The Equivalence Of Gravitational Field And Time From The Standpoint Of The General Theory Of Relativity

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Abstract: In general, the modification of our understanding of space and time undergone through Einstein's relativity theory is indeed a profound one. But even Einstein's relativity theory does not give satisfactory answers to a lot of questions. One of these questions is the problem of the 'true' tensor of the gravitational field. The purpose of this publication is to provide some new and basic fundamental insights by the proof that the gravitational field and time is equivalent even under conditions of the general theory of relativity.

Key words: Quantum theory, relativity theory, unified field theory, causality.

1. Introduction

Einstein’s successful geometrization of the gravitational field in his general theory of relativity does not include a geometrized theory of the electromagnetic field too. The theoretical physicists working in the field of the general theory of relativity were not able to succeed in finding a convincing geometrical formulation of the gravitational and electromagnetic field. Still, electromagnetic fields are not described by Riemannian metrics. More serious from the conceptual point of view, in order to achieve unification, with the development of quantum theory any conceptual unification of the gravitational and electromagnetic field should introduce a possibility that the fields can be quantized. In our striving toward unification of the foundations of physics a relativistic field theory we are looking for should therefore be an extension of the general theory of relativity and equally and of no less importance a generalization of the theory of the gravitational field. In the attempt to solve these problems one meets at least with another difficulty. Einstein was demanding that

" the symmetrical tensor field must be replaced by a non-symmetrical one. This means that the condition $g_{ik} = g_{ki}$ for the field components must be dropped. " [1]

Evidently, following up these train of thoughts and in view of all these difficulties, the following theory is based on a (gravitational) field of more complex nature. Still, in our attempt to obtain a deeper knowledge of the foundations of physics the new and basic concepts are in accordance with general relativity theory from the beginning but with philosophy too. In general, energy, time and space are deeply related and interacting like the one with its own other and vice versa.
2. Definitions

2.1. Definition. The Tensor Of Energy $\mathcal{E}_{ae}$

Following Einstein’s special theory of relativity, mass and energy are equivalent.

"Die Masse eines Körpers ist ein Maß für dessen Energieinhalt" [2]

In general relativity, energy is described by the stress-energy tensor of energy.

“Da Masse und Energie nach den Ergebnissen der speziellen Relativitätstheorie das Gleiche sind und die Energie formal durch den symmetrischen Energietensor (T\textsubscript{\mu\nu}) beschrieben wird, so besagt dies, daß das G-Geld [gravitational field, author] durch den Energietensor der Materie bedingt und bestimmt ist.” [3]

In general we define the tensor of energy $\mathcal{E}_{ae}$ as

$$\mathcal{E}_{ae} = \frac{4\times2\times\pi\times\gamma}{c\times c\times c\times c} 	imes T_{ae}$$

(1)

2.2. Definition. The Tensor Of Matter $\mathcal{M}_{ae}$

Matter and energy are not absolutely the same. Einstein himself defines matter in relation to gravitational field. Due to Einstein’s understanding of matter and gravitational field, all but the gravitational field has to be treated as matter. Consequently, matter as such includes matter in the ordinary sense and the electromagnetic field as well. In other words, there is no third between matter and gravitational field. Einstein himself wrote:

"Wir unterscheiden im folgenden zwischen 'Gravitationsfeld' und 'Materie', in dem Sinne, daß alles außer dem Gravitationsfeld als 'Materie' bezeichnet wird, also nicht nur die 'Materie' im üblichen Sinne, sondern auch das elektro-magnetische Feld. " [4]

Einstein’s writing translated into English:

>> We make a distinction hereafter between 'gravitational field' and 'matter' in this way, that we denote everything but the gravitational field as 'matter', the word matter therefore includes not only matter in the ordinary sense, but the electromagnetic field as well. <<

In general we define the tensor of matter $\mathcal{M}_{ae}$ as

$$\mathcal{M}_{ae} = \frac{\mathcal{E}_{ae}}{c\times c} = \frac{4\times2\times\pi\times\gamma}{c\times c\times c\times c\times c \times c} 	imes T_{ae}$$

(2)
2.3. Definition. The Tensor Of Time $R_{t\alpha\epsilon}$

We define the second covariant tensor of time of preliminary unknown structure as

$$R_{t\alpha\epsilon}.$$  \hfill (3)

2.4. Definition. The Tensor Of Gravitational Field $R_{g\alpha\epsilon}$

We define the second covariant tensor of the gravitational field of preliminary unknown structure as

$$R_{g\alpha\epsilon}.$$  \hfill (4)

2.5. Definition. The Tensor Of Space $R_{s\alpha\epsilon}$

We define the second covariant tensor of space of preliminary unknown structure as

$$R_{s\alpha\epsilon} = R_{E\alpha\epsilon} + R_{t\alpha\epsilon}.$$  \hfill (5)

Scholium.

