Electro-Magnetic Field Equation and Lorentz gauge in Rindler spacetime

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ABSTRACT

In the general relativity theory, we find the electro-magnetic field transformation and the electro-magnetic field equation (Maxwell equation) in Rindler spacetime. We treat Lorentz gauge transformation in Rindler spacetime. Specially, this article say the uniqueness of the accelerated frame because the accelerated frame can treat electro-magnetic field equation.

PACS Number:**04,04.90.+e,03.30, 41.20**

Key words:General relativity theory,

Rindler spacetime,

Electro-magnetic field transformation,

Electro-magnetic field equation

Lorentz gauge transformation

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1. Introduction

In the general relativity theory, our article's aim is that we find the electro-magnetic field equation in Rindler space-time.

The Rindler coordinate is

$$ct = \left(\frac{c^2}{a_0} + \xi^1\right)\sinh\left(\frac{a_0\xi^0}{c}\right)$$

$$x = \left(\frac{c^2}{a_0} + \xi^1\right)\cosh\left(\frac{a_0\xi^0}{c}\right) - \frac{c^2}{a_0} \quad , y = \xi^2, z = \xi^3 \tag{1}$$

In this time, the tetrad $e^a{}_\mu$ is

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dx^2 + dy^2 + dz^2]$$

$$= -\frac{1}{c^2}\eta_{ab}\frac{\partial x^a}{\partial \xi^\mu}\frac{\partial x^b}{\partial \xi^\upsilon}d\xi^\mu d\xi^\upsilon$$

$$= -\frac{1}{c^2}\eta_{ab}e^a{}_\mu e^b{}_\upsilon d\xi^\mu d\xi^\upsilon = -\frac{1}{c^2}g_{\mu\upsilon}d\xi^\mu d\xi^\upsilon \quad , \quad e^a{}_\mu = \frac{\partial x^a}{\partial \xi^\mu} \tag{2}$$

$$e^\alpha{}_0(\xi^0) = \frac{\partial x^\alpha}{c\partial \xi^0} = \left(\left(1 + \frac{a_0}{c^2}\xi^1\right)\cosh\left(\frac{a_0\xi^0}{c}\right), \left(1 + \frac{a_0}{c^2}\xi^1\right)\sinh\left(\frac{a_0\xi^0}{c}\right), 0, 0\right) \tag{3}$$

About y -axis's and z -axis's orientation

$$e^\alpha{}_2(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^2} = (0,0,1,0) \quad , \quad e^\alpha{}_3(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^3} = (0,0,0,1) \tag{4}$$

The other unit vector $e^\alpha{}_1(\xi^0)$ is

$$e^\alpha{}_1(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^1} = \left(\sinh\left(\frac{a_0\xi^0}{c}\right), \cosh\left(\frac{a_0\xi^0}{c}\right), 0, 0\right) \tag{5}$$

Therefore,

$$cdt = c\cosh\left(\frac{a_0\xi^0}{c}\right)d\xi^0\left(1 + \frac{a_0}{c^2}\xi^1\right) + \sinh\left(\frac{a_0\xi^0}{c}\right)d\xi^1$$

$$dx = c\sinh\left(\frac{a_0\xi^0}{c}\right)d\xi^0\left(1 + \frac{a_0}{c^2}\xi^1\right) + \cosh\left(\frac{a_0\xi^0}{c}\right)d\xi^1, dy = d\xi^2, \ dz = d\xi^3 \tag{6}$$

The vector transformation is

$$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} V^{\alpha}, \qquad U'_{\mu} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} U_{\alpha} \tag{7}$$

Therefore, the transformation of the electro-magnetic 4-vector potential $(\phi, \vec{A}) = A^{\alpha}$ is

$$A^{\alpha} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} A'^{\mu} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} A_{\xi}^{\ \mu} = e^{\alpha}{}_{\mu} A_{\xi}^{\ \mu} \ , \ e^{\alpha}{}_{\mu} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}}$$

$$dx^{\alpha} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} dx'^{\mu} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} d\xi^{\mu} = e^{\alpha}{}_{\mu} d\xi^{\mu} \ , e^{\alpha}{}_{\mu} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} \tag{8}$$

$$(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2)\phi = 4\pi\rho$$

$$(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2)\vec{A} = \frac{4\pi}{c}\vec{j}$$

$$\text{4-vector } (c\rho, \vec{j}) = \rho_0 \frac{dx^{\alpha}}{d\tau} \tag{9}$$

Lorentz gauge transformation is in Rindler spacetime,

$$A^{\mu} \to A^{\mu} + \partial^{\mu}\Lambda = A^{\mu} + g^{\mu\upsilon}\partial_{\upsilon}\Lambda \quad , \ \Lambda \text{ is a scalar function.}$$

$$g^{00} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}, g^{11} = g^{22} = g^{33} = 1$$

Hence,

$$\phi_{\xi} \to \phi_{\xi} - \frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}, \ \vec{A}_{\xi} \to \vec{A}_{\xi} + \vec{\nabla}_{\xi}\Lambda, \ \Lambda \text{ is a scalar function.} \tag{10}$$

