

# Nuclear Waves (Strong Interaction Wave and Weak Interaction Wave) and Related Problems

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**Abstract:** Based on fractal landscapes of various waves, and fifteen kinds of waves caused by four fundamental interactions (forces), this paper discusses nuclear (force) waves (strong interaction wave and weak interaction wave) and related problems; such as: application of fractal method, properties of nuclear waves, certainty-uncertainty principles, conservation of parity and nonconservation of parity, a revision to Gödel's incompleteness theorem, partial and temporary unified theory of four fundamental interactions so far, partial and temporary unified theory of nuclear (force) waves so far, and partial and temporary unified theory of natural science so far.

**Key words:** Nuclear force, strong interaction, weak interaction, nuclear (force) wave, strong interaction wave, weak interaction wave, fractal, certainty-uncertainty principles, conservation of parity and nonconservation of parity, Gödel's incompleteness theorem, partial and temporary unified theory so far

## Introduction

As well-known, electromagnetic wave (E wave) is caused by electromagnetic interaction, and gravitational wave (G wave) is caused by gravity; besides electromagnetic wave and gravitational wave, in reference [1] we propose that strong interaction wave (S wave) is caused by strong interaction, and weak interaction wave (W wave) is caused by weak interaction. This paper discusses nuclear (force) waves (strong interaction wave and weak interaction wave) and related problems with fractal theory and method, as well as other theories and methods.

### 1 Fractal landscapes of various waves

According to fractal theory and method, the different scale "structures" of the universe include: the universe itself, galaxy (such as the Milky Way, and the like), star (celestial body), material (object), molecule, atom, nuclear, electron, and the like. All of these "structures" can produce waves, such as: universe wave, galaxy wave, star wave, material wave, molecule wave, atom wave, nuclear wave, electron wave, and the like.

In addition, a variety of coupling (joint) structures can also be formed by the fractal structures of various scales, such as: universe and galaxy coupling (joint) structure, universe and star coupling (joint) structure, universe and material coupling (joint) structure, universe and molecule coupling (joint) structure, universe and atom coupling (joint) structure, universe and nuclear coupling (joint) structure, universe and electron coupling (joint) structure, and the like. These structures generate various coupling (joint) waves: universe and galaxy coupling (joint) wave, universe and star coupling (joint) wave, universe and material coupling (joint) wave, universe and molecule coupling (joint) wave, universe and atom coupling (joint) wave, universe and nuclear coupling (joint) wave, universe and electron coupling (joint) wave, and the like.

From the viewpoint of four fundamental interactions (forces), in reference [2] we propose that a total of 15 kinds of waves are caused by four fundamental interactions (forces): gravitational wave (G wave), electromagnetic wave (E wave), weak interaction wave (W wave), strong interaction wave (S wave), GE wave, GS wave, GW wave, ES wave, EW wave, SW wave, EGS wave, EGW wave, ESW wave, GSW wave, and EGSW wave.

## 2 Properties of nuclear (force) waves (strong interaction wave and weak interaction wave)

As well-known, the main properties of gravitational wave are as follows: gravitational wave is the shear wave, carrying energy, and in the vacuum its speed equals the speed of light. Referring to the main properties of gravitational wave, in reference [1] we propose that, strong interaction wave and weak interaction wave are also the shear wave, carrying energy, and in the vacuum their speeds equal the speed of light.

In reference [2], it is pointed out that, the original uncertainty principle is improper, considering all the possible situations (including the case that people can create laws), the "certainty-uncertainty principles" with general form and variable dimension fractal form should be presented. Therefore, this paper also presents that, for some properties of strong interaction wave and weak interaction wave (such as shape, energy and the like), there may be three situations: the shape, energy and the like of strong interaction wave and weak interaction wave can be known simultaneously; cannot be known simultaneously; and the neutral (fuzzy) situation that whether or not these properties can be known simultaneously will be undetermined. On this issue, we will discuss it in detail below.

## 3 How to find strong interaction wave and weak interaction wave

There are two main ways to find nuclear (force) waves (strong interaction wave and weak interaction wave).

The first way: similar to the reason that gravitational wave is impossible to exist alone as pointed out in reference [3], the nuclear wave can not exist alone too, and it can only exist in the form of coupling (joint) wave of nuclear (force) wave and electromagnetic wave; therefore, people can only through the detection and research of coupling (joint) wave of nuclear (force) wave and electromagnetic wave to indirectly detect and research nuclear (force) waves (strong interaction wave and weak interaction wave).