The tensor of space is determined by the tensor of energy and the tensor of time. Clearly, all but energy is treated as time. Under conditions of general theory of relativity, there are circumstances where the tensor of space is equivalent with the Ricci tensor $R_{ae}$.

2.6. Definition. The Tensor $R_{n\alpha\epsilon}$

We define the second covariant tensor $R_{n\alpha\epsilon}$ as

$$R_{n\alpha\epsilon} = R_{S\alpha\epsilon} = R_{M\alpha\epsilon} + R_{g\alpha\epsilon}.$$  \hfill (6)

There is no third between matter and gravitational field. In terms of set theory we would obtain the following picture.

\begin{center}
\begin{tikzpicture}
\node (M) at (0,0) {Matter $R_{M\alpha\epsilon}$};
\node (G) at (2,0) {Gravitational field $R_{g\alpha\epsilon}$};
\end{tikzpicture}
\end{center}

This approach to the relationship between matter and gravitational field is sometimes also referred to as the *de Broglie hypothesis* since all matter can exhibit wave-like behavior. Thus far, so called “matter waves”, a concept having been proposed by Louis de Broglie in 1924.
3. Theorems

3.1. Theorem. The Equivalence Of Time And Gravitational Filed

Claim.
The relationship between time and gravitational field is determined as

\[ R \, t_{ae} = c^2 \times R \, g_{ae} \quad (7) \]

Proof.
Our starting point is (lex identitatis) the claim that

\[ +1 = +1 . \quad (8) \]

Multiplying this equation by \( R \, E_{ae} + R \, t_{ae} \) we obtain

\[ R \, E_{ae} + R \, t_{ae} = R \, E_{ae} + R \, t_{ae} . \quad (9) \]

Due to our definition, it \( R \, S_{ae} \equiv R \, E_{ae} + R \, t_{ae} \) and \( R \, N_{ae} \equiv \frac{R \, S_{ae}}{c \times c} \equiv R \, M_{ae} + R \, g_{ae} \) we obtain

\[ R \, E_{ae} + R \, t_{ae} = c^2 \times \left( R \, M_{ae} + R \, g_{ae} \right) \quad (10) \]

Rearranging equation, it is

\[ R \, E_{ae} + R \, t_{ae} = c^2 \times R \, M_{ae} + c^2 \times R \, g_{ae} . \quad (11) \]

Due to the relationship \( R \, M_{ae} \equiv \frac{R \, E_{ae}}{c \times c} \equiv \frac{4 \times 2 \times \pi \times \gamma}{c \times c \times c \times c \times c} \times T_{ae} \), it follows that

\[ R \, E_{ae} + R \, t_{ae} = c^2 \times R \, g_{ae} \quad (12) \]

The equivalence of time and gravitational field follows in general as

\[ R \, t_{ae} = c^2 \times R \, g_{ae} \quad (13) \]

Quod erat demonstrandum.
3.2. **Theorem. The Tensor Of Time** _r_t ae_

**Claim.**
Under conditions where the tensor of space _r_S ae_ is equivalent to the Ricci tensor _R ae_, the tensor of time _r_t ae_ follows as

\[ _r_t ae = \frac{R}{2} \times g_{ae} - \Lambda \times g_{ae} \]  \hspace{1cm} (14)

**Proof.**
Our starting point (**lex identitatis**) is the claim that

\[ +1 = +1. \]  \hspace{1cm} (15)

Multiplying this equation by Einstein's stress energy tensor \( \frac{4 \times 2 \times \pi \times \gamma}{c \times c \times c \times c} \times T_{ae} \) we obtain

\[ \frac{4 \times 2 \times \pi \times \gamma}{c \times c \times c \times c} \times T_{ae} = \frac{4 \times 2 \times \pi \times \gamma}{c \times c \times c \times c} \times T_{ae} \]  \hspace{1cm} (16)

which is equivalent with Einstein's field equation as

\[ R_{ae} = \frac{R}{2} \times g_{ae} + \Lambda \times g_{ae} = \frac{4 \times 2 \times \pi \times \gamma}{c \times c \times c \times c} \times T_{ae} \]  \hspace{1cm} (17)