Lorentz gauge fixing condition is in Rindler spacetime,

$$A^{\mu}{}_{;\mu} = \frac{\partial A^{\mu}}{\partial \xi^{\mu}} + \Gamma^{\mu}{}_{\mu\rho}A^{\rho},$$

$$\Gamma^{\mu}{}_{\mu\rho} = \Gamma^{0}{}_{01} = \frac{1}{2}g^{00}(\frac{\partial g_{00}}{\partial \xi^1}) = \frac{a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}$$

$$g^{00} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}, g^{11} = g^{22} = g^{33} = 1 \tag{11}$$

$$0=\frac{\partial\phi}{c\partial t}+\vec{\nabla}\cdot\vec{A}=\frac{\partial\phi_\xi}{c\partial\xi^0}+\vec{\nabla}_\xi\cdot\vec{A}_\xi+\frac{A_{\xi^1}a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}$$

$$A^\mu\to A^\mu+\partial^\mu\Lambda=A^\mu+g^{\mu\upsilon}\partial_\upsilon\Lambda$$

$$A^\mu{}_{;\mu}=\frac{\partial A^\mu}{\partial\xi^\mu}+\Gamma^\mu{}_{\mu\rho}A^\rho\to\partial_\mu(A^\mu+g^{\mu\upsilon}\partial_\upsilon\Lambda)+\Gamma^0{}_{01}(A^1+\frac{\partial\Lambda}{\partial\xi^1})$$

$$=\partial_\mu A^\mu+(g^{\mu\upsilon}\partial_\mu\partial_\upsilon)\Lambda+\Gamma^0{}_{01}(A^1+\frac{\partial\Lambda}{\partial\xi^1})$$

$$0=\frac{\partial\phi_\xi}{c\partial\xi^0}+\vec{\nabla}_\xi\cdot\vec{A}_\xi+\frac{A_{\xi^1}a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}$$

$$\to\frac{1}{c}\frac{\partial\phi_\xi}{\partial\xi^0}+\vec{\nabla}_\xi\cdot\vec{A}_\xi-[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]\Lambda$$

$$+\frac{A_{\xi^1}a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}+\frac{\partial\Lambda}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]\Lambda-\frac{\partial\Lambda}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0$$

(12)

Hence, the transformation of the electro-magnetic 4-vector potential $(\phi,\vec{A})$ in inertial frame and the eletro-magnetic 4-vector potential $(\phi_\xi,\vec{A}_\xi)$ in uniformly accelerated frame is

$$\phi=\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi+\sinh(\frac{a_0\xi^0}{c})A_{\xi^1}$$

$$A_x=\sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi+\cosh(\frac{a_0\xi^0}{c})A_{\xi^1}$$

$$A_y=A_{\xi^2},A_z=A_{\xi^3}\qquad(13)$$

$$g=\begin{pmatrix} -(1+\frac{a_0\xi^1}{c^2})^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \eta=\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$e^a{}_\mu e_b{}^\mu = \delta^a{}_b \ , \ e^a{}_\mu e_a{}^\upsilon = \delta_\mu{}^\upsilon$$

$$e^a{}_\mu e^b{}_\upsilon \eta_{ab} = g_{\mu\upsilon} \ \rightarrow \ A^T \eta A = g$$

$$e_a{}^\mu e_b{}^\upsilon g_{\mu\upsilon} = \eta_{ab} \rightarrow \ (A^T)^{-1} g A^{-1} = (A^T)^{-1} A^T \eta A A^{-1} = \eta$$

$$e^a{}_\mu = \eta^{ab} g_{\mu\upsilon} e_b{}^\upsilon \rightarrow \eta^{-1}(A^T)^{-1} A^T \eta A = A = \eta^{-1}(A^T)^{-1} g \tag{14}$$

$$\begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2}) & \sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ \sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ d\xi^3 \end{pmatrix}$$

$$= A \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ d\xi^3 \end{pmatrix}$$

$$= \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c}) & \sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ \sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cd\hat{\xi}^0 \\ d\hat{\xi}^1 \\ d\hat{\xi}^2 \\ d\hat{\xi}^3 \end{pmatrix}$$

(15)

$$e_{\mu}{}^{\alpha} = \frac{\partial \xi^{\alpha}}{\partial x^{\mu}} = A^{-1} = \begin{pmatrix} \frac{c\partial \xi^0}{c\partial t} & \frac{c\partial \xi^0}{\partial x} & \frac{c\partial \xi^0}{\partial y} & \frac{c\partial \xi^0}{\partial z} \\ \frac{\partial \xi^1}{c\partial t} & \frac{\partial \xi^1}{\partial x} & \frac{\partial \xi^1}{\partial y} & \frac{\partial \xi^1}{\partial z} \\ \frac{\partial \xi^2}{c\partial t} & \frac{\partial \xi^2}{\partial x} & \frac{\partial \xi^2}{\partial y} & \frac{\partial \xi^2}{\partial z} \\ \frac{\partial \xi^3}{c\partial t} & \frac{\partial \xi^3}{\partial x} & \frac{\partial \xi^3}{\partial y} & \frac{\partial \xi^3}{\partial z} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0 \xi^1}{c^2})} & -\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0 \xi^1}{c^2})} & 0 & 0 \\ -\sinh(\frac{a_0 \xi^0}{c}) & \cosh(\frac{a_0 \xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{16}$$

$$\begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = (A^{-1})^T \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix} = (A^T)^{-1} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0 \xi^1}{c^2})} & -\sinh(\frac{a_0 \xi^0}{c}) & 0 & 0 \\ -\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0 \xi^1}{c^2})} & \cosh(\frac{a_0 \xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix}$$

$$= \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c}) & -\sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ -\sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial\hat{\xi}^0} \\ \frac{\partial}{\partial\hat{\xi}^1} \\ \frac{\partial}{\partial\hat{\xi}^2} \\ \frac{\partial}{\partial\hat{\xi}^3} \end{pmatrix} \tag{17}$$

$$\frac{1}{c}\frac{\partial}{\partial t} = \frac{c\partial\xi^0}{c\partial t}\frac{1}{c}\frac{\partial}{\partial\xi^0} + \frac{\partial\xi^1}{c\partial t}\frac{\partial}{\partial\xi^1}$$

$$= \frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}$$

$$\frac{\partial}{\partial x} = \frac{c\partial\xi^0}{\partial x}\frac{1}{c}\frac{\partial}{\partial\xi^0} + \frac{\partial\xi^1}{\partial x}\frac{\partial}{\partial\xi^1}$$

$$= -\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial z} = \frac{\partial}{\partial\xi^3}$$

