A Comparison of Combined Overlap Block Fuzzy Cognitive Maps (COBFCM) and Combined Overlap Block Neutrosophic Cognitive Map (COBNCM) in finding the hidden patterns and indeterminacies in Psychological Causal Models: Case Study of ADHD

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Abstract

In spite of researchers’ concerns to find causalities, reviewing the literature of psychological studies one may argue that the classical statistical methods applied in order to find causalities are unable to find uncertainty and indeterminacies of the relationships between concepts.
In this paper, we introduce two methods to find effective solutions by identifying “hidden” patterns in the patients’ cognitive maps. Combined Overlap Block Fuzzy Cognitive Map (COBFCM) and Combined Overlap Block Neutrosophic Map (COBNCM) are effective when the number of concepts can be grouped and are large in numbers. In the first section, we introduce COBFCM, COBNCM, their applications, and the advantages of COBNCM over COBFCM in some cases. In the second section, we explain eight overlapped cognitive concepts related to ADHD in children and apply COBNCM and COBFCM to analyze the modeled data, comparing their results. Conclusions, limitations, and implications for applying COBNCM in other psychological areas are also discussed.

Keywords

1 Introduction

A portfolio of project is a group of project that share resources creating relation among them of complementarity, incompatibility or synergy [1]. The interdependency modeling and analysis have commonly been ignored in project portfolio management [2].

Identifying causalities is one of the most important concerns of researchers, one may find out reviewing the literature of psychological research. Although there are some statistical methods to investigate this issue, all, or majority, rely on quantitative data. Less attention was directed towards scientific qualitative knowledge and experience. In some methods based on theoretical basics such as structural equation modeling (SEM), there is no chance to find optimal solutions, hidden patterns and indeterminacies (possibilities) of causal relationships between variables, which are common in psychological research. Therefore, for linking quantitative and qualitative knowledge, it seems an urge to use methods as fuzzy cognitive maps or neutrosophic cognitive maps in psychological research. The two methods are rooted in cognitive map (CM). The cognitive maps for representing social scientific knowledge and describing the methods that is used for decision-making were introduced by Axelrod in 1976. The fuzzy cognitive map (FCM) was proposed by Kosko (1986) to present the causal relationship between concepts and analyze inference patterns. Kosko (1986, 1988, 1997) considered fuzzy degree of inter relationships between concepts, its nodes corresponding to a relevant node and the edges stating the relation between two nodes, denoted by a sign. A positive sign implies a positive relation; moreover, any increase in its source value leads to increase in its target value. A negative sign stages a negative
relation and any increase or decrease in its source value leads to reverse effect to its target value. If there is no edge between two nodes in a cognitive map, it means that there is no relation between them (Zhang et al., 1998). In a simple fuzzy cognitive map, the relation between two nodes is determined by taking a value in interval [-1, 1].

While -1 corresponds to the strongest negative value, +1 corresponds to strongest positive value. The other values express different levels of influence (Lee, et al., 2003). Fuzzy cognitive maps are important mathematical models representing the structured causality knowledge for quantitative inferences (Carvalho & Tome, 2007). FCM is a soft computing technique that follows an approach similar to the human reasoning and decision-making process (Markinos, et al., 2004). Soft computing is an emerging field that combines and synergies advanced theories and technologies such as Fuzzy Logic, Neural Networks, Probabilistic reasoning and Genetic Algorithms. Soft computing provides a hybrid flexible computing technology that can solve real world problems. Soft computing includes not only the previously mentioned approaches, but also useful combinations of its components, e.g. Neurofuzzy systems, Fuzzy Neural systems, usage of Genetic Algorithms in Neural Networks and Fuzzy Systems, and many other hybrid methodologies (Stylios & Peter, 2000). FCM can successfully represent knowledge and human experiences, introduce concepts to represent the essential elements, cause and effect relationships among the concepts, to model the behavior of a system (Kandasamy, 1999, 2004). This method is a very simple and powerful tool that is used in numerous fields (Thiruppathi, et al. 2010). When dataset is an unsupervised one and there is uncertainty within the concepts, this method is very useful. The FCM give us the hidden patterns; this method is one effective method, providing a tool for unsupervised data. In addition, using this method, one can analyze the data by directed graphs and connection matrices where nodes represent concepts and edges - strength of relationships (Stylios & Groumpos, 2000). FCM works on the opinion of experts or another uncertainty results like the obtained results using structural equation modeling (SEM). FCM clarify optimal solution by using a simple way, while other causal models such as SEM are complicated. They do not perform well to clarify what-if scenario, for example, their results do not clarify what happens to marital satisfaction if Alexithymia is very high and Family intimacy is very low. Another advantage of FCM is its functioning on experts’ opinions (Thiruppathi et al. 2010). FCM is a flexible method used in several models to display several types of problems (Vasantha Kandasamy & Devadoss, 2004; Vasantha Kandasamy & Kisho, 1999). Although by using this method we are able to study uncertainty and find hidden patterns, the FCM is unable to investigate indeterminate relationships, which is a limitation in psychological causal
models. A solution to overcome this limitation is the Neutrosophic Cognitive Map (NCM).