The second way: to create nuclear (force) waves (strong interaction wave and weak interaction wave) with artificial method.

For the first way, we can refer to the manner to indirectly detect and research gravitational wave proposed in reference [3].

Why gravitational wave cannot be existed independently? The reason is quite simple, any object which temperature is greater than absolute zero will radiate electromagnetic wave, however in universe the object which temperature is equal to or lower than absolute zero cannot be existed. In other words, any object will radiate electromagnetic wave at any time. Therefore, for the macroscopic object and microscopic object, gravitational wave cannot be existed independently, while the gravitational-electromagnetic coupling

wave (GE wave) is existed.

At present, all the experiments to detect the independent gravitational wave will end in failure.

It should be noted that, the GE wave may be a new kind of wave, instead of the simple superposition of gravitational wave and electromagnetic wave. To illustrate this viewpoint, we can see the example of elastic-plastic theory. For a thin circular plate heated in its center, when the temperature is reached to a certain extent, three zones will be existed: plastic zone, elastic-plastic zone and elastic zone. Plastic problem can be solved by using the plastic theory, and elastic problem can be solved by using the elastic theory, however, the coupling elastic-plastic problem cannot be solved by simply using the superposition of the elastic result and the plastic result, in order to solve this problem, a new theory, namely elastic-plastic theory must be established. For the same reason, we can assume that the GE wave may be a new kind of wave, and the related new theory should be established. For this reason, the difficulty to detect and research GE wave is also increased.

At present there may be two main programs to detect nuclear wave: direct detection and indirect detection.

For the direct detection program, generally to eliminate noise is required. Although we believe that nuclear wave may also exist in noise, eliminating noise will lose the chance of detecting nuclear wave. The correct program should be as follows: firstly detecting the coupling (joint) wave of nuclear wave and electromagnetic wave in the noise, then through the coupling (joint) wave of nuclear wave and electromagnetic wave to research the nature of nuclear wave. Of course, such detection and research are also very difficult.

For the indirect detection program, we can refer to the most successful case is the observation of double stars PSR1913+16 (it obtains the so-called indirect evidence of gravitational radiation, and the two observers R. A. Hulse and J. H. Taylor won the 1993 Nobel Prize). But according to our point of view, for the observation of double stars PSR1913+16, it obtains the indirect evidence of GE wave, instead of gravitational wave. Similarly, we can research the property of nuclear wave through the indirect evidence of the coupling (joint) wave of nuclear wave and electromagnetic wave. Of course, such research is very difficult too.

In addition, besides two kinds of coupling (joint) wave of nuclear wave and electromagnetic wave (ES wave and EW wave), nuclear waves can also create other eight kinds of coupling (joint) wave: GS wave, GW wave, SW wave, EGS wave, EGW wave, ESW wave, GSW wave, and EGSW wave. To detect and research these eight kinds of coupling (joint) wave, may pave the new way to detect and research the nuclear waves.

For the second way, we can refer to the possible manner to create instantaneous magnetic monopole proposed in reference [4].

The conventional viewpoint considers that man cannot create law. This is a one-sided viewpoint. In some cases, man can create law, including change the rule into law. So the laws can be divided into at least three kinds: the objective law, the man created subjective law, as well as the synthetic law formed by the above mentioned two kinds of laws.

Now we discuss various man created laws (man-made laws).

In the social science: (1)in stock market the banker created the law of stock, (2)for various goods, the wholesale price calculation formula is decided by the owner, (3) the laws of Chinese new year firecrackers and the Mid-Autumn Festival cake.

In the natural science: (1)the law of gravity and the theory of general relativity were created by Newton and Einstein respectively, (2)some geometries built from a set of axioms, (3)various carry-systems in mathematics, (4)the operation of fountain with man created law, (5)the temperature law of the greenhouse.

In thinking science: one divides into two or one divides into three (such as the three worlds) and one divides into five (such as the five elements in Chinese ancient times), and the different laws to learn the knowledge such as the sequence of easy-difficult or difficult-easy.

In the virtual world (the laws don't need to be tested by practice): (1)in science fiction the Hubble constant can be given arbitrarily as well as the speed of airship can reach ten thousand times of the speed of light, (2)in the ancient Chinese novel "The Pilgrimage to the West", Tang Monk's law to punish the Monkey King, (3)in artistic works the law of the hero and the beauty, (4)the law to steal vegetables from the online game.

