An Equivalent Circuit Modeling Self Heating Induced InterModulation Distortion in Microwave Interconnects

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Abstract
Self-heating is an undesirable phenomenon, causing intermodulation distortion in RF frequencies. The theories pertinent to passive electrothermal theory are studied, and the phenomenon giving rise to self-heating is represented as an RC equivalent circuit. After validation of the model with established experimental observations, the model is applied to various microwave transmission line configurations (coplanar and microstrip) considering various material configurations. After extracting certain useful inferences, the model is applied to smart materials such as Titanium Dioxide.

Keywords: Self Heating, Intermodulation Distortion, Equivalent Circuit, Microwave Interconnects, Smart Materials

1. Introduction
As the frequency of operation increases, especially in the Radio Frequency (RF) and microwave range (MegaHertz - GigaHertz), current that flows through resistive elements in integrated circuits (IC) causes collision of charge carriers, resulting in an increase in the temperature of the IC [1]. This phenomenon, known widely as self-heating has become of late, one of the hot topics of the electronics industry [1, 2].

The physics of self-heating can be given as follows: when a current at RF frequencies passes through a resistive element, collision of charge carriers occurs that causes a change in the temperature of resistive element. This is independent of ambient temperature and hence, appropriately called ‘self’ heating [3]. This heating causes a change in resistivity, which then affects the time constants of the model, and hence causes undesired frequency components to appear in the output. This is an undesired effect because of appearance of undesired frequency components, and self heating effects become more prominent as the devices are scaled down, especially towards 90nm and smaller technology nodes [1, 3, 4].

The problem of self heating can be accounted for, by coupling thermal and electrical domains and then developing a comprehensive model [5, 6, 7]. In this light, the present work purports to the development of an equivalent circuit representation of the self-heating effect in interconnects such as microstrips and coplanar waveguides and their contribution to the intermodulation distortion [8]. SPICE implementations of the model is then validated for the accuracy and reliability using experimental results obtained from literature, and intermodulation distortion for typical microstrip structures are studied [3, 5]. Finally, using the developed equivalent circuit model, self-heating effects for smart materials such as Titanium Dioxide (TiO$_2$) and Zinc Oxide (ZnO) microstrip structures are explored [9, 10, 11]. The results obtained in the paper facilitate a computationally efficient equivalent circuit model based study of one of the most prominent problems in semiconductor technology - self-heating induced intermodulation distortion.

2. Interconnects and Transmission Lines
Typically, in VLSI circuits, in order to minimize chip size, one often goes for multilayered interconnects where, there are various metal layers one above the other, separated with insulators in between [12, 13]. These interconnects can be modeled using standard RLC based transmission line models [14].

The transmission line models used most commonly to represent interconnects are the coplanar model (within a metal layer), and the microstrip model (between two metal layers of different heights) [14, 15]. The model parameters used in this work are given in Fig. (1).

3. Equivalent circuits of transmission line models
Both coplanar and microstrip models can be modeled using passive RLC (resistor inductor capacitor) elements as shown in Fig. (2). The values of R, L and C are obtained from the device dimensions, permittivity values and
characteristic impedance using empirical relations [14, 15]. For instance, the L and C values of a microstrip structure are obtained as follows [14, 15, 16]:

\[
Z_0 = \frac{87}{\sqrt{\varepsilon_r + 1}} \ln\left(\frac{5.98H}{0.8W + T}\right)
\]  

(1)

\[
C = \frac{2.64 \times 10^{-11}(\varepsilon_r + 1.41)}{\ln\left(\frac{5.98H}{0.8W + T}\right)}
\]  

(2)

\[
L = C Z_0^2
\]  

(3)

Here \(H\) denotes substrate height, \(T\) denotes conductor thickness, \(W\) denotes conductor width. The models used are lossy line models, including the resistive losses given by \(R = \rho l/A\) where \(\rho\) is the resistivity and \(l\) and \(A\) denote the length and cross sectional area of the conductor [17]. It is in this equivalent resistor that self heating plays a vital role.

