Quantum Gravity Explanation

of the

de Broglie Pilot Wave

F. Winterberg

University of Nevada, Reno

USA

June 2015

The particle-wave duality of the de Broglie pilot wave is explained by the emission of watt-less gravitational waves from Schrödinger's "Zitterbewegung" of elementary particles obeying the Dirac equation.

In particular is shown that the quantum potential of the Madelung-transformed Schrödinger Equation is gravitational in it's origin.

1. Quantum gravity Origin of the Quantum potential

In accordance with the general theory of relativity we take as the most simple particle a Planck mass particle $m_p = \sqrt{hc/G}$ (G Newton's constant), with its radius equal the event horizon, that is the Planck length¹ $r_p = \sqrt{hG/c^3}$. If its kinetic energy is still below the Planck energy $m_pc^2 \cong 10^{19}$ GeV, we can describe it with sufficient accuracy by the non-relativistic Schrödinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m_p} \nabla^2 \Psi + U \Psi$$
(1)

where U is the potential of an externally applied force. Making for (1) the Madelung transformation:

$$\Psi = \sqrt{n} e^{iS}$$
(2)
$$\Psi^* = \sqrt{n} e^{-iS}$$

where S is the Hamilton action function and $n = \Psi^* \Psi$, the Planck mass particle number density, one obtains two coupled equations:

$$h\frac{\partial \mathbf{S}}{\partial t} + U + \frac{h^2}{2m_p} (\nabla \mathbf{S})^2 + \frac{h^2}{2m_p} \frac{\nabla^2 \sqrt{n}}{\sqrt{n}} = 0$$

$$\left. \right\}$$

$$\left. \frac{\partial \mathbf{n}}{\partial t} + \frac{h}{m_p} \nabla (\mathbf{n} \mathbf{S}) = 0 \right\}$$

$$(3)$$

where

$$\frac{h^2}{2m_p} \frac{\nabla^2 \sqrt{n}}{\sqrt{n}} = Q \tag{4}$$

 $^{^1}$ henceforth h stands for Planck constant divided by 2 π

is the so called quantum potential. Setting, as in the Hamilton-Jacobi theory of classical mechanics $\mathbf{v}=(h/m_p) \nabla S$, one obtains from (3)

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \frac{\partial \mathbf{v}}{\partial \mathbf{r}} = -\frac{1}{m_p} \frac{\partial}{\partial \mathbf{r}} [\mathbf{U} + \mathbf{Q}]$$

$$\frac{\partial \mathbf{n}}{\partial t} + \operatorname{div}(\mathbf{n}\mathbf{v}) = 0$$
(5)

the Euler and continuity equation for a frictionless fluid with ordinary U and quantum potential Q. Setting Q = 0 and making the inverse Madelung transformation of (3) one finds that

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m_p} \nabla^2 \Psi + U\Psi + \frac{\hbar^2}{2m_p} \frac{\nabla^2 |\Psi|}{|\Psi|}$$
(6)

Unlike the Schrödinger equation (1) it is nonlinear. This simple fact shows it is the quantum potential which makes the Schrödinger equation a linear wave equation.

The meaning of the quantum potential can best be seen by setting in (5) U = 0 and writing it down in Lagrangian co-moving coordinates:

$$m_{p}\frac{d\mathbf{v}}{dt} = -\nabla Q \tag{7}$$

where by an order of magnitude estimate one has

$$|\mathbf{Q}| \simeq \frac{h^2}{m_p r_p^2} = m_p c^2 = \sqrt{hc^5/G} \simeq 10^{19} \,\text{GeV}$$
(8)

This shows that it is through gravity that the quantum potential enters the Schrödinger equation. And it already gives us important information about gravity in the structure of matter: For $G \rightarrow 0$ the quantum potential becomes overwhelming. For $G \rightarrow \infty$ one has $m_p \rightarrow 0$, and there can be no matter as we know it. For these reasons nature has chosen G in between.

It is at this point to propose the existence of negative Planck masses, permitted in an exactly nonrelativistic theory where the particle number operator commutes with the Hamilton operator. There then a Fermion obeying the Dirac equation can be seen as composed of a large positive and likewise large negative mass (the latter due to the negative energy = negative mass states of the Dirac equation), leading to the "Zitterbewegung" (quivering motion) of such a composed particle, discovered by Schrödinger [1]. This Zitterbewegung must lead to the emission of gravitational waves, and with the existence of negative masses must be a watt-less "empty" wave, as the sum of a positive and phase shifted negative space curvature wave, to explain why Fermions do not decay by the emission of such waves.

It will be shown that this guess can reproduce the quantum potential of the Madelung – transformed Schrödinger equation.

A greatly simplified version of the Dirac equation, under the name pole-dipole particle has been given by Hönl and Papapetrou [2]. This model explicitly assumes that a particle obeying the Dirac equation is made up of a large positive and a likewise large negative mass particle, where the observed positive mass of the composed particle is the positive mass of the gravitational field made up of the mass dipole. Without this positive mass a mass dipole would be selfaccelerating, but with the inclusion of this positive mass the self-acceleration takes place on a circle around the center of mass dipole, which is still positioned on the line connection the two masses but not in between.

Analyzing the internal motion of the pole-dipole particle we assume that it consists of two Planck masses $\pm m_p$, but that the kinetic energy is still below the Planck energy. This permits to use Newton's law of gravity and Bohr's correspondence principle. With this simplifying assumption the positive gravitational field energy E and field mass $m = E/c^2$

$$\frac{E}{c^2} = \mathbf{m} = \frac{Gm_p^2}{c^2 r} \tag{9}$$

Where r is the distance between the two masses of oppsite sign. To determine R and m we use Bohr's correspondence principle where

$$m_{p}rc = h \tag{10}$$

With $Gm_p^2 = hc$, we find from (9) and (10) that

$$\begin{array}{c} m = m_p \\ r = r_p \end{array} \right\} \tag{11}$$

To compute the energy loss by gravitational radiation we use the formula by Eddington [3] for a spinning dumb bell of two masses:

$$-\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{32\,G}{5c^5}\,(mr^2)^2\,\omega^6\tag{12}$$

where we put $m = m_p/2$ because we are only interested in the energy loss of $\frac{1}{2}$ of the mass of a dumb-bell and further set $r = r_p$ and $\omega = c/r_p$:

$$-\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{8}{5} \frac{Gm_p^2 c}{r_p^2} \tag{13}$$

To obtain the integrated positive and likewise compensating negative energy loss in the time r_p/c of a sinosoidally undulating positive-negative space curvature gravitational wave, we have to multiply (13) by \pm (½) (r_p/c), hence,

$$\varepsilon = \frac{4}{5} \frac{Gm_p^2}{r_p} = \frac{4}{5} \frac{Gm_p^2}{r_p} = \frac{4}{5} m_p c^2$$
(14)

Because of (8) it is tempting to postulate that

$$|\mathbf{Q}| = |\varepsilon| \tag{15}$$

which means that the emission of the "empty" gravitational wave by a Dirac particle Fermion is the cause of the quantum potential in the Madelung – transformed Schrödinger equation. With the scaling of $\omega = c/r_p$ to $\omega = c/r$ for $r > r_p$, and because $m_p r_p c = mrc$, equation (15) remains unchanged by

replacing $\varepsilon = m_p c^2$ with $\varepsilon = mc^2$ (m<m_p). Take for example an electron where $\varepsilon = mc^2 \simeq 0.5$ MeV, implying a rather large amplitude of the "empty" gravitational wave in an electron two slit interference experiment.

If confirmed, the insight expressed by equation (15) would be of fundamental importance in all attempts to formulate a correct theory of quantum gravity.

2. Experimental Verification

An independent verification of the made guess might be provided by an experimentally verifiable disturbance of the "empty" gravitational wave emitted by the Zitterbewegung of a Fermion, through the disturbance of a gravitational wave from an astrophysical source. Such waves come from the magnetohydrodynamic dynamo in the center of the sun, as it has recently been conjectured by the author [4]. There then, the gravitational waves from the sun, amplified in the center of the lunar shadow during a total solar eclipse, could conceivably influence the empty wave of a two-slit experiment.

References

- 1. E. Schrödinger, Berl. Berichte 1930, 416; 1931, 418.
- H. Hönl and A. Papapetrou, Z. Physik **112**, 512 (1939), **114**, 478 (1939), **116**, 153 (1940).
- A.S. Eddington, Proceedings of the Royal Society (London) A 102, 268 (1923).
- 4. F. Winterberg, Z. Naturforsch. 2015; aop.