Nonlinear Analysis in Radio Astronomy

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Abstract:
One of the exciting prospects in radio astronomy is the observation of the ‘sounds’ of the universe, owing to the fact that radio signals recorded largely falling within the audible frequency range. To this end, the present work purports to the proposal and implementation nonlinear analysis of such radio astronomical data, using qualitative techniques such as time series, spectrum, bispectrum, phase portrait, Poincare section, polar plot, spatiotemporal analysis, wavelet analysis and distance plots, as well as quantitative analysis techniques including the Lyapunov exponent, Kolmogorov entropy and fractal dimension. The analyses results are illustrated for a select assortment of the radio astronomical signals. The analysis techniques and results presented here, coupled with theory backed rigorous investigations into the astronomical mechanisms may serve as a valuable tool for unlocking the secrets of the universe.

Keywords: Radio Astronomy, Nonlinear Analysis, Signals, Chaos Theory

1. Introduction
Radio Astronomy, a subfield of astronomy studies celestial objects using radio frequency signals, and has resulted in innumerable discoveries of the presence and nature of several stars, galaxies, quasars, pulsars and evidences for Big Bang Theories such as the Cosmic Microwave Background [1,2]. Thus, it is no surprise that this field has its own amateur technologies and communications being developed over the years [3]. Owing to the frequencies of radio electromagnetic waves falling within the audible range (20Hz-20kHz), it is possible to hear the ‘sounds’ of several celestial objects by playing the received signals on a speaker [4].

The nature of the received radio signals reveals key information about the concerned celestial object and the underlying mechanisms [1-2]. It is for this purpose that characterization of these signals plays a crucial role. This forms the motivation for the present work, where a set of tools and techniques, collectively termed “Nonlinear Analysis” is performed to a selection of radio astronomical data corresponding to various aspects such as geomagnetic activity, Jovian and Saturn activity, solar flares and so on. The quantitative analysis, performed using Kolmogorov Entropy, Lyapunov Exponents and Fractal Dimension is tabulated. It is seen that almost all the signals are indicative of underlying nonlinearity and chaos in the emission mechanisms. This information, coupled with thorough investigations into the astronomical theory, potentially reveals an enormous amount of information pertaining to radio astronomy.
2. Nonlinear Analysis - Methodology

In this section, various aspects of the nonlinear analysis techniques used in the present work are briefly presented. In general, the techniques are grouped into two categories – qualitative and quantitative, as listed below:

**Qualitative Techniques**

1. **Time Series**: Here the obtained signal is plotted as a function of time. Apart from the general ‘ups’ and ‘downs’, the time series analysis enables one to discern the presence of peaks, increases and decreases in the signal amplitude [5].

2. **Spectrum**: Plotted using the Fast Fourier Transform, the Spectrum highlights periodicities and other repetitive elements in the time series data [5].

3. **Bispectrum**: In order to study the effects of frequency mixing and other related nonlinearities such as intermodulation and cross modulation, the bispectrum is plotted. The bispectrum is the spectrum obtained from the third order cumulant, and for any two frequencies f1 and f2, gives the corresponding components as well as the coupled components of the form f1+f2 [6].

4. **Phase Portrait**: This is a plot of time derivative of the signal in terms of the signal, illustrating the phase space dynamics and qualitatively serving as a tool to assess sensitivity and ergodicity. The detection of ornamental and rich patterns in a phase portrait is a clear indicator of the presence of chaos [7].

5. **Poincare Section**: This involves plotting a signal with a delayed version of itself, and similar to the phase portrait, indicates the phase space dynamics of the signal along with a visualization of its autocorrelation properties [7].

6. **Polar Plot**: Plotted as a function of magnitude and phase, the polar plot helps to understand the component-wise and collective phase distributions in the signal, distinguishing between the noise floor and chaotic components present therein [5].

7. **Spatiotemporal Propagation**: This plot is obtained as a contour illustrating the propagation of the time series signal across a 200-cell lattice with the spatial rule given as a difference profile \( c(i)=|c(i-1)-c(i-2)| \). This plot shows how the energy of the signal disperses with space.

8. **Distance Map**: The main premise in the concept of distance plot is that most natural processes possess recurrent behavior in the form of periodicities and irregular cyclicities. Here, a recurrence is defined as a condition where states in the system are arbitrarily close after some time of divergence. On this concept, the distance plot (DP) is defined for a discrete signal with N samples denoted by \( x(n), n<=N \) as the distance between the ith and jth point \( D(i,j) \) is given by \( D(i,j) = \|x(i)-x(j)\| \). The collection of all the distance points \( D(i,j) \) for all \( i,j<N \) form the Distance Matrix \( D \), a plot of which is termed the Distance Plot (DP) [8].

