_{a_0 \rightarrow 0} \int d\xi^0 = \lim_{a_0 \rightarrow 0} \xi^0 = t$$

$$\lim_{a_0 \rightarrow 0} \hat{\xi}^1 = \lim_{a_0 \rightarrow 0} \xi^1 = x, \quad y = \xi^2 = \hat{\xi}^2, \quad z = \xi^3 = \hat{\xi}^3$$

Therefore, electro-magnetic wave function is

$$\begin{aligned} \lim_{a_0 \rightarrow 0} \Phi^1 &= \lim_{a_0 \rightarrow 0} \omega^1 \left(\hat{\xi}^0 - l' \frac{\hat{\xi}^1}{c} - m' \frac{\hat{\xi}^2}{c} - n' \frac{\hat{\xi}^3}{c} \right) = \omega^1 \left(t - l' \frac{x}{c} - m' \frac{y}{c} - n' \frac{z}{c} \right) \\ &= \omega^1 \left(t - l' \frac{x}{c} - m' \frac{y}{c} - n' \frac{z}{c} \right) = \Phi \end{aligned}$$

$$\omega = \omega^1, \quad l' = l, \quad m' = m, \quad n' = n$$

$$l'^2 + m'^2 + n'^2 = 1, \quad l^2 + m^2 + n^2 = 1 \quad (39)$$

Hence,

$$\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^1}^2 \right] E_{\xi^1}$$

$$= \left[\frac{1}{c^2} \left(\frac{\partial}{\partial \hat{\xi}^0}\right)^2 - \nabla_{\hat{\xi}^1}^2 \right] E_{\xi^1} = 0$$

$$\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] B_{\xi^1}$$

$$= \left[\frac{1}{c^2} \left(\frac{\partial}{\partial \hat{\xi}^0}\right)^2 - \nabla_{\hat{\xi}^2}^2 \right] B_{\xi^1} = 0$$

$$\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^3}^2 \right] E_y$$

$$= \left[\frac{1}{c^2} \left(\frac{\partial}{\partial \hat{\xi}^0}\right)^2 - \nabla_{\hat{\xi}^3}^2 \right] E_y = 0$$

$$\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^1}^2 \right] B_y$$

$$= \left[\frac{1}{c^2} \left(\frac{\partial}{\partial \hat{\xi}^0}\right)^2 - \nabla_{\hat{\xi}^1}^2 \right] B_y = 0$$

$$\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] E_z$$

$$= \left[\frac{1}{c^2} \left(\frac{\partial}{\partial \hat{\xi}^0}\right)^2 - \nabla_{\hat{\xi}^2}^2 \right] E_z = 0$$

$$\begin{aligned}
& \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2 \right] B_z \\
& = \left[\frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2 \right] B_z = 0
\end{aligned} \tag{40}$$

The electro-magnetic wave equation is in vacuum

$$\begin{aligned}
& \vec{\nabla}_{\xi} \times \left(1 + \frac{a_0}{c^2} \xi^1\right) \vec{\nabla}_{\xi} \times \left\{ \vec{E}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} \\
& = \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{\nabla}_{\xi} \times \left\{ \vec{E}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} + \left(1 + \frac{a_0}{c^2} \xi^1\right) \vec{\nabla}_{\xi} \times \vec{\nabla}_{\xi} \times \left\{ \vec{E}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} \\
& = \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{E}_{\xi} \\
& + \left(1 + \frac{a_0}{c^2} \xi^1\right) \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{\nabla}_{\xi} \times \vec{E}_{\xi} \\
& + \left(1 + \frac{a_0}{c^2} \xi^1\right) \vec{\nabla}_{\xi} \times \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{E}_{\xi} \\
& + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \vec{\nabla}_{\xi} \times \vec{\nabla}_{\xi} \times \vec{E}_{\xi} \\
& = \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{E}_{\xi} + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \vec{\nabla}_{\xi} \times \vec{\nabla}_{\xi} \times \vec{E}_{\xi} \\
& = \left[\vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \cdot \vec{E}_{\xi} \right] \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) - \left[\vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \cdot \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right] \vec{E}_{\xi} \\
& + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \left[\vec{\nabla}_{\xi} (\vec{\nabla}_{\xi} \cdot \vec{E}_{\xi}) - \nabla_{\xi}^2 \vec{E}_{\xi} \right] \\
& = -\frac{1}{c} \frac{\partial}{\partial \xi^0} \left[\vec{\nabla}_{\xi} \times \left\{ \vec{B}_{\xi} \left(1 + \frac{a_0 \xi^1}{c^2}\right) \right\} \right] = -\frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \vec{E}_{\xi},
\end{aligned}$$

$$\text{In this time, } \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) = \left(\frac{a_0}{c^2}, 0, 0\right) \tag{41}$$