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{1}{c^2(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2$$

$$\vec{\nabla} = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}), \quad \vec{\nabla}_\xi = (\frac{\partial}{\partial\xi^1}, \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial\xi^3}) \tag{18}$$

2. Electro-magnetic Field in the Rindler space-time

The electro-magnetic field $(\vec{E},\vec{B})$ is in the inertial frame,

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial\vec{A}}{c\partial t}, \vec{B} = \vec{\nabla}\times\vec{A} \tag{19}$$

$$E_x = -\frac{\partial\phi}{\partial x} - \frac{\partial A_x}{c\partial t}$$

$$=-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2})\phi_\xi+\sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[\sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2})\phi_\xi+\cosh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$=-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial A_{\xi^1}}{c\partial\xi^0}-(1+\frac{a_0\xi^1}{c^2})\frac{\partial\phi_\xi}{\partial\xi^1}-2\phi_\xi\frac{a_0}{c^2}$$

$$=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\xi^1}[(1+\frac{a_0}{c^2}\xi^1)^2\phi_\xi]-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^1}}{c\partial\xi^0} \qquad (20)$$

$$E_y=-\frac{\partial\phi}{\partial y}-\frac{\partial A_y}{c\partial t}=-\frac{\partial}{\partial\xi^2}[\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi+\sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^2}$$

$$=-(1+\frac{a_0\xi^1}{c^2})\cosh(\frac{a_0\xi^0}{c})\frac{\partial\phi_\xi}{\partial\xi^2}-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\cosh(\frac{a_0\xi^0}{c})\frac{\partial A_{\xi^2}}{c\partial\xi^0}$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^2}}{\partial\xi^1}-\frac{\partial A_{\xi^1}}{\partial\xi^2}]$$

$$=\cosh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2]-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^2}}{c\partial\xi^0}]$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^2}}{\partial\xi^1}-\frac{\partial A_{\xi^1}}{\partial\xi^2}] \qquad (21)$$

$$E_z=-\frac{\partial\phi}{\partial z}-\frac{\partial A_z}{c\partial t}=-\frac{\partial}{\partial\xi^3}[\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi+\sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^3}$$

$$=-(1+\frac{a_0\xi^1}{c^2})\cosh(\frac{a_0\xi^0}{c})\frac{\partial\phi_\xi}{\partial\xi^3}-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\cosh(\frac{a_0\xi^0}{c})\frac{\partial A_{\xi^3}}{c\partial\xi^0}$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^3}}{\partial\xi^1}-\frac{\partial A_{\xi^1}}{\partial\xi^3}]$$

$$=\cosh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2]-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^3}}{c\partial\xi^0}]$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^3}}{\partial\xi^1}-\frac{\partial A_{\xi^1}}{\partial\xi^3}]$$

(22)

$$B_x=\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}=\frac{\partial A_{\xi^3}}{\partial\xi^2}-\frac{\partial A_{\xi^2}}{\partial\xi^3} \quad (23)$$

$$B_y=\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}=\frac{\partial A_x}{\partial\xi^3}-\frac{\partial A_{\xi^3}}{\partial x}$$

$$=\frac{\partial}{\partial\xi^3}[\sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi+\cosh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^3}$$

$$=\cosh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^1}}{\partial\xi^3}-\frac{\partial A_{\xi^3}}{\partial\xi^1}]$$

$$-\sinh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2]-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^3}}{c\partial\xi^0}] \quad (24)$$

$$B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = \frac{\partial A_{\xi^2}}{\partial x} - \frac{\partial A_x}{\partial \xi^2}$$

$$= [-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^3}$$

$$-\frac{\partial}{\partial\xi^2}[\sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \cosh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^2}}{\partial\xi^1} - \frac{\partial A_{\xi^1}}{\partial\xi^2}]$$

$$+\sinh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^2}}{c\partial\xi^0}] \quad (25)$$

Hence, we can define the electro-magnetic field $(\vec{E}_\xi, \vec{B}_\xi)$ in Rindler spacetime.

$$\vec{E}_\xi = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{A}_\xi}{c\partial\xi^0}$$

$$\vec{B}_\xi = \vec{\nabla}_\xi \times \vec{A}_\xi$$

In this time, $\vec{\nabla}_\xi = (\frac{\partial}{\partial\xi^1}, \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial\xi^3}), \vec{A}_\xi = (A_{\xi^1}, A_{\xi^2}, A_{\xi^3})$ (26)