Rearranging equation, we obtain

\[ R_{ae} = \frac{4 \times 2 \times \pi \times \gamma}{c \times c \times c \times c} \times T_{ae} + \frac{R}{2} \times g_{ae} - \Lambda \times g_{ae} \]  \hspace{1cm} (18)

Under conditions of general relativity, the tensor of space _r_S ae_ is equivalent with the Ricci tensor _R ae_. Thus far we equate _r_S ae = R ae_ and do obtain

\[ _r_S ae = \frac{4 \times 2 \times \pi \times \gamma}{c \times c \times c \times c} \times T_{ae} + \frac{R}{2} \times g_{ae} - \Lambda \times g_{ae} \]  \hspace{1cm} (19)

In general, it is \( _r S_{ae} = _r E_{ae} + _r t_{ae} \) and \( _r E_{ae} = \frac{4 \times 2 \times \pi \times \gamma}{c \times c \times c \times c} \times T_{ae} \). Rearranging equation it is

\[ _r E_{ae} + _r t_{ae} = _r E_{ae} + \frac{R}{2} \times g_{ae} - \Lambda \times g_{ae} \]  \hspace{1cm} (20)

The tensor of time under conditions of general theory of relativity follows as

\[ _r t_{ae} = \frac{R}{2} \times g_{ae} - \Lambda \times g_{ae} \]  \hspace{1cm} (21)

**Quod erat demonstrandum.**
3.3. **Theorem. The Tensor Of The Gravitational Field \( R_{ae} \)**

**Claim.**
Under conditions where the tensor of space \( R_{Sae} \) is equivalent to the Ricci tensor \( R_{ae} \), the tensor of the gravitational field \( R_{gae} \) follows as

\[
R_{gae} = \frac{R}{2 \times c^2} \times g_{ae} - \frac{\Lambda}{c^2} \times g_{ae}
\]  

(22)

**Proof.**
Our starting point (\textit{lex identitatis}) is the claim that

\[
1 + 1 = +1.
\]  

(23)

Multiplying this equation by the tensor of time \( R_{t ae} \), we obtain

\[
R_{t ae} = R_{t ae}
\]  

(24)

Due to our theorem above (under conditions where the tensor of space \( R_{Sae} \) is equivalent to the Ricci tensor \( R_{ae} \)) we obtain

\[
R_{t ae} = \frac{R}{2} \times g_{ae} - \Lambda \times g_{ae}
\]  

(25)

Due to the equivalence of time and gravitational field \( R_{t ae} = c^2 \times R_{gae} \) we obtain

\[
c^2 \times R_{gae} = \frac{R}{2} \times g_{ae} - \Lambda \times g_{ae}
\]  

(26)

The tensor of the gravitational field under conditions of the theory of general relativity (i.e. where the equivalence \( R_{Sae} = R_{ae} \) is valid) follows as

\[
R_{gae} = \frac{R}{2 \times c^2} \times g_{ae} - \frac{\Lambda}{c^2} \times g_{ae} = \frac{R - 2 \times \Lambda}{2 \times c^2} \times g_{ae}
\]  

(27)

**Quod erat demonstrandum.**

**Scholium.**
The relationship is valid under conditions of general relativity. Theoretically, it is possible that the tensor of space is not all the time identical with the Ricci tensor \( R_{ae} \) (i.e. Weyl curvature tensor). Consequently, we would have to accept something like \( R_{Sae} = R_{ae} + X_{abe} \). The metric tensor of general relativity \( g_{ae} \) describes more or less the gravitational potential but not the gravitational field as such. Under conditions of general relativity where \( \frac{R - 2 \times \Lambda}{2 \times c^2} = 1 \) we obtain \( R_{gae} = g_{ae} \) but not in general.
4. Discussion

Time appear to be closely related to the gravitational field itself. In general, it is known that clocks moving with respect to an inertial system of observation are found to be running more slowly. Thus far, a clock closer to a gravitational mass (i.e. deeper in its “gravity well”) go more slowly than a clock much more distant from the same mass (energy). The gravitational time dilation has been confirmed experimentally several times. Altogether, the gravitational field and time as such are linked somehow. In general, the relationship between gravitational field and time is similar to the relationship between mass and energy. Finally, we are force to accept the gravitational field-time equivalence even under conditions of the general theory of relativity.

5. Conclusion

In general, the gravitational field is equivalent to time and vice versa even [5] under conditions of the general theory of relativity.

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Appendix

None.

References