Hence,

$$\begin{aligned}
& \vec{\nabla}_{\xi} \times \left(1 + \frac{a_0}{c^2} \xi^1\right) \vec{\nabla}_{\xi} \times \left\{ \vec{E}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} + \frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \vec{E}_{\xi} \\
& = \left[\vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \cdot \vec{E}_{\xi} \right] \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) - \left[\vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \cdot \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right] \vec{E}_{\xi} \\
& + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \left[\vec{\nabla}_{\xi} (\vec{\nabla}_{\xi} \cdot \vec{E}_{\xi}) - \nabla_{\xi}^2 \vec{E}_{\xi} \right] + \frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \vec{E}_{\xi}
\end{aligned}$$

$$\begin{aligned}
&= \frac{a_0^2}{c^4} (E_{\xi^1}, 0, 0) - \frac{a_0^2}{c^4} (E_{\xi^1}, E_{\xi^2}, E_{\xi^3}) + (1 + \frac{a_0}{c^2} \xi^1)^2 \left[\frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 - \nabla_{\xi}^2 \right] \vec{E}_{\xi} \\
&= \frac{a_0^2}{c^4} (0, -E_{\xi^2}, -E_{\xi^3}) + (1 + \frac{a_0}{c^2} \xi^1)^2 \left[\frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 - \nabla_{\xi}^2 \right] \vec{E}_{\xi} \\
&= \frac{a_0^2}{c^4} (0, -E_{\xi^2}, -E_{\xi^3}) + (1 + \frac{a_0}{c^2} \xi^1)^2 \left[\frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 - \nabla_{\xi}^2 \right] \vec{E}_{\xi} \\
&= \vec{0}
\end{aligned} \tag{42}$$

Hence, the magnetic wave equation is in vacuum

$$\begin{aligned}
&\vec{\nabla}_{\xi} \times \left(1 + \frac{a_0}{c^2} \xi^1 \right) \vec{\nabla}_{\xi} \times \left\{ \vec{B}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1 \right) \right\} + \frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 \vec{B}_{\xi} \\
&= \left[\vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1 \right) \cdot \vec{B}_{\xi} \right] \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1 \right) - \left[\vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1 \right) \cdot \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1 \right) \right] \vec{B}_{\xi} \\
&\quad + \left(1 + \frac{a_0}{c^2} \xi^1 \right)^2 \left[\vec{\nabla}_{\xi} \left(\vec{\nabla}_{\xi} \cdot \vec{B}_{\xi} \right) - \nabla_{\xi}^2 \vec{B}_{\xi} \right] + \frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 \vec{B}_{\xi} \\
&= \frac{a_0^2}{c^4} (0, -B_{\xi^2}, -B_{\xi^3}) + (1 + \frac{a_0}{c^2} \xi^1)^2 \left[\frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 - \nabla_{\xi}^2 \right] \vec{B}_{\xi} \\
&= \frac{a_0^2}{c^4} (0, -B_{\xi^2}, -B_{\xi^3}) + (1 + \frac{a_0}{c^2} \xi^1)^2 \left[\frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 - \nabla_{\xi}^2 \right] \vec{B}_{\xi} \\
&= \vec{0}
\end{aligned} \tag{43}$$

The electromagnetic wave function, Eq(38),Eq(39) satisfy the electromagnetic wave equation, Eq(42),Eq(43).

5. Conclusion

We find the electro-magnetic field transformation and the electro-magnetic equation in uniformly accelerated frame.

Generally, the coordinate transformation of accelerated frame is

$$\begin{aligned}
\text{(I)} \quad ct &= \left(\frac{c^2}{a_0} + \xi^1 \right) \sinh \left(\frac{a_0 \xi^0}{c} \right) \\
x &= \left(\frac{c^2}{a_0} + \xi^1 \right) \cosh \left(\frac{a_0 \xi^0}{c} \right) - \frac{c^2}{a_0}, \quad y = \xi^2, \quad z = \xi^3
\end{aligned} \tag{44}$$

$$(II) \quad ct = \frac{c^2}{a_0} \exp\left(\frac{a_0}{c^2} \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$x = \frac{c^2}{a_0} \exp\left(\frac{a_0}{c^2} \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) - \frac{c^2}{a_0}, y = \xi^2, z = \xi^3 \quad (45)$$

Hence, this article say the accelerated frame is Rindler coordinate (I) that can treat electro-magnetic field equation.

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