Abstract
Radio astronomy plays a crucial role in revealing vital information about the early universe and celestial structures. A novel nonlinear analysis technique using the polar and phase plane plots for analyzing radio astronomical data is proposed, and the analysis for a pulsar B1933 is performed. It is observed that the analysis reveals information regarding the angle of emissions/beaming of 30 degrees, and this forms the novelty of the present work.

Keywords: B1933 Pulsars, Nonlinear Analysis, Polar Plots, Phase Portrait

1. Introduction
Of the very many disciplines of science, astronomy occupies a significant niche, mostly due to the fact that astronomy helps to validate some of the most crucial theories of general physics [1, 2, 3]. Many experiments have been conducted and many space missions have been launched to collect data from the stars, comets, black holes and other celestial bodies [4, 5]. The fact that these celestial objects emit radiation almost in every region of the electromagnetic spectrum is an added advantage [1, 2, 3, 4, 5]. Radio astronomy deals in particular with those radiations falling in the audible range of frequencies (20Hz to 20kHz) [6, 7, 8]. These data, varying in the millisecond range, can be easily converted to audio signals and the ‘sound’ of our universe can be recorded [9, 10, 11].

Over the years of scientific and technological advancement, numerous signal analysis techniques have emerged [12]. The right choice of data analysis technique depends heavily on the data and the type of information one is looking for. In this context, radio astronomical data is complex [13]. It is rich, varied, carries a lot of complexity and often is shrouded in background ‘noise’ [14]. A lot of factors ranging from the celestial object properties to the atmospheric variations in earth affect the data received [6, 7, 8, 9, 10, 11]. It is in such situations that nonlinear data analysis is significant [15, 16]. Here the phase of the signal has an equal footing as the amplitude, and this helps to extract a lot of information, most of which cannot be achieved with conventional linear data analytic techniques [17, 18].

The present work purports to the nonlinear data analysis obtained from experimental observation of pulsars, and after deriving various plots from the data, uncovers fresh information with regards to the pulsar and the way it emits radiations [19, 20, 21]. The results obtained from the present work could reveal hitherto obscure beaming and phase related information about the pulsar. Though the analysis techniques mentioned here are no substitute for a rigorous theoretical study taking into account all the relativistic and astronomical principles, the proposed nonlinear analysis does play a humble role in expanding mankind’s understanding of the universe.

2. Pulsars - An Overview
Pulsars appear as earth-sized neutron stars, extremely dense with a mass comparable to that of the sun [19]. Emitting Short pulses of radiation between 1.4 ms and 8.5s, pulsars are the ideal candidates for radio astronomy data analysis [19, 20]. The mean density $\rho$ of a pulsar satisfies the relation

$$\rho > \frac{3\pi}{G P^2}$$  \hspace{1cm} (1)

where G is the gravitational constant and P is the rotation period [20]. The study of pulsars and their emission properties is significant for the following reasons [20]:

1. Neutron stars exhibit deep gravitational potentials combined with extremely high densities (often exceeding those of the atomic nucleus) and very high magnetic field strengths of the order of $10^{14}$ Gauss.
2. The temporal data of a pulsar reveals vital information such as gravitational perturbations from planetary mass objects orbiting it.
3. The magnetosphere of a pulsar extends to the light cylinder, thus acting as a platform to observe relativistic effects. The emission mechanism of a pulsar is illustrated in Fig. (1).

![Figure 1: A simplified illustration of the Emission Mechanism of a Pulsar with respect to the rotation and magnetic axes](image)

The mechanism of signal emission in a pulsar can be conceptually outlined as follows. Firstly, the spinning magnetic dipole of a neutron star acts as a magnetic unipolar generator. Charges in the magnetic equatorial region redistribute themselves by moving along closed field lines. In due course, they build up electrostatic field large enough to cancel the magnetic force with the voltage induced at about 1MV. But the co-rotating field lines emerging from the polar caps cross the light cylinder and these field lines cannot close. Electrons in the polar cap are magnetically accelerated to very high energies along the open but curved field lines, where the acceleration resulting from the curvature causes them to emit curvature radiation that is strongly polarized in the plane of curvature. As the radio beam sweeps across the line-of-sight, the plane of polarization is observed to rotate by up to 180 degrees, a purely geometrical effect. High-energy photons produced by curvature radiation interact with the magnetic field and lower-energy photons to produce electron-positron pairs that radiate more high-energy photons. The final results of this cascade process are bunches of charged particles that emit at radio wavelengths [20, 21, 22].

Building on the above explained pulsar mechanism, extensive analytical studies have been performed to understand the theory behind pulsar emission and propose models for the same, with some of this work even substantiated by measurement data [23, 24, 25, 26]. However, the present work takes a different approach. The present work uses methods well established in the fields of nonlinear analysis, such as phase portraits, bispectrum and polar plots, and apply these techniques to radio astronomical data of pulsars. In order to highlight the potential capabilities of such nonlinear analysis, the present work focuses on those results alone, although corroboration and correlation with the above mentioned models would enrich the understanding of pulsar mechanism even further. Additionally, it is also highlighted that the platform of nonlinear analysis and signal processing used in the present work contains capabilities not only for pulsar data analysis, but also as a general tool for analyzing any radio astronomical data without significant loss of generality.