9. **Wavelet Analysis**: Comprising of the shift-scale contour plot obtained by matching various scaled versions of a compact mother wavelet signal with the time series signal, the wavelet analysis enables the identification of bursts and discontinuities in the signal. In the present work, the mother wavelet is obtained by setting the father wavelet, or the scaling function to a hyperbolic secant function (sech), whose smooth and compact nature yields to effective detection of burst type signals [9].
### Quantitative Techniques

1. **Largest Lyapunov Exponent (LLE):** Denoting the sensitive dependence of the system to initial conditions, the LLE is a definite and assertive test of chaos\(^7\). In the present work, the Rosenstein’s algorithm is used to calculate the Lyapunov exponent from the time series [10,11].

2. **Fractal Dimension (D2):** The Fractal Dimension (D2) is calculated using the Minkowski Bouligand Box Counting Algorithm and indicates the dimensionality of the chaos; higher the dimensionality, higher the instability [12].

3. **Kolmogorov Entropy (K2):** The Kolmogorov entropy (K2), measured in information units of nats per symbol denotes the entropy and thus the uncertainty present in the signal, and large values indicate more dynamic and unpredictable behavior [10].

### 3. Results and Discussion

Using the nonlinear analysis techniques listed above, the analysis of radio astronomical data obtained from the Luxorion project website (http://www.astrosurf.com/luxorion/qsl-audiofiles.htm) is performed. The quantitative analyses results for a select assortment of the signals are tabulated as follows:

<table>
<thead>
<tr>
<th>Description</th>
<th>K2 (nats/symbol)</th>
<th>D2</th>
<th>LLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jupiter hiss recording at 25.55 and 25.67 MHz by Altaïr</td>
<td>0.968</td>
<td>0.79</td>
<td>0.12</td>
</tr>
<tr>
<td>Lightning in Jupiter's atmosphere recorded by plasma wave instrument onboard Voyager's spaceprobes</td>
<td>0.886</td>
<td>0.66</td>
<td>-0.61</td>
</tr>
<tr>
<td>Jovian electron cyclotron emission</td>
<td>1.008</td>
<td>0.99</td>
<td>7.66</td>
</tr>
<tr>
<td>Mixed of Jupiter L-burst and Io B-storm recorded in stereo on March 7, 2001 at 0247 UTC by Thomas Ashcraft</td>
<td>0.928</td>
<td>0.90</td>
<td>-0.21</td>
</tr>
<tr>
<td>The sound of Saturn rotation, recorded over a five day interval, from 2-7 June 2004 during Cassini-Huygens approach to Saturn. These low frequencies radio emissions called Saturn Kilometric Radiation (SKR) are generated by charged particles whose motions are controlled by the planetary magnetic field.</td>
<td>0.896</td>
<td>0.85</td>
<td>23.49</td>
</tr>
<tr>
<td>Cosmic background noise recorded in the hydrogen line on 1420.40575 MHz. Recorded in the framework of SETI project Argus.</td>
<td>0.843</td>
<td>0.79</td>
<td>-0.47</td>
</tr>
<tr>
<td>Earthquake Recording</td>
<td>1.026</td>
<td>0.66</td>
<td>6.94</td>
</tr>
<tr>
<td>A type III solar burst on November 4, 2003 by Cassini spaceprobe, produced by very energetic (1 to 100 keV) electrons emitted by a solar flare.</td>
<td>0.883</td>
<td>0.95</td>
<td>0.23</td>
</tr>
<tr>
<td>&quot;Saucers&quot; recorded by Dynamics Explorer spacecraft.</td>
<td>1.173</td>
<td>0.66</td>
<td>0.29</td>
</tr>
<tr>
<td>Dawn chorus with evident magnetic field micro-pulsations (undulations in the chorus trains).</td>
<td>1.03</td>
<td>0.66</td>
<td>2.25</td>
</tr>
</tbody>
</table>

It is seen from the table that some of the signals such as the Saturn rotation, Jovian electron cyclotron and Earthquake recordings show highly chaotic nature, as seen by the positive LLE value. The nonlinear analysis results including qualitative and quantitative analyses are plotted in Fig. 1-9.
4. Conclusion
The present work proposes and implements nonlinear analysis of radio astronomical data, using both qualitative and quantitative techniques. The analyses results are illustrated for a select assortment of the radio astronomical signals. It is hoped that the analysis techniques and results presented here, coupled with theory backed rigorous investigations into the astronomical mechanisms may serve as a valuable tool for unlocking the secrets of the universe.

Figures

Figure 1 Nonlinear Analysis of Jupiter Hiss

Figure 2 Nonlinear Analysis of Jovian Lightning
Figure 3: Nonlinear Analysis of Jovian Electron Cyclotron emission

Figure 4: Nonlinear Analysis of Saturn Rotation

Figure 5: Nonlinear Analysis of Cosmic Background Noise
Figure 6 Nonlinear Analysis of Earthquake

Figure 7 Nonlinear Analysis of type III Solar Burst

Figure 8 Nonlinear Analysis of Saucers
Figure 9 Nonlinear Analysis of Dawn Chorus

References