Lorentz gauge transformation is in Rindler spacetime,

$\phi_\xi \to \phi_\xi - \frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}$, $\vec{A}_\xi \to \vec{A}_\xi + \vec{\nabla}_\xi\Lambda$, Λ is a scalar function. (27)

$$\vec{E}_\xi = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} + \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\frac{\partial\Lambda}{c\partial\xi^0}$$

$$-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{A}_\xi}{c\partial\xi^0} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}\vec{\nabla}_\xi\Lambda$$

$$= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{A}_\xi}{c\partial\xi^0}$$

$$\vec{B}_\xi = \vec{\nabla}_\xi \times \vec{A}_\xi + \vec{\nabla}_\xi \times \vec{\nabla}_\xi\Lambda = \vec{\nabla}_\xi \times \vec{A}_\xi \quad (28)$$

Lorentz gauge fixing condition is in Rindler spacetime,

$$0 = \frac{\partial\phi_\xi}{c\partial\xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi + \frac{A_{\xi^1}a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}$$

$$\rightarrow \frac{1}{c}\frac{\partial\phi_\xi}{\partial\xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi - [\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2]\Lambda$$

$$+ \frac{A_{\xi^1}a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)} + \frac{\partial\Lambda}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})} = 0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2]\Lambda - \frac{\partial\Lambda}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})} = 0$$

$$\phi - \frac{1}{c}\frac{\partial\Lambda}{\partial t} = \cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\{\phi_\xi - \frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\}$$

$$+\sinh(\frac{a_0\xi^0}{c})(A_{\xi^1} + \frac{\partial\Lambda}{\partial\xi^1})$$

$$A_x + \frac{\partial\Lambda}{\partial x} = \sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\{\phi_\xi - \frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\}$$

$$+\cosh(\frac{a_0\xi^0}{c})(A_{\xi^1} + \frac{\partial\Lambda}{\partial\xi^1})$$

$$A_y + \frac{\partial\Lambda}{\partial y} = A_{\xi^2} + \frac{\partial\Lambda}{\partial\xi^2}, A_z + \frac{\partial\Lambda}{\partial z} = A_{\xi^3} + \frac{\partial\Lambda}{\partial\xi^3} \quad (29)$$

We obtain the transformation of the electro-magnetic field.

$$E_x = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\xi^1}\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^1}}{c\partial\xi^0} = E_{\xi^1} \quad ,$$

$$E_y = E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c}),$$

$$E_z = E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

$$B_x = B_{\xi^1},$$

$$B_y = B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$$

$$B_z = B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c}) \qquad (30)$$

Hence,

$$E_x = E_{\xi^1}, B_x = B_{\xi^1},$$

$$\begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix} = H \begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix}$$

$$H = \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 & \sinh(\frac{a_0\xi^0}{c}) \\ 0 & \cosh(\frac{a_0\xi^0}{c}) & -\sinh(\frac{a_0\xi^0}{c}) & 0 \\ 0 & -\sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 \\ \sinh(\frac{a_0\xi^0}{c}) & 0 & 0 & \cosh(\frac{a_0\xi^0}{c}) \end{pmatrix} \qquad (31)$$

The inverse-transformation of the electro-magnetic field is

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$\begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix} = H^{-1} \begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix}$$

$$H^{-1} = \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 & -\sinh(\frac{a_0\xi^0}{c}) \\ 0 & \cosh(\frac{a_0\xi^0}{c}) & \sinh(\frac{a_0\xi^0}{c}) & 0 \\ 0 & \sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 \\ -\sinh(\frac{a_0\xi^0}{c}) & 0 & 0 & \cosh(\frac{a_0\xi^0}{c}) \end{pmatrix} \quad (32)$$

$$E_{\xi^1} = E_x , B_{\xi^1} = B_x$$

$$E_{\xi^2} = E_y \cosh(\frac{a_0\xi^0}{c}) - B_z \sinh(\frac{a_0\xi^0}{c}),$$

$$B_{\xi^2} = B_y \cosh(\frac{a_0\xi^0}{c}) + E_z \sinh(\frac{a_0\xi^0}{c})$$

$$E_{\xi^3} = E_z \cosh(\frac{a_0\xi^0}{c}) + B_y \sinh(\frac{a_0\xi^0}{c})$$

$$B_{\xi^3} = B_z \cosh(\frac{a_0\xi^0}{c}) - E_y \sinh(\frac{a_0\xi^0}{c}) \quad (33)$$

3. Electro-magnetic Field Equation(Maxwell Equation) in the Rindler space-time

Maxwell equation is

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \quad (34\text{-i})$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{c\partial t} + \frac{4\pi}{c}\vec{j} \quad (34\text{-ii})$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (34\text{-iii})$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{c\partial t} \quad (34\text{-iv})$$

1. $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$

$E_x = E_{\xi^1}$,

$$E_y = E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c}),$$

$$E_z = E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

$$4\pi\rho = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$= [-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]E_{\xi^1}$$

$$+\frac{\partial}{\partial\xi^2}[E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$+\frac{\partial}{\partial\xi^3}[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$= \cosh(\frac{a_0}{c}\xi^0)(\vec{\nabla}_\xi\cdot\vec{E}_\xi) + \sinh(\frac{a_0}{c}\xi^0)[\frac{\partial B_{\xi^3}}{\partial\xi^2} - \frac{\partial B_{\xi^2}}{\partial\xi^3} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial E_{\xi^1}}{c\partial\xi^0}] \qquad (35)$$