4. Two-tone test and Intermodulation Distortion

Typically, to test for nonlinear effects in line models, certain test signals are used. One such signal that is commonly used is the two-tone signal which is the sum of 2 sinusoids of different frequencies [1, 8]. This is represented as

\[
x(t) = a_1 \cos(2\pi f_1 t) + a_2 \cos(2\pi f_2 t)
\]  

(4)

The present work uses this 2 tone signal with the frequency of separation \(f_2 - f_1\) ranging from tens of Megahertz to tens of Gigahertz. When the 2 tone signal is passed through a nonlinear model a wide range of frequencies, created by the sum and difference of the fundamental frequencies and their harmonics are formed.
Hence if the input tones are defined by \( f_1 \) and \( f_2 \), a third order nonlinearity yields \( f_1, f_2, 2f_1, 2f_2, f_1 - f_2, f_1 + f_2, 3f_1, 3f_2, 2f_1 - f_2, 2f_1 + f_2, 2f_2 - f_1 \) and \( 2f_2 + f_1 \). Out of these components, all frequencies except \( f_1, f_2, 2f_2 - f_1 \), and \( 2f_1 - f_2 \) are called out-of-band products and can be easily filtered out. The in-band frequencies are \( f_1, f_2, 2f_1 - f_2 \), and \( 2f_2 - f_1 \). Out of these \( f_1 \) and \( f_2 \) are the desired output frequencies. The distortion caused by the remaining frequencies \( (2f_1 - f_2, \text{ and } 2f_2 - f_1) \) are called ‘InterModulation Distortion’ (IMD) \([1, 3, 8]\). IMD is the most critical form of distortion as these frequencies can neither be filtered out nor be ignored. It is the Third Order IMD (IMD3) i.e \( 2f_2 - f_1 \) and \( 2f_1 - f_2 \) that cause much of the problem with regards to self heating.

5. Electro-Thermal Theory of Self-Heating

It is well established in literature that self heating causes InterModulation Distortion and for passive components such as interconnects this is called ET-PIM (electro-thermal passive intermodulation distortion). The concept is explained in the by Wilkerson et al. and is outlined briefly as follows \([3]\). The collision of charge carriers in a resistive element causes change in temperature and this change is periodic, with a baseband range. When a 2 tone input signal is given as input, the power spectrum consists of the sum \((f_1 + f_2)\) and the difference \((f_1 - f_2, \text{ also called envelope or beat frequency})\). If the beat frequency happens to fall in the thermal baseband range, the thermal effects become prominent, periodically varying the resistance. In effect, this creates a parasitic passive mixer effect producing intermodulation distortion through upconversion of the envelope frequencies at baseband to RF frequencies. Thus, these frequencies arising from the IMD3 (third order intermodulation distortion) \([3]\).

6. Equivalent Circuit model of self-heating

The principal objective of the present work is to obtain a equivalent circuit model describing the IMD obtained using self-heating. As a starting step, the temperature dependent resistivity is written as a power series as follows \([9, 10, 11, 17]\):

\[
\rho(T) = \rho_0(1 + \alpha T + \beta T^2 + \ldots)
\]

where \( \rho_0 \) is the base resistivity and \( \alpha \) and \( \beta \) are first and second order temperature coefficients of resistance. By incorporating this power series into the expression for heat generation given by \( Q=J^2\rho \) where \( J \) denotes the current density, a differential equation describing the heat conduction is obtained as follows \([3]\):

\[
\nabla.(\frac{\nabla(\rho(T))}{\Delta T}) - C_v \frac{\partial T}{\partial t} = J^2[\rho_0(1 + \alpha T + \beta T^2 + \ldots)]
\]

(6)

Here \( C_v = c_v\rho_0 V \) where \( c_v \) is the volume specific heat capacity of the conductor, \( \rho_0 \) is the density and \( V \) is the conductor volume. Using fractional calculus based procedures, Wilkerson et al. obtained a solution to the heat conduction equation describing temperature evolution of the following form \([3, 18]\):

\[
T = Ae^{-kx}\cos(\omega t - kx); k = \sqrt{\omega/2\kappa}
\]

(7)

where \( \kappa \) denotes the thermal diffusivity of the conductor.

It is noteworthy that the temperature evolution describes a low-pass filtering baseband effect consistent with the earlier description \([3, 5]\). This low-pass effect is represented as an equivalent circuit of a ‘thermal resistance’ \( R_{th} \) and a ‘thermal capacitance’ \( C_{th} \) in parallel as shown in Fig. (3), \( Ta \) representing ambient temperature. The corresponding expressions for the thermal R and C values are obtained as follows:

\[
R_{th} = \frac{1}{\kappa C_v}
\]

(8)

\[
C_{th} = C_v R_{th}
\]

(9)

7. Transmission line models including self-heating effects

The transmission line models described in Fig. (2) coupled with the self heating equivalent circuit of Fig. (3) are implemented using PSPICE simulations, for a Microstrip made of Aluminium SOI, with the 2 tones at \( f_1=600\text{MHz} \) and \( f_2=700\text{MHz} \). and the input and output waveforms are shown in Fig. (4). The Fourier spectrum of the output waveform is shown in Fig. (5).