Vasantha Kandasamy and Smarandache (2003) proposed the neutrosophic cognitive maps, making it possible to mitigate the limitation of fuzzy cognitive maps, which cannot represent the indeterminate relations between variables. The capability of neutrosophic cognitive maps to represent indetermination facilitates the apprehension of systems complexity, and thus elucidates and predicts their behaviors in the absence of complete information.

Neutrosophic Cognitive Map (NCM) relies on Neutrosophy. Neutrosophy is a new branch of philosophy introduced by Smarandache in 1995 as a generalization of dialectics, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophic Cognitive Map (NCM) is the generalization and combination of the Fuzzy Cognitive Map in which indeterminacy is included. Fuzzy theory only measures the grade of membership or the non-existence of a membership in a revolutionary way, but failing to attribute the concept when the relationship between concepts in debate are indeterminate (Vasantha Kandasamy & Smarandache, 2007). A Neutrosophic Cognitive Map is a neutrosophic directed graph with concepts like policies, events etc. as nodes and causalities, or indeterminacies as edges. It represents the causal relationship between concepts defined by Smarandache (2001) and Vasantha Kandasamy (2007). Fuzzy cognitive maps deals with the relation / non-relation between two nodes or concepts, but it declines to attribute the relation between two conceptual nodes when the relation is an indeterminate one. In Neutrosophic Logic, each proposition is estimated to have the percentage of truth in a subset $T$, the percentage of indeterminacy in a subset $I$, and the percentage of falsity in a subset $F$. Every logical variable $x$ is described by an ordered triple $x = (T, I, F)$, where $T$ is the degree of truth, $F$ is the degree of false and $I$ - the level of indeterminacy. Neutrosophy means that any proposition has a percentage of truth, a percentage of indeterminacy and a percentage of falsity (some of these percentages may be zero). Neutrosophy also makes distinctions between absolute truth (a proposition true in all possible worlds), which is denoted by 1, and relative truth (a proposition which is true in at least one world, but not in all), which is denoted by I (Smarandache & Liu, 2004). Sometimes, in psychological and educational research, the causality between the two concepts, i.e. the effect of $C_i$ on $C_j$ is indeterminate. Chances of indeterminacy are possible and frequent in case of unsupervised data. Therefore, the NCM is a flexible and effective method based on fuzzy cognitive map for investigating the relations of psychological casual models in which indeterminate relationships are not unusual. We describe the basic components in detail to explain differences between the two methods.
2 Combined Overlap Block Fuzzy Cognitive Maps (COBFCM) and Combined Overlap Block Neutrosophic Cognitive Map (COBNCM)

We can combine arbitrarily FCM and NCM connection matrices $F_1, F_2, ..., F_k$ by adding augmented FCM and NCM matrices, $F_1, ..., F_k$. Each augmented matrix $F_i$ has $n$-rows and $n$-columns; $n$ equals the total number of distinct concepts used by the experts. We permute the rows and columns of the augmented matrices to bring them into mutual coincidence. Then we add the $F_i$’s point wise to yield the combined FCM and NCM matrix $F$, $F = \Sigma F_i$. We can then use $F$ to construct the combined FCM and NCM directed graph. The combination can be in disjoint or overlapping blocks.