2. $\vec{\nabla}\times\vec{B} = \frac{\partial\vec{E}}{c\partial t} + \frac{4\pi}{c}\vec{j}$

$$B_x = B_{\xi^1}$$

$$B_y = B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$$

$$B_z = B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

X-component) $\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}$

$$= \frac{\partial}{\partial\xi^2}[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$-\frac{\partial}{\partial\xi^3}[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3}] + \sinh(\frac{a_0\xi^0}{c})[\frac{\partial E_{\xi^2}}{\partial \xi^2} + \frac{\partial E_{\xi^3}}{\partial \xi^3}]$$

$$= \frac{\partial E_x}{c\partial t} + \frac{4\pi}{c} j_x$$

$$= [\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} \frac{\partial}{c\partial \xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}]E_{\xi^1} + \frac{4\pi}{c} j_x$$

Hence,

$$\frac{4\pi}{c} j_x$$

$$= \sinh(\frac{a_0\xi^0}{c})(\vec{\nabla}_\xi \cdot \vec{E}_\xi) + \cosh(\frac{a_0\xi^0}{c})[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial E_{\xi^1}}{c\partial \xi^0}] \qquad (36)$$

Y-component) $\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}$

$$= \frac{\partial B_{\xi^1}}{\partial \xi^3}$$

$$-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}] \cdot [B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$= \frac{\partial E_y}{c\partial t} + \frac{4\pi}{c} j_y$$

$$= [\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}] \cdot [E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$+ \frac{4\pi}{c} j_y$$

$$\frac{4\pi}{c} j_y = \frac{\partial B_{\xi^1}}{\partial \xi^3} - \frac{\partial B_{\xi^3}}{\partial \xi^1} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}B_{\xi^3} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^2}}{c\partial \xi^0}$$

$$=\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}\{B_{\xi^1}(1+\frac{a_0}{c^2}\xi^1)\}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{B_{\xi^3}(1+\frac{a_0\xi^1}{c^2})\}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^2}}{c\partial\xi^0}$$

(37)

Z-component) $\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}$

$$=[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c})-E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$-\frac{\partial B_{\xi^1}}{\partial\xi^2}$$

$$=\frac{\partial E_z}{c\partial t}+\frac{4\pi}{c}j_z$$

$$=[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$+\frac{4\pi}{c}j_z$$

$$\frac{4\pi}{c}j_z=\frac{\partial B_{\xi^2}}{\partial\xi^1}-\frac{\partial B_{\xi^1}}{\partial\xi^2}+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}B_{\xi^2}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^3}}{c\partial\xi^0}$$

$$=\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{B_{\xi^2}(1+\frac{a_0}{c^2}\xi^1)\}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}\{B_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^3}}{c\partial\xi^0}$$

(38)

3. $\vec{\nabla}\cdot\vec{B}=0$

$$\vec{\nabla}\cdot\vec{B}=\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}$$

$$=[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]B_{\xi^1}$$

$$+\frac{\partial}{\partial\xi^2}[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c})-E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$+\frac{\partial}{\partial\xi^3}[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c})+E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$=\cosh(\frac{a_0\xi^0}{c})(\vec{\nabla}_\xi\cdot\vec{B}_\xi)+\sinh(\frac{a_0\xi^0}{c})[-(-\frac{\partial E_{\xi^2}}{\partial\xi^3}+\frac{\partial E_{\xi^3}}{\partial\xi^2})-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^1}}{c\partial\xi^0}]=0 \tag{39}$$

4. $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{c\partial t}$

$E_x=E_{\xi^1}$,

$E_y=E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$,

$E_z=E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$

X-component) $\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}$

$$=\frac{\partial}{\partial\xi^2}[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$-\frac{\partial}{\partial\xi^3}[E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$=\cosh(\frac{a_0}{c}\xi^0)[\frac{\partial E_{\xi^3}}{\partial\xi^2}-\frac{\partial E_{\xi^2}}{\partial\xi^3}]-\sinh(\frac{a_0\xi^0}{c})[\frac{\partial B_{\xi^2}}{\partial\xi^2}+\frac{\partial B_{\xi^3}}{\partial\xi^3}]$$

$$=-\frac{\partial B_x}{c\partial t}$$

$$= -[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]B_{\xi^1}$$

Hence,

$$-\sinh(\frac{a_0\xi^0}{c})(\vec{\nabla}_\xi \cdot \vec{B}_\xi) + \cosh(\frac{a_0\xi^0}{c})\,[(\frac{\partial E_{\xi^3}}{\partial\xi^2} - \frac{\partial E_{\xi^2}}{\partial\xi^3}) + \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial B_{\xi^1}}{c\partial\xi^0}] = 0 \qquad (40)$$

Y-component) $\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}$

$$= \frac{\partial E_{\xi^1}}{\partial\xi^3}$$

$$-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$= -\frac{\partial B_y}{c\partial t}$$

$$= -[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$\frac{\partial E_{\xi^1}}{\partial\xi^3} - \frac{\partial E_{\xi^3}}{\partial\xi^1} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}E_{\xi^3} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^2}}{c\partial\xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}\{E_{\xi^1}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{E_{\xi^3}(1+\frac{a_0\xi^1}{c^2})\} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^2}}{c\partial\xi^0}$$

$$= 0$$

(41)

Z-component) $\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}$

$$= [-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$-\frac{\partial E_{\xi^1}}{\partial\xi^2}$$

$$= -\frac{\partial B_z}{c\partial t}$$

$$= -[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$\frac{\partial E_{\xi^2}}{\partial\xi^1} - \frac{\partial E_{\xi^1}}{\partial\xi^2} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}E_{\xi^2} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^3}}{c\partial\xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{E_{\xi^2}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}\{E_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^3}}{c\partial\xi^0}$$

$$= 0 \tag{42}$$

Therefore, we obtain the electro-magnetic field equation by Eq (35)-Eq(42) in Rindler spacetime .

