Polignac conjecture is true for every n=p+1 where p is the lower of a pair of twin primes

Diego Liberati
Consiglio Nazionale delle Ricerche

Good candidates to be pairs of consecutive primes with gap n=p+1 are the pairs in the set [k(p#)-p k(p#)-1] where p is the lower of a couple of twin primes, p# its primorial and k a multiplying natural: in fact:

- neither prime up to p could sieve either one in the pair
- each odd in between is instead sieved by one of the primes up to p.

Not all the primes with gap n need to belong to such set, nor all the pairs in such set are pair of primes, but it is sufficient to prove that infinitely many of them are both primes.

For that purpose, we need that both k(p#)-p and k(p#)-1 are primes for infinitely many k.

In order not to be primes, they need to be sieved by at least one prime q greater than p.

In fact, every prime q higher than p does sieve one of such candidate pairs, in at least one of its two positions, twice every q candidates, thus not sieving q-2 pairs every q candidates.

Then, in order to compute how many candidate pairs are not sieved by any of such infinitely many primes greater than p, one needs to compute the product of the infinitely many fractions q-2/q over all the primes q greater than p.

When q tends to the infinity, both the numerator and the denominator of such product tend to the infinity with the same strength (even if their fraction tends to be infinitesimal towards zero, quite slowly indeed, because q-2/q tends to increase toward 1 when q increases).

Thus, when the denominator tends to the infinity, also the numerator does, proving that there are infinitely many pairs of primes with gap n=p+1 not sieved by any lower prime, then proving the conjecture.