A Proof of Fermat Last Theorem

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Abstract. We give a proof of Pierre de Fermat’s Last Theorem using that Beal Conjecture is true [1].

1. Introduction

In 1621, Pierre de Fermat published the following theorem (called the Fermat Last Theorem):

**Theorem 1.** There are no solutions of:

\[ A^m + B^m = C^m \]

with \( A, B, C, m \), be positive integers with \( m > 2 \).

In this paper, we give a proof of this theorem using that Beal Conjecture [1] is true.

2. The Proof

**Proof.** We suppose that for \( m \in \mathbb{N}^* \), \( m > 2 \), there is a solution \((A, B, C) \in \mathbb{N}^3\) of (1). Using Beal conjecture [1], then \( A, B \) and \( C \) have a common factor. Let \( \mu \in \mathbb{N}^* \) be the great common factor that divides \( A, B, C \). Then we write:

\[ A = \mu A_1 \]
\[ B = \mu B_1 \]
\[ C = \mu C_1 \]

with \((A_1, B_1, C_1)\) are co-prime. The equation (1) becomes:

\[ A_1^m + B_1^m = C_1^m \]

In the following, we suppose that \( A_1 > B_1 \).
2.1. $B_1 > 1$. As $m > 2$, we use the Beal Conjecture, then $A_1, B_1, C_1$ have a common factor $>1$ which is a contradiction with $A_1, B_1, C_1$ co-prime. Then the equation (1) has no integer solutions.

2.2. $B_1 = 1$. Then we obtain:

$$A_1^m + 1 = C_1^m \Rightarrow C_1 > A_1$$

We write $C_1 = A_1 + c, c \in \mathbb{N}^*$, then:

$$(A_1 + c)^m = A_1^m + 1 \Rightarrow c^m + \sum_{k=0}^{m-1} C_k A_1^{m-k} = 1$$

which is impossible.

Q.E.D

References


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