$$\vec{\nabla}_\xi\cdot\vec{E}_\xi = 4\pi\rho_\xi(1+\frac{a_0\xi^1}{c^2}) \tag{43-i}$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\times\{\vec{B}_\xi(1+\frac{a_0\xi^1}{c^2})\} = \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{E}_\xi}{c\partial\xi^0} + \frac{4\pi}{c}\vec{j}_\xi \tag{43-ii}$$

$$\vec{\nabla}_\xi\cdot\vec{B}_\xi = 0 \tag{43-iii}$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\times\{\vec{E}_\xi(1+\frac{a_0\xi^1}{c^2})\} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{B}_\xi}{c\partial\xi^0} \tag{43-iv}$$

$\vec{E}_\xi = (E_{\xi^1}, E_{\xi^2}, E_{\xi^3}), \vec{B}_\xi = (B_{\xi^1}, B_{\xi^2}, B_{\xi^3})$,

$$\vec{\nabla}_{\xi} = (\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3})$$

Hence, the transformation of 4-vector $(c\rho, \vec{j}) = \rho_0 \dfrac{dx^{\alpha}}{d\tau}$ is

$$\rho = \rho_{\xi}(1 + \frac{a_0 \xi^1}{c^2})\cosh(\frac{a_0 \xi^0}{c}) + \frac{j_{\xi^1}}{c}\sinh(\frac{a_0 \xi^0}{c})$$

$$j_x = j_{\xi^1}\cosh(\frac{a_0 \xi^0}{c}) + c\rho_{\xi}(1 + \frac{a_0}{c^2}\xi^1)\sinh(\frac{a_0 \xi^0}{c}), \quad j_y = j_{\xi^2}, j_z = j_{\xi^3}$$

In this time, 4-vector $(c\rho_{\xi}, \vec{j}_{\xi}) = \rho_0 \dfrac{d\xi^{\alpha}}{d\tau}$ (44)

Generally, the continuity equation is in Rindler spacetime,

$$0 = j^{\mu}{}_{;\mu} = \frac{\partial j^{\mu}}{\partial \xi^{\mu}} + \Gamma^{\mu}{}_{\mu\rho} j^{\rho},$$

$$\Gamma^{\mu}{}_{\mu\rho} = \Gamma^{0}{}_{01} = \frac{1}{2} g^{00}(\frac{\partial g_{00}}{\partial \xi^1}) = \frac{a_0}{c^2}\frac{1}{(1 + \frac{a_0}{c^2}\xi^1)}$$

$$g^{00} = -\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})^2}, g^{11} = g^{22} = g^{33} = 1$$

$$0 = \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = \frac{\partial \rho_{\xi}}{\partial \xi^0} + \vec{\nabla}_{\xi} \cdot \vec{j}_{\xi} + \frac{j_{\xi^1} a_0}{c^2}\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \tag{45}$$

We treat Lorentz gauge transformation about the electro-magnetic field equation in Rindler spacetime. Eq(43-i) is

$$\vec{\nabla}_{\xi} \cdot \vec{E}_{\xi} = \vec{\nabla}_{\xi} \cdot \{-\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})}\vec{\nabla}_{\xi}\{\phi_{\xi}(1 + \frac{a_0 \xi^1}{c^2})^2\} - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})}\frac{\partial \vec{A}_{\xi}}{c\partial \xi^0}\}$$

$$= -\vec{\nabla}_{\xi}\{\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})}\} \cdot [\vec{\nabla}_{\xi}\{\phi_{\xi}(1 + \frac{a_0 \xi^1}{c^2})^2\} + \frac{\partial \vec{A}_{\xi}}{c\partial \xi^0}]$$

$$-\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})}[\nabla_{\xi}{}^2\{\phi_{\xi}(1 + \frac{a_0 \xi^1}{c^2})^2\} + \frac{\partial}{c\partial \xi^0}(\vec{\nabla}_{\xi} \cdot \vec{A}_{\xi})]$$

$$= \frac{a_0}{c^2} \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2} [\frac{\partial}{\partial\xi^1} \{\phi_\xi (1+\frac{a_0\xi^1}{c^2})^2\} + \frac{\partial A_{\xi^1}}{c\partial\xi^0}]$$

$$- \frac{1}{(1+\frac{a_0\xi^1}{c^2})} [\nabla_\xi{}^2 - \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} (\frac{\partial}{\partial\xi^0})^2] \{\phi_\xi (1+\frac{a_0\xi^1}{c^2})^2\}$$

$$- \frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{\partial}{c\partial\xi^0} [- \frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{A_{\xi^1} a_0}{c^2}] \qquad ,$$

$$\frac{1}{c}\frac{\partial\phi_\xi}{\partial\xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi = - \frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{a_0}{c^2} A_{\xi^1}$$

$$= - \frac{a_0}{c^2} \frac{1}{(1+\frac{a_0\xi^1}{c^2})} E_{\xi^1} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})} [\nabla_\xi{}^2 - \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} (\frac{\partial}{\partial\xi^0})^2] \{\phi_\xi (1+\frac{a_0\xi^1}{c^2})^2\}$$

$$+ \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2} \frac{\partial A_{\xi^1}}{c\partial\xi^0} \frac{a_0}{c^2}$$

$$= 4\pi\rho_\xi (1+\frac{a_0\xi^1}{c^2}) \qquad (46)$$

If we apply Lorentz gauge transformation to Eq (46),

$\phi_\xi \to \phi_\xi - \frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0} \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}$, $\vec{A}_\xi \to \vec{A}_\xi + \vec{\nabla}_\xi \Lambda$, Λ is a scalar function.