As can be seen in Fourier analysis of output, there is significant amplitude of the desired components, 600 and 700 MHz (the 2 tones). In addition there are components at 800 and 500 MHz which are the IMD3 frequencies. There are components in other frequencies as well. For example, 400 and 900 MHz But these components are far apart from the desired frequency (600 and 700 MHz) and hence can be easily filtered out using appropriate band pass filters.
Figure 3: Equivalent Circuit representing IMD3 induced due to self-heating, showing the expressions for thermal resistance and capacitance.

Figure 4: Output (top) and input (bottom) waveforms of Aluminium microstrip equivalent circuit model with self-heating component included.

Figure 5: Output Spectrum showing IMD3.
8. Validation of the proposed model

The equivalent circuit model developed needs to be verified and checked for consistency. For this purpose, a the self-heating based IMD3 results obtained experimentally by Edouard Rocas et al. are considered [5]. In order to verify the present model, a simulation of the gold coplanar transmission line of the dimensions as used in the experiment (Fig. (6)) is performed [5].

![Figure 6: Geometry of the Gold Coplanar Waveguide used as the Benchmark](image)

The frequency separation (MHz) vs. IMD3 (dBm) of the simulated gold CPW model, shown alongside the corresponding curve obtained by Eduard Rocas et al (denoted as A-CPW) is shown in Fig. (7) [5].

![Figure 7: Overlaying of the Frequency-IMD3 curves of theoretical model and experiment on Gold coplanar waveguide](image)

The above curves assert without doubt the validity and accuracy of the proposed model, thus establishing it as a credible alternative to measuring the IMD3 values experimentally.

9. Modeling of BEOL Interconnects including self-heating effects

The next step is to model the self heating in typical IC back end of line (BEOL) interconnects [12, 13]. These are usually made of aluminium or occasionally tantalum which has a negative temperature coefficient of resistance [12, 13]. Thus the BEOL can be modeled as microstrip with the same geometry given earlier but with the conductor replaced by tantalum or aluminium. Five key cases are considered as follows:
Table 1: Frequency-IMD3 Obtained for various Interconnect Configurations

<table>
<thead>
<tr>
<th>Conductor Structure</th>
<th>Material Geometry</th>
<th>Tantalum Coplanar</th>
<th>Aluminium Coplanar</th>
<th>Tantalum Microstrip</th>
<th>Aluminium Microstrip</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-Heating Effect</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Frequency (MHz)</td>
<td>IMD3 (dBm)</td>
<td>IMD3 (dBm)</td>
<td>IMD3 (dBm)</td>
<td>IMD3 (dBm)</td>
<td>IMD3 (dBm)</td>
</tr>
<tr>
<td>0.001</td>
<td>-180</td>
<td>-35</td>
<td>-32.17</td>
<td>-78.36</td>
<td>-80</td>
</tr>
<tr>
<td>0.01</td>
<td>-180</td>
<td>-49</td>
<td>-43.67</td>
<td>-80.59</td>
<td>-82</td>
</tr>
<tr>
<td>0.1</td>
<td>-180</td>
<td>-63</td>
<td>-58</td>
<td>-83</td>
<td>-84</td>
</tr>
<tr>
<td>1</td>
<td>-180</td>
<td>-77</td>
<td>-61</td>
<td>-95</td>
<td>-100</td>
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<tr>
<td>10</td>
<td>-180</td>
<td>-100</td>
<td>-81</td>
<td>-118</td>
<td>-145</td>
</tr>
<tr>
<td>100</td>
<td>-180</td>
<td>-120</td>
<td>-103</td>
<td>-128.29</td>
<td>-150</td>
</tr>
<tr>
<td>1000</td>
<td>-180</td>
<td>-128.5</td>
<td>-123</td>
<td>-133</td>
<td>-167</td>
</tr>
</tbody>
</table>

1. Linear Case: Aluminium/Tantalum without thermal model, yielding lowest possible value of SNR (-180dB) for both Microstrip and Coplanar structures.
2. Tantalum Coplanar Structure including thermal model
3. Aluminium Coplanar Structure including thermal model
4. Tantalum Microstrip Structure including thermal model
5. Aluminium Microstrip Structure including thermal model

For each case, PSPICE simulations are performed and for frequency separation values from 1kHz to 1GHz, the IMD3 values are noted, and tabulated as shown in Table 1.

One infers the following important points from Table 1.