Combined overlap block fuzzy cognitive maps (COBFCM) were introduced and applied in social sciences by Vasantha Kandasamy et al. (2004), and combined overlap block neutrosophic cognitive map (COBNCM) - by Vasantha Kandasamy & Smarandache (2007). In these two methods, finite number of NCM and FCM can be combined together to produce the joint effect of all NCM and FCM. In NCM method, $N(E_1), N(E_2), ..., N(E_p)$ are considered the neutrosophic adjacency matrices, with nodes $C_1, C_2, ..., C_n$ and $E_1, E_2, ..., E_p$ are the adjacency matrices of FCM with nodes $C_1, C_2, ..., C_n$. The combined NCM and the combined FCM are obtained by adding all the neutrosophic adjacency matrices $N(E_1) + N(E_2) + ... + N(E_p)$ and adjacency matrices by $E_1, E_2, ..., E_p$ respectively. We denote the Combined NCM adjacency neutrosophic matrix by $N(E) = N(E_1) + N(E_2) + ... + N(E_p)$ and the Combined FCM adjacency matrix by $E = E_1 + E_2 + ... + E_p$. Both models $\{C_1, C_2, C_3, ..., C_n\}$ contain $n$ concepts associated with $P$ (a given problem). We divide the number of concepts $\{C_1, C_2, C_3, ..., C_n\}$ into $K$ classes $S_1, S_2, S_3, ..., S_K$, where the classes are such that $S_i \cap S_{i+1} \neq \phi$, $U S_i = \{C_1, C_2, ..., C_n\}$ and $|S_i| \neq |S_j|$, if $i \neq j$ in general. To introduce these methods in detail, we explain their basic components below.

3 Concepts and edges

In Combined Overlap Block Fuzzy Cognitive Maps (COBFCM) and Combined Overlap Block Neutrosophic Cognitive Map (COBNCM), the edges are qualitative concepts considered as nodes and causal influences. Concept nodes possess a numeric state, which denotes qualitative measures of the concepts present in the conceptual domain. When the nodes of FCM are a fuzzy set, they are called fuzzy nodes. Fuzzy means the concepts are not quantitative, they are uncertain, and we have to study them using linguistic variables, such as “very high”, “high”, “middle”, etc. The nodes or concepts are presented by $C_1, C_2$.
The state of concepts is portrayed as a vector. In COBNCM, we assume each node is a neutrosophic vector from neutrosophic vector space \( V \). Let \( C_1, C_2, \ldots, C_n \) denote \( n \) nodes. So a node \( C_i \) will be represented by \((x_1, \ldots, x_n)\), where \( x_k \)'s - zero or one or \( I \) (\( I \) is the indeterminate) and \( x_k = 1 \) means that the node \( C_k \) is in the \( \text{ON} \) state, and \( x_k = 0 \) means the node is in the \( \text{OFF} \) state, and \( x_k = I \) means the nodes state is an \( \text{indeterminate} \) at that time or in that situation. Let \( C_1, C_2, \ldots, C_n \) be the nodes of COBNCM and let \( A = (a_1, a_2, \ldots, a_n) \), where \( a_i \in \{0, 1, I\} \). \( A \) is called the instantaneous state neutrosophic vector and it denotes the \( \text{ON} - \text{OFF} - \text{indeterminate} \) state position of the node at an instant:

\[
\begin{align*}
a_i &= 0 \text{ if } a_i \text{ is off (no effect)}, \\
a_i &= 1 \text{ if } a_i \text{ is on (has effect)}, \\
a_i &= I \text{ if } a_i \text{ is indeterminate (effect cannot be determined)},
\end{align*}
\]

for \( i = 1, 2, \ldots, n \).

In COBNCM, the nodes \( C_1, C_2, \ldots, C_n \) are nodes and not indeterminate nodes, because they indicate the concepts which are well known. But the edges connecting \( C_i \) and \( C_j \) may be indeterminate, i.e. an expert may not be in the position to say that \( C_i \) has some causality on \( C_j \), either he will be in the position to state that \( C_i \) has no relation with \( C_j \); in such cases, the relation between \( C_i \) and \( C_j \) which is indeterminate, is denoted by \( I \). The COBFCM with edge weights or causalities from the set \( \{-1, 0, 1\} \) are called simple, and COBNCM with edge weight from \( \{-1, 0, 1, I\} \) are called simple COBNCM. In COBFCM, the edges \((e_{ij})\) take values in the fuzzy causal interval \([-1, 1]\), \( e_{ij} = 0, e_{ij} > 0 \) and \( e_{ij} < 0 \) indicate no causality, positive and negative causality, respectively. In simple FCM, if the causality occurs, it occurs to a maximal positive or negative degree. Every edge in COBNCM is weighted with a number in the set \( \{-1, 0, 1, I\} \). \( e_{ij} \) is the weight of the directed edge \( C_i \rightarrow C_j \), \( e_{ij} \in \{-1, 0, 1, I\} \). \( e_{ij} = 0 \) if \( C_i \) does not have any effect on \( C_j \), \( e_{ij} = 1 \) if increase (or decrease) in \( C_i \) causes increase (or decrease) in \( C_j \), \( e_{ij} = -1 \) if increase (or decrease) in \( C_i \) causes decrease (or increase) in \( C_j \). \( e_{ij} = I \) if the relation or effect of \( C_i \) on \( C_j \) is an indeterminate. In such cases, it is denoted by dotted lines in the model.

### 4 Adjacency Matrix

In COBFCM and COBNCM, the edge weights are presented in a matrix. This matrix is defined by \( E = (e_{ij}) \), where \( e_{ij} \) indicates the weight of direct edge \( C_i \rightarrow C_j \) and \( e_{ij} \in \{0, 1, -1\} \), and by \( N(E) = (e_{ij}) \), where \( e_{ij} \) is the weight of the directed edge \( C_i \rightarrow C_j \), where \( e_{ij} \in \{0, 1, -1, I\} \). We denote by \( N(E) \) the neutrosophic adjacency matrix of the COBNCM. It is important to note that all matrices used
in these methods are always a square matrix with diagonal entries as zeros. All off-diagonal entries are edge weights that link adjacent nodes to each other. A finite number of FCM and NCM can be combined together to produce the joint effect of all FCM and NCM. Suppose $E_1, E_2, E_3, \ldots, E_p$ and $N(E_1), N(E_2), N(E_3) \ldots N(E_p)$ are adjacency matrices of FCM and neutrosophic adjacency matrix of NCM, respectively, with nodes $C_1, C_2, C_3, \ldots, C_n$. Then combined FCM and NCM are obtained by adding all the adjacency matrices (Vasantha Kandasamy & Smarandache, 2003). In combined overlap FCM and NCM, all entries of all different overlapped matrices are put in a whole matrix and added to each other.

5 Inference process

The states of concepts are rendered as vectors. Therefore, the inference process of FCM and NCM can be represented by an iterative matrix calculation process. Let $V_0$ be the initial state vector, $V_n$ be the state vector after $n$th iterative calculation, and $W$ be the causal effect degree matrix; then the inference process can be defined as a repeating calculation of Equation 1 until the state vector converges to a stable value or fall in to an infinite loop. Suppose $X_1 = [1 \ 0 \ 0 \ 0 \ldots 0]$ is the input vector and $E$ is the associated adjacency matrix. $X_1 E$ is obtained by multiplying $X_1$ by the matrix $E$. We obtain $X_1 E = [x_1, x_2, x_3, \ldots, x_n]$ by replacing $x_i$ by 1, if $x_i > c$, and $x_i$ by 0, if $x_i < c$ (c is a suitable positive integer).

After updating the thresholding concept, the concept is included in the updated vector by making the first coordinate as 1 in the resulting vector. Suppose $X_1 E \rightarrow X_2$, then $X_2 E$ is considered; the same procedure is repeated until it gets limit cycle or a fixed point (Thiruppathi, et al., 2010).

$$V_{n+1} = f(V_n \times W + V_n),$$

(1)

where the $f$ is usually simply defined as $f(x) = f_0(x) = 1 \ (x \geq 1), \ 0 \ (1 > x > -1)$ and $-1 \ (-1 \leq x)$.

If the equilibrium state of a dynamical system is a unique state vector, then it is called a fixed point. Consider FCM and NCM with $C_1, C_2, \ldots, C_n$ as nodes. For example, let us start the dynamical system by switching on $C_1$. Let us assume that NCM and FCM settle down with $C_1$ and $C_n$ ON, i.e. the state vector remains as $(1, 0, \ldots, 1)$; this state vector $(1,0,\ldots,0,1)$ is called the fixed point; if FCM and NCM settle down with a state vector repeating in the form $A_1 \rightarrow A_2 \rightarrow \ldots \rightarrow A_i \rightarrow A_1$, then this equilibrium is called a limit cycle of NCM and FCM (Tabar, 1991).

Let $C_1, C_2, \ldots, C_n$ be the vector of FCM and NSM. Let $E$ be the associated adjacency matrix. Let us find the hidden pattern when $x_1$ is switched on when an input is given as the vector $A_1 = (1, 0, 0, \ldots, 0)$; the data should pass through the
neutrosophic matrix $N(E)$; this is done by multiplying $A_1$ by the matrix $N(E)$. Let $A_1 N(E) = (a_1, a_2, \ldots, a_n)$ with the threshold operation, by replacing $a_i$ by 1, if $a_i > k$, and $a_i$ by 0, if $a_i < k$, and $a_i$ by $I$, if $a_i$ is not an integer.

$$f(k) = \begin{cases} 
    a_i < k \rightarrow a_i = 0 \\
    a_i > k \rightarrow a_i = 1 \\
    a_i = b + c \times I \rightarrow a_i = b \\
    a_i = c \times I \rightarrow a_i = I 
\end{cases} \quad (k \text{ depends on researcher’s opinion, for example } K=1 \text{ or } 0.5).$$

Note that $(a_1, a_2, \ldots, a_n)$ and $(a'_1, a'_2, \ldots, a'_n)$ are two neutrosophic vectors. We say $(a_1, a_2, \ldots, a_n)$ is equivalent to $(a'_1, a'_2, \ldots, a'_n)$ denoted by $(a_1, a_2, \ldots, a_n) \sim (a'_1, a'_2, \ldots, a'_n)$, if we get $(a'_1, a'_2, \ldots, a'_n)$ after thresholding and updating the vector $(a_1, a_2, \ldots, a_n)$, after passing through the neutrosophic adjacency matrix $N(E)$. The initial state vector in FCM and NCM is included 0 and 1 only (OFF and ON states, respectively). But after it passes through the adjacency matrix, the updating resultant vector may have entries from (0 and 1) in FCM and from (0, 1, I) in NCM, respectively. In this case, we cannot confirm the presence of that node (ON state), nor the absence (OFF state). Such possibilities are present only in the case of NCM.

6 Cyclic and acyclic FCM and NCM

If FCM and NCM possess a directed cycle, it is said to be cyclic (to have a feedback) and we call it a dynamical system. FCM and NCM are acyclic if they do not possess any directed cycle.

7 FCM versus NCM

Vasantha Kandasamy and Smarandache (2003) summarize the differences between FCM and NCM:

[1] FCM indicates the existence of causal relation between two concepts, and if no relation exists, it is denoted by 0.

[2] NCM does not indicate only the existence or absence of causal relation between two concepts, but also gives representation to the indeterminacy of relations between any two concepts.

[3] We cannot apply NCM for all unsupervised data. NCM will have meaning only when relation between at least two concepts $C_i$ and $C_j$ are indeterminate.
The class of FCM is strictly contained in the class of NCM. All NCM can be made into FCM by replacing I in the connection matrix by 0.

The directed graphs in case of NCM are called neutrosophic graphs. In the graphs, there are at least two edges, which are related by the dotted lines, meaning the edge between those two vertices is an indeterminate.

All connection matrices of the NCM are neutrosophic matrices. They have in addition to the entries 0, 1, –1, the symbol I.

The resultant vectors, i.e. the hidden pattern resulting in a fixed point or a limit cycle of a NCM, can also be a neutrosophic vector, signifying the state of certain conceptual nodes of the system to be an indeterminate; indeterminate relation is signified by I.

Because NCM measures the indeterminate, the expert of the model can give careful representation while implementing the results of the model.

In case of simple FCM, we have the number of instantaneous state vectors to be the same as the number of resultant vectors, but in the case of NCM the number of instantaneous state vectors is from the set \{0, 1\}, whereas the resultant vectors are from the bigger set \{0, 1, I\}.

Neutrosophic matrix \(N(E)\) converts to adjacency matrix \(E\) by easily recoding I to 0.

8 **Case study:** The comparison of COBFCM and COBNCM to find solution for ADHD

Attention-Deficit/Hyperactivity Disorder (ADHD) is not only the most common neuro-developmental disorder of childhood today, but also the most studied. Literature reviews report very different prevalence estimates. The DSM-IV states that the prevalence of ADHD is about 3–5% among school-age children [American Psychiatric Association, 1994]. Some of consequences of untreated ADHD children are social skills deficits, behavioral disinhibition and emotional skills deficits. Therefore, early diagnosis of ADHD is very important. The purpose of this paper is the comparison of application of COBFCM and COBNCM to identify the risk groups. When data is an unsupervised one and based on experts’ opinions and there is uncertainty in the concepts, COBFCM is the best option, and when data is an unsupervised one and there is indeterminacy in the concepts, COBNCM is a preferred method. The comparison of these methods clarifies this fundamental point and the relationship of to-be-determined and not-to-be-determined between the concepts, including the effect on results in casual models in psychological research.
Based on experts’ opinions (five child and developmental psychologists) and the corresponding literature, we determined eight cognitive concepts related to ADHD:

1. C₁: Mother’s harmful substance use;
2. C₂: Mother’s low physical self-efficacy;
3. C₃: Mother’s bad nutrition;
4. C₄: Mother’s depression;
5. C₅: Family conflict;
6. C₆: Father’s addiction;
7. C₇: Child’s emotional problems;

9 Combined Overlap Block NCM

We divide these concepts in to 3 equal length classes; each class has just four concepts in the following manner:

S₁={C₁,C₂,C₃,C₄}, S₂={C₂,C₄,C₅,C₆} and S₃={C₄,C₅,C₇,C₈}

These three classes are offered to experts in order to determine relationships and the strength. In addition, we asked them to delineate edges that have indeterminate effects by dotted lines in the figures and by I in the corresponding matrices. The directed graph and relation matrix for the S₁, S₂ and S₃ given by the expert is as follow:

Figure 1

Figure 2

Figure 3

The combined overlap block connection matrix of NCM is given by E(N).
The combined overlap block connection matrix of FCM is given by E.

\[
E(N) = \begin{bmatrix}
0, 0, 0, 1, 0, 0, 0, 0 \\
0, 0, 0, 2, 0, 0, 0, 0 \\
1, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 2, 0, 0, 1, 1 \\
0, 0, 0, 1, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 1, 0 \\
0, 0, 0, 0, 0, 0, 1, 0
\end{bmatrix}
\]

10 Hidden Patterns

Now, using the combined matrix E(N), we can determine any hidden patterns embedded in the matrix. Suppose the concept \( C_4 \) (Mother’s depression) is in the ON state. So, initial vector for studying the effects of these concepts on the dynamical system E is \( A = [0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0] \). Let A state vector depicting the ON state of Mother’s depression passing the state vector A in to the dynamical system E(N):

\[
A = [0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0] \]

\[
AE(N) = [0, 0, 1, 0, 1, 1, 0, 1] \rightarrow [0, 0, 1, 1, 1, 0] = A_1
\]

\[
A_1E(N) = [1, 0, 1, 2*I^2 + 3, I^2 + 1, I, 2, 2] \rightarrow [I, 0, 1, 1, 1, 1, 1] = A_2
\]

\[
A_2E(N) = [I, I^2, 2*I^2 + I + 3, I^2 + 1, I, 3, 2] \rightarrow [I, I, 1, 1, 1, 1, 1] = A_3
\]

\[
\]
Since $A_4 = A_3$ (we have reached the fixed point of the dynamical system), $A_3$ is determined to be a hidden pattern. Now again using the COBFCM we can determine hidden patterns embedded in the matrix $(E)$, such as COBNCM, here initial vector considered $A= [0 0 0 1 0 0 0 0]$, i.e. we suppose the Mother’s depression is high. The results obtained are as following:

$$AE=[0 0 0 1 1 0 0 0] = A_1$$

$$A_1E=[0 0 0 1 1 0 1 0] = A_2$$

$$A_2E=[0 0 0 1 1 0 1 1] = A_3 = A_2$$

By $A_3 = A_2$ we have reached the fixed point of the dynamical system. $A_2$ is determined to be a hidden pattern using the COBFCM.

11 Weighted Method

We can use the weighted method to clarify the results, when there is a tie between the concepts inputs. Suppose the resultant vector be $A= [10 0 1 1 1 0]$, i.e., the half of the concepts suggest that the given problem exists, but other three suggest that the problem is not justified on the basis of available concept. In this case, we can adopts a simple weighted approach where in each of the concepts can be assigned weights based on experts’ opinions. For example, $C_1=20\%$, $C_2=10\%$, $C_3=10\%$, $C_4=60\%$, $C_5=25\%$, $C_6=30\%$, $C_7=20\%$. The ON-OFF state for each Concept in $A$ vector leads to a weighted average score of the corresponding concepts. Suppose the initial vector is $A= [0 0 0 0 0 1 0]$, based on the resultant vector and the experts’ weights for the concepts, we can find a weighted average score. In this case, Geometric mean is an accurate and appropriate measure for calculating average score, because the data are expressed in percentage terms. The resulting of the example equals to 30\% (which tends towards absence of the problem (since this is <50\%, the point of no difference).

The results based on the COBNCM indicated when a mother suffering from depression, i.e. the $C_4$ is in the ON state; there will be family conflict, child’s emotional problems, Child’s hyper activity and also there may be Mother’s harmful substance use, Mother’s low physical self-efficacy, Mother’s bad nutrition and Father’s addiction. Based on the results of this study using the COBFCM, when a mother is depressed, there will be child’s hyperactivity, emotional problems, and family conflict. Although, based on the results of the two models mother’s depression being the main cause of ADHD, based on the COBFCM we cannot determine the occurrence of possibilities of some corresponding concepts in developing ADHD.
12 Discussion

It is important to note that in COBFCM $e_{ij}$ measures only absence or presence of influence of the node $C_i$ on $C_j$, but until now any researcher has not contemplated the indeterminacy of any relation between two nodes $C_i$ and $C_j$. When researchers deal with unsupervised data, there are situations when no relation can be determined between two nodes (Vasantha Kandasamy & Smarandache, 2005). The presence of I in any coordinate implies the expert cannot tell the presence of that node, i.e. on state after passing through N (E), nor can we say the absence of the node, i.e. off state - the effect on the node after passing through the dynamical system is indeterminate, so it is represented by $I$. Thus, only in case of NCM we can identify that the effect of any node on other nodes can also be indeterminate. Such possibilities and analysis is totally absent in the case of FCM. Therefore, the COBFCM only indicates that what happens for $C_j$ when $C_i$ is in an ON state, but it cannot indicate the effects of the concepts on each other in neutral states. In other words, by using COBFCM, some of the latent layers of the relationships between the concepts are not discovered. Thus, only the COBNCM helps in such conditions.

The core of psychology and education is theoretical. Theories themselves consist of constructs, concepts and variables, which are expressed by linguistic propositions - to describe, explain and predict the phenomena. For these characteristics of theory, Smarandache (2001) believes that no theory is exempted from paradoxes, because of language imprecision, metaphoric expression, various levels or meta-levels of understanding/interpretation, which might overlap. These propositions do not mean a fixed-valued components structure and it is dynamic, i.e. the truth value of a proposition may change from one place to another place and from one time to another time, and it changes with respect to the observer (subjectivity). For example, the proposition "Family conflict leads to divorce" does not mean a fixed-valued components structure; this proposition may be stated 35% true, 45% indeterminate, and 45% false at time $t_1$; but at time $t_2$ may change at 55% true, 49% indeterminate, and 32% false (according with new evidences, sources, etc.); or the proposition "Jane is depressed " can be (.76, .56, .30) according to her psychologist, but (.85, .25, .15) according to herself, or (.50, .24, .35) according to her friend, etc. Therefore, considering the indeterminacies in investigating the causal relationships in psychological and educational research is important, and it is closer to the human mind reasoning. A good method in this condition is using the NCM, as seen before, using the FCM leads to ignoring indeterminacies (by converting the $e_{ij}=I$ to $e_{ij}=0$), and this ignoring itself leads to the covering the latent effects of the concepts of the causal models. It is recommended that in the conditions that indeterminacies are important, researchers use the NCM method.
13 References


A Comparison of COBFCM and COBNCM; Psychological Causal Models


