An algorithm for producing Benjamin Franklin’s magic squares

J. S. Markovitch
Saint Augustine, FL 32084
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An algorithm is presented that produces, in six steps, Benjamin Franklin’s best known magic squares, one $8 \times 8$, and one $16 \times 16$. This same algorithm is then used to produce three related magic squares, dimensioned $4 \times 4$, $32 \times 32$, and $64 \times 64$.

I. GENERATING BENJAMIN FRANKLIN’S MAGIC SQUARES

Starting from a consecutively numbered $4 \times 4$ square, an algorithm uses six steps to produce a $4 \times 4$ magic square, as seen in Fig. 1. This same algorithm is then used to produce Benjamin Franklin’s two best known magic squares, one $8 \times 8$, as seen in Fig. 2, and one $16 \times 16$, as seen in Figs. 3 and 4. These are the magic squares made famous by Van Doren’s 1938 biography of Franklin [1]. The algorithm then produces a $32 \times 32$ magic square in Figs. 5 through 10, and a $64 \times 64$ magic square in Figs. 11 and 12, where size considerations prevent showing the steps producing this largest square.

The algorithm’s steps either shift, or reflect about a vertical or horizontal midline, a rectangle of numbers belonging to a square that is initially consecutively numbered. For steps in which the rectangle is shifted, the rectangle’s numbers are highlighted in orange. So, in Fig. 1, a row is shifted up in step 3 → 4; and a column is shifted left in step 6 → 7. For steps in which the rectangle is reflected about a midline, the rectangle’s numbers are highlighted in orange and green, where orange numbers swap with orange, and green with green. So, in Fig. 1, rows are reflected about the vertical midline in steps 2 → 3 and 5 → 6; and columns are reflected about the horizontal midline in steps 1 → 2 and 4 → 5.

It is an obvious point, but worth mentioning, that Franklin possessed a printer’s skill at using a composing stick (see figure on right [2]). This may have aided Franklin’s discovery of his famous $8 \times 8$ and $16 \times 16$ magic squares, as he was likely capable of rapidly moving blocks of numbers about physically and mentally, the exact skills needed to apply an algorithm of the above kind.

The algorithm employed in this article relates to that given by Pasles [3], which focuses on Franklin’s step-by-step reasoning, and which works by building the magic square column by column, while employing shifts similar to those used here. See [4–6] for an overview of Franklin’s magic squares.

*Electronic address: jsmarkovitch@gmail.com

FIG. 1: Producing a $4 \times 4$ magic square in six steps using the same algorithm used to produce Franklin’s larger magic squares. The colors orange and green highlight the numbers that change with each step. Horizontal and vertical axes of rotation are included.
FIG. 2: Making Franklin’s 8 × 8 magic square in six steps. The colors orange and green highlight the numbers that change with each step.
FIG. 3: Making Franklin’s 16 × 16 magic square in six steps (first figure of two). The colors orange and green highlight the numbers that change with each step.
FIG. 4: Making Franklin’s 16 × 16 magic square in six steps (second figure of two). The colors orange and green highlight the numbers that change with each step.
FIG. 5: Producing a $32 \times 32$ magic square in six steps using the same algorithm used to produce Franklin’s smaller magic squares (first figure of six). In this step, orange numbers swap with orange, and green numbers with green, along an unseen axis of rotation.
FIG. 6: Producing a 32 × 32 magic square in six steps using the same algorithm used to produce Franklin’s smaller magic squares (second figure of six). In this step, orange numbers swap with orange, and green numbers with green, along an unseen axis of rotation.
squares (fourth figure of six). In this step, orange numbers swap with orange, and green numbers with green, along an unseen axis of rotation.

FIG. 8: Producing a 32 × 32 magic square in six steps using the same algorithm used to produce Franklin’s smaller magic squares (fourth figure of six). In this step, orange numbers swap with orange, and green numbers with green, along an unseen axis of rotation.
FIG. 9: Producing a 32 × 32 magic square in six steps using the same algorithm used to produce Franklin’s smaller magic squares (fifth figure of six). In this step, orange numbers swap with orange, and green numbers with green, along an unseen axis of rotation.
FIG. 12: A 64 × 64 magic square created using the same algorithm used to produce Franklin’s smaller magic squares (second figure of two).