1. Self heating does have a significant impact on IMD as the IMD3 values of aluminium/microstrip show a significant increase when self heating is present.
2. There are anomalies in the trend at high frequencies (around 10 to 1000 MHz). But in this region aluminium microstrip shows much better performance than the tantalum counterpart.
3. Among the two geometries, microstrip structures with Aluminium conductors exhibit the lowest IMD3 values, followed by Tantalum Microstrips.
4. Though the temperature coefficient of Tantalum is negative, this does not drastically affect the IMD3 performance.
5. As predicted by the electro thermal theory [3], higher separation frequencies indeed result in lower IMD3 values, consistent with the fact that the thermal response, being a relatively slow process is more pronounced at baseband frequencies.

10. Extension to smart materials

The equivalent circuit model developed for self-heating can be readily extended to model thin-film coatings involving smart materials such as Titanium coated on TiO₂ (Rutile), and ZnO [9, 10, 11]. These materials find significant applications in photovoltaic cells, and with research reporting high amounts of inefficiency of solar cells (close to 50 percent) being attributed to thermal loss, the IMD3 obtained through self-heating can act as an instrument to understanding the thermal losses encountered and hence appropriate selection of material [9, 10, 11]. To start with, the self-heating model was applied to a microstrip formed by coating a 2um layer of Titanium on TiO₂, and the IMD3 was analysed and compared with values obtained earlier. The structure is as depicted in Fig. 8 and the dependence of IMD3 values on separation frequencies are illustrated in Fig. 9 for the smart materials vis-a-vis the aluminium and tantalum microstrips.

The most significant inference from Fig. 9 is that the IMD3 of the smart material configuration (Titanium conductor on Rutile substrate) shows significantly higher IMD3 values than the conventional materials. This is a clear assertion that in smart material based microwave transmission line structures and interconnects, self-heating plays a critical role in contributing to IMD3 and hence, the overall signal distortion. The results developed from this model could be utilized to design appropriate compensation circuits.

11. Conclusion

An equivalent circuit modeling self-heating effects and subsequent IMD has been designed and PSPICE simulations and validations are performed.
Figure 8: Tabulation of material properties and IMD3 values for select microstrip structures involving conventional and smart materials

<table>
<thead>
<tr>
<th>PROPERTY</th>
<th>ALUMINIUM</th>
<th>GOLD</th>
<th>TANTALUM</th>
<th>TITANIUM</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUBSTRATE</td>
<td>SILICA</td>
<td>SAPPHIRE</td>
<td>SILICA</td>
<td>RUTILE</td>
</tr>
<tr>
<td>LENGTH</td>
<td>1.00E-03</td>
<td>9.93E-03</td>
<td>1.00E-03</td>
<td>1.50E-02</td>
</tr>
<tr>
<td>WIDTH</td>
<td>6.00E-06</td>
<td>3.00E-05</td>
<td>6.00E-06</td>
<td>1.00E-02</td>
</tr>
<tr>
<td>THICKNESS</td>
<td>1.00E-07</td>
<td>4.80E-07</td>
<td>1.00E-07</td>
<td>2.00E-02</td>
</tr>
<tr>
<td>SUB HEIGHT</td>
<td>1.00E-04</td>
<td>2.00E-04</td>
<td>1.00E-04</td>
<td>5.00E-05</td>
</tr>
<tr>
<td>SUB PERMITTIVITY</td>
<td>3.90E+00</td>
<td>9.10E+00</td>
<td>3.90E+00</td>
<td>8.50E+01</td>
</tr>
<tr>
<td>VOLUME</td>
<td>6.00E-16</td>
<td>1.43E-13</td>
<td>6.00E-16</td>
<td>3.00E-06</td>
</tr>
<tr>
<td>IMD3 IN dBm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1MHz</td>
<td>-100.00</td>
<td>-102.00</td>
<td>-95.00</td>
<td>-85.00</td>
</tr>
<tr>
<td>10MHz</td>
<td>-145.00</td>
<td>-110.00</td>
<td>-118.00</td>
<td>-90.00</td>
</tr>
<tr>
<td>100MHz</td>
<td>-150.00</td>
<td>-120.00</td>
<td>-128.00</td>
<td>-73.00</td>
</tr>
<tr>
<td>1GHz</td>
<td>-167.00</td>
<td>-140.00</td>
<td>-133.00</td>
<td>-75.00</td>
</tr>
</tbody>
</table>

Figure 9: Comparison of IMD3 (dBm) as a function of separation frequency (MHz) for select microstrip structures involving conventional and smart materials
Also, certain novel and smart materials were studied for self-heating induced distortion effects, and inferences were made with regards to the effect of thermal conductivity and diffusivity on the IMD3.

The self-heating problem is a very significant problem in industries dealing with RF frequencies. This causes most of the products to malfunction because of distortions and hence it has become a very hot topic of many academic and industrial research ventures. The present work offers a small but a very vital contribution towards solving this problem by modeling self-heating induced IMD3.

References