Finally the optimum synthetic law formed by subjective law and objective law, such as Earth's best seasonal variation, can be created by people.

In physics, the man-made laws have not been paid enough attention. However, some scholars have presented some issues connected with man-made laws. For example, some scholars say that "magnetic monopole" can exist. While, "magnetic monopole can exist" is a man-made law, because in nature "magnetic monopole" does not exist.

For more information about man-made laws, see reference [1].

Now, we give an artificial method to create "man-made instantaneous magnetic monopole".

As shown in figure 1, suppose there is a long uniform rectangular-shaped magnet, along its middle cross section (the demarcation section of N-pole and S-pole) to cut it at very high speed, then the disconnected two pieces will quickly turn into two new magnets. However, this process takes time. While before the changing, namely as the disconnected instant moment, one half of the magnet is the pure N-pole, and the other half is the pure S-pole.

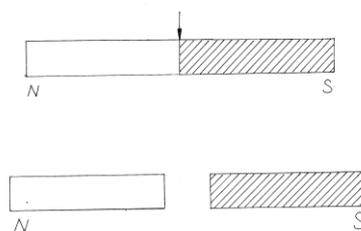


Figure 1 Instantaneous magnetic monopole

Figure 1 only shows one scheme to realize instantaneous magnetic monopole. Other options may be also exist.

Referring to the scheme to realize instantaneous magnetic monopole, we can also present the imagine to realize "man-made nuclear (force) waves (strong interaction wave and weak interaction wave) ". But the special program cannot be given so far.

#### 4 Application of fractal method

As analyzing nuclear wave, we can refer to reference [6], in which several solar data (sunspot numbers, Penticton 2800MHz solar flux, and THULE cosmic ray indices) are analyzed and predicted with fractal method. These data and the analysis methods may be related to nuclear waves, or to provide a useful reference.

##### 4.1 Constant and Variable Dimension Fractal Prediction Methods

Recently, fractal method has been successfully used in many fields, and it is used for finding out the deeply hidden organized structure in complex phenomena. The quantity for reflecting the character of organized structure is called the fractal dimension, expressed with the value of  $D$ . In the fractal methods for general application at present, the fractal dimension  $D$  is a constant. For example the values of fractal dimension  $D$  for different coastlines may be taken as 1.02, 1.25 and so on. The fractal model<sup>[7]</sup> reads

$$N = \frac{C}{r^D} \quad (1)$$

where  $r$  is the characteristic scale, such as time, length, coordinates and so on;  $N$  is the object number or quantity related with the value of  $r$ , such as sunspot number, solar flux, cosmic ray index, price, temperature, the value to be predicted and so on;  $C$  is a constant to be determined,  $D$  is the fractal dimension.

In the recent application of fractal method,  $D$  is the constant, may be called constant dimension fractal. It is a straight line in the double logarithmic coordinates. According to arbitrary two data points  $(N_i, r_i)$  and  $(N_j, r_j)$  on this straight line, the fractal parameters of this straight line, i.e., the fractal dimension  $D_{ij}$  and the constant  $C_{ij}$ , can be determined; Substituting the coordinates of the two data points into Eq.(1), they can be solved

$$D_{ij} = \frac{\ln(N_i/N_j)}{\ln(r_j/r_i)} \quad (2)$$

$$C_{ij} = N_i r_i^{D_{ij}} \quad (3)$$

For the straight line functional relation in the double logarithmic coordinates, it is able to process prediction and calculation with the constant dimension fractal directly.

But for the non-straight line functional relation in the double logarithmic coordinates, it is unable to process the prediction and calculation with the constant dimension fractal. Many questions belong to this situation. In order to overcome this difficulty, the concept of variable dimension fractal in references [8-10] is introduced, namely the fractal dimension  $D$  is the function of characteristic scale  $r$ .

$$D = F(r) \quad (4)$$

Now how to carry on prediction and calculation with this fractal model is to be discussed.

For the sake of convenience, let  $r$  denote the serial number of time, it will stipulate some year for the first year, then  $r_1 = 1$ , for the second year,  $r_2 = 2$ , and so on.

Let  $N$  denote the given value and the value to be predicted, for example, taking  $N_1$  as the value of the first year,  $N_2$  as the value of the second year, and so on.

Now supposing that  $n$  data points are given, the values for the first year to the  $n^{\text{th}}$  year are known, thereupon the question becomes how to predict the values for the