3. Nonlinear Data Analysis of the Pulsar Signal

The present work purports to the nonlinear data analysis of the radio astronomical data of the pulsar B1933+16, obtained from [27]. This pulsar has a period of approximately 359ms. The time series data is obtained and the waveform plotted is shown in Fig. (2). A zoomed in version of the waveform plot is shown in Fig. (3), which helps to understand the shape of the pulse emitted.

As evident from the time series waveform, the pulsars pulses have a Gaussian temporal profile which are distinguishable from the noise floor [28]. This is also seen from an exponentially tapering spectral profile in the spectrum plot, as shown in Fig. (4).

In order to understand the delicate phase relationships between the various frequency components of the pulsar signal, the bispectrum is plotted, and is shown in Fig. (5). The bispectrum is the spectrum obtained from the third order cumulant, and for any two given frequencies \( f_1 \) and \( f_2 \), the bispectrum of a signal displays the frequency components \( f_1 \) and \( f_2 \) and the cross coupling components \( f_1 + f_2 \) [12].
Figure 2: The Time Series Waveform of the B1933+16 Pulsar, showing the presence of discernible emission pulses.

Figure 3: Time Series Waveform of the B1933+16 Pulsar - Zoomed In. The Gaussian profile of the emitted pulses can be seen.

Figure 4: Spectral Profile of the B1933+16 Pulsar. The exponentially tapering spectral profile is in agreement with the Gaussian temporal profile.

Figure 5: Bispectrum of the B1933+16 Pulsar. The magnitude bispectrum shows the presence of cross harmonic products, whereas the phase bispectrum shows an interesting pattern being formed.
The above plots give a significant indication of the nature of the pulses emitted from the pulsar. In order to study the complexity of the pulsar data, a phase portrait is plotted in Fig. (6).

Figure 6: Phase portrait of the B1933+16 Pulsar. Closed loops depicting ergodic behavior can be seen, attesting to the presence of chaos.

As is seen from the phase portrait, the pulsar data forms a discernible pattern amidst the background noise, and these patterns are characterized by closed loops displaying ergodic behavior. This indicates the presence of chaos, though the reasons for chaos and underlying mechanism can be revealed only by a thorough theoretical study of the pulsar mechanism. This can be further ascertained by plotting the Minkowski Bouligand Fractional dimension computed using the box counting method [29]. In this method, various square ‘boxes’ of different sizes $e$ are formed and for each size $e$, the number of boxes $N(e)$ required to cover the entire set is computed. The fractal dimension $D$ is then given by

$$D = \lim_{e \to 0} \frac{\log(N(e))}{\log(e)}$$

(2)

A non integer fractional dimension obtained in the range of 0.65 attests to the presence of chaos in the pulsar data. The chaotic nature of the signal $A(t)$ is assertively established by calculating the largest Lyapunov Exponent, a measure of a system’s sensitive dependence on initial conditions [30, 31]. Rosenstein’s algorithm is used to compute the Lyapunov Exponents $\lambda_i$, where the sensitive dependence is characterized by the divergence samples $d_j(i)$ between nearest trajectories represented by $j$ given as follows, $C_j$ being a normalization constant:

$$d_j(i) = C_j e^{\lambda_i(i\delta t)}$$

(3)

The Largest Lyapunov exponent thus obtained for the chaotic signal $A(t)$ is 4.551 [30, 31]. Finally, to understand the arrival times of the pulsar beams, the polar plot is plotted. Mathematically this can be obtained by converting the Cartesian coordinates of Time and Amplitude to the Polar coordinates of Magnitude and Angle. The polar plot of the pulsar data is as shown in Fig. (7).

Figure 7: Polar Plot of the B1933+16 Pulsar. The ‘peaks’ observed in certain angles may correspond to the emission angles of the pulsar.

The polar plot indicates certain angles of beaming, and a distinction between the beams and the noise floor. Since the magnitude and angles of the polar plot are obtained from the time and amplitude values, the angles at which the beaming occurs are directly related to the position within the pulsar cycle (358ms), at which the beams are received on earth. In other words, the angles of beaming are related to the arrival times of pulsar pulses. These arrival times would correlate with the angles of rotation at which the pulsar emits the radiation, since radiation from different angles correspond to different arrival times [9, 10, 11, 19, 20, 21]. Thus the polar plot has identified the angles of rotation at which the pulsar emits the radiation. This is a feature that is not directly available from the magnitude based analyses techniques outlined...
earlier. The various angles of beaming would then correspond to the various spatial harmonic components contained in the pulsar signal [32].

4. Conclusion

Using various nonlinear data analysis techniques, the radiated pulse characteristics and the pulsar emission characteristics are studied. The use of the polar plot as a tool to understanding the phase-related information directly indicating pulsar emission angles forms the novelty of the present work. Future steps in this direction include development of a detailed analysis regarding the polar plot phase data and correlations of such data with the angular emission properties of multiple pulsars. Nevertheless, the present work takes a small yet significant step in harnessing the power of nonlinear signal processing to reveal vital astrophysical data.

References