$$= - \frac{a_0}{c^2} \frac{1}{(1+\frac{a_0\xi^1}{c^2})} E_{\xi^1} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})} [\nabla_\xi{}^2 - \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} (\frac{\partial}{\partial\xi^0})^2] \{\phi_\xi (1+\frac{a_0\xi^1}{c^2})^2\}$$

$$+ \frac{1}{(1+\frac{a_0\xi^1}{c^2})} [\nabla_\xi{}^2 - \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} (\frac{\partial}{\partial\xi^0})^2] \frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}$$

$$+ \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2} \frac{\partial A_{\xi^1}}{c\partial\xi^0} \frac{a_0}{c^2} + \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2} \frac{a_0}{c^2} \frac{\partial}{c\partial\xi^0} \frac{\partial\Lambda}{\partial\xi^1}$$

$$= -\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}E_{\xi^1} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2 - \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}\{[\nabla_\xi{}^2 - \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\Lambda + \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\frac{\partial\Lambda}{\partial\xi^1}\}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{\partial A_{\xi^1}}{c\partial\xi^0}\frac{a_0}{c^2} \tag{47}$$

In this time,

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2]\Lambda - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\frac{\partial\Lambda}{\partial\xi^1} = 0 \tag{48}$$

Hence, Eq(43-i) is

$$\vec{\nabla}_\xi \cdot \vec{E}_\xi$$

$$= -\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}E_{\xi^1} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2 - \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{\partial A_{\xi^1}}{c\partial\xi^0}\frac{a_0}{c^2}$$

$$= 4\pi\rho_\xi(1+\frac{a_0\xi^1}{c^2}) \tag{49}$$

Eq(43-i) is invariant about Lorentz gauge transformation in Rindler spacetime.

Eq (43-ii) is

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi \times \{\vec{B}_\xi(1+\frac{a_0\xi^1}{c^2})\}$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi \times \{\vec{\nabla}_\xi \times \vec{A}_\xi(1+\frac{a_0\xi^1}{c^2})\}$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1) \times \{\vec{\nabla}_\xi \times \vec{A}_\xi\} + \vec{\nabla}_\xi \times \vec{\nabla}_\xi \times \vec{A}_\xi$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}(1,0,0)\times\vec{B}_\xi + \{-\nabla_\xi{}^2\vec{A}_\xi + \vec{\nabla}_\xi(\vec{\nabla}_\xi \cdot \vec{A}_\xi)\}$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}(0,-B_{\xi^3}, B_{\xi^2}) + \{-\nabla_\xi{}^2\vec{A}_\xi + \vec{\nabla}_\xi(\vec{\nabla}_\xi \cdot \vec{A}_\xi)\}$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{E}_\xi}{c\partial\xi^0} + \frac{4\pi}{c}\vec{j}_\xi$$

$$= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{\partial}{c\partial\xi^0}[\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{A}_\xi + \frac{4\pi\vec{j}_\xi}{c}$$

$$= -\frac{\partial}{c\partial\xi^0}\vec{\nabla}_\xi\phi_\xi - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0) - \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{A}_\xi + \frac{4\pi\vec{j}_\xi}{c}$$

(50)

Therefore,

$$\frac{4\pi}{c}\vec{j}_\xi$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}(0,-B_{\xi^3}, B_{\xi^2}) + \{-\nabla_\xi{}^2\vec{A}_\xi + \vec{\nabla}_\xi(\vec{\nabla}_\xi \cdot \vec{A}_\xi)\}$$

$$+ \frac{\partial}{c\partial\xi^0}\vec{\nabla}_\xi\phi_\xi + \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0) + \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{A}_\xi$$

$$= \frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3}, B_{\xi^2}) + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)$$

$$+ [-\nabla_\xi{}^2 + \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_\xi + \vec{\nabla}_\xi[-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}A_{\xi^1}]$$

$$\frac{1}{c}\frac{\partial\phi_\xi}{\partial\xi^0}+\vec{\nabla}_\xi\cdot\vec{A}_\xi=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}A_{\xi^1} \tag{51}$$

$$\frac{4\pi}{c}\vec{j}_\xi$$
$$=\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2})+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)$$
$$+[-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_\xi$$
$$+\vec{\nabla}_\xi[-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}A_{\xi^1}] \tag{52}$$

If we apply Lorentz gauge transformation to Eq (52),

$\phi_\xi\rightarrow\phi_\xi-\frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}$, $\vec{A}_\xi\rightarrow\vec{A}_\xi+\vec{\nabla}_\xi\Lambda$, Λ is a scalar function.

$$\frac{4\pi}{c}\vec{j}_\xi$$
$$=\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2})+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)$$
$$-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^3}\frac{2a_0}{c^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\Lambda(1,0,0)$$
$$+[-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_\xi+[-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{\nabla}_\xi\Lambda$$
$$+\vec{\nabla}_\xi[-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}A_{\xi^1}]+\vec{\nabla}_\xi[-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\frac{\partial\Lambda}{\partial\xi^1}]$$
$$=\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2})+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)$$

$$-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^3}\frac{2a_0}{c^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\Lambda(1,0,0)$$

$$+[-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_\xi+[-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{\nabla}_\xi\Lambda$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{a_0{}^2}{c^4}A_{\xi^1}(1,0,0)-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\vec{\nabla}_\xi A_{\xi^1}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{a_0{}^2}{c^4}\frac{\partial\Lambda}{\partial\xi^1}(1,0,0)-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\frac{\partial}{\partial\xi^1}\vec{\nabla}_\xi\Lambda$$

(53)

In this time,

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]\Lambda-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\frac{\partial\Lambda}{\partial\xi^1}=0$$

$$0=\vec{\nabla}_\xi[\{-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\frac{\partial}{\partial\xi^1}\}\Lambda]$$

$$=\vec{\nabla}_\xi\{\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\}(\frac{\partial}{\partial\xi^0})^2\Lambda+[-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{\nabla}_\xi\Lambda$$

$$-\vec{\nabla}_\xi\{\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\}\frac{\partial\Lambda}{\partial\xi^1}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}\frac{\partial}{\partial\xi^1}\vec{\nabla}_\xi\Lambda$$

$$=-\frac{2}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^3}\frac{a_0}{c^2}(\frac{\partial}{\partial\xi^0})^2\Lambda(1,0,0)+[-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{\nabla}_\xi\Lambda$$

$$+\frac{a_0{}^2}{c^4}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\frac{\partial\Lambda}{\partial\xi^1}(1,0,0)-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}\frac{\partial}{\partial\xi^1}\vec{\nabla}_\xi\Lambda \tag{54}$$

Therefore,

$$
\begin{aligned}
&\frac{4\pi}{c}\vec{j}_{\xi} \\
&= \frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2}) + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_{\xi}}{c\partial\xi^0}(1,0,0) \\
&\quad - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^3}\frac{2a_0}{c^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\Lambda(1,0,0) \\
&+[-\nabla_{\xi}{}^2 + \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_{\xi} + \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{a_0{}^2}{c^4}A_{\xi^1}(1,0,0) \\
&- \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\vec{\nabla}_{\xi}A_{\xi^1} + \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{a_0{}^2}{c^4}\frac{\partial\Lambda}{\partial\xi^1}(1,0,0) - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\frac{\partial}{\partial\xi^1}\vec{\nabla}_{\xi}\Lambda \\
&+\vec{\nabla}_{\xi}[\{-\nabla_{\xi}{}^2 + \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\frac{\partial}{\partial\xi^1}\}\Lambda] \\
&+\frac{2}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^3}\frac{a_0}{c^2}(\frac{\partial}{\partial\xi^0})^2\Lambda(1,0,0) - \frac{a_0{}^2}{c^4}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\frac{\partial\Lambda}{\partial\xi^1}(1,0,0) \\
&+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}\frac{\partial}{\partial\xi^1}\vec{\nabla}_{\xi}\Lambda \\
&= \frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2}) + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_{\xi}}{c\partial\xi^0}(1,0,0) \\
&\quad +[-\nabla_{\xi}{}^2 + \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_{\xi} \\
&+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{a_0{}^2}{c^4}A_{\xi^1}(1,0,0) - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\vec{\nabla}_{\xi}A_{\xi^1} \qquad (55)
\end{aligned}
$$

Hence, Eq(43-ii) is invariant about Lorentz gauge transformation in Rindler spacetime.

Eq (43-iii) is

$$\vec{\nabla}_{\xi} \cdot \vec{B}_{\xi} = \vec{\nabla}_{\xi} \cdot (\vec{\nabla}_{\xi} \times \vec{A}_{\xi} + \vec{\nabla}_{\xi} \times \vec{\nabla}_{\xi}\Lambda) = \vec{\nabla}_{\xi} \times \vec{\nabla}_{\xi} \cdot \vec{A}_{\xi} = 0 \qquad (56)$$

Eq (43-iv) is

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_{\xi} \times \{\vec{E}_{\xi}(1+\frac{a_0\xi^1}{c^2})\}$$

$$= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_{\xi} \times [\vec{\nabla}_{\xi}\{\phi_{\xi}(1+\frac{a_0\xi^1}{c^2})^2\} - \vec{\nabla}_{\xi}(\frac{\partial\Lambda}{c\partial\xi^0}) + \frac{\partial\vec{A}_{\xi}}{c\partial\xi^0} + \frac{\partial}{c\partial\xi^0}(\vec{\nabla}_{\xi}\Lambda)]$$

$$= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_{\xi} \times \frac{\partial\vec{A}_{\xi}}{c\partial\xi^0} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial(\vec{\nabla}_{\xi} \times \vec{A}_{\xi})}{c\partial\xi^0} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{B}_{\xi}}{c\partial\xi^0} \qquad (57)$$

Hence, Eq (43-iii), Eq (43-iv) are invariant about Lorentz gauge transformation in Rindler spacetime. Hence, the electro-magnetic field equations(Maxwell Equations) in Rindler spacetime are invariant about Lorentz gauge transformation.

4. Conclusion

We find the electro-magnetic field transformation and the electro-magnetic equation in uniformly accelerated frame.

Generally, the coordinate transformation of accelerated frame is

(I) $$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})$$

$$x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0} \quad , y = \xi^2, z = \xi^3 \qquad (58)$$

(II) $$ct = \frac{c^2}{a_0}\exp(\frac{a_0}{c^2}\xi^1)\sinh(\frac{a_0\xi^0}{c})$$

$$x = \frac{c^2}{a_0}\exp(\frac{a_0}{c^2}\xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0}, y = \xi^2, z = \xi^3 \qquad (59)$$

Hence, this article say the accelerated frame is Rindler coordinate (I) that can treat electro-magnetic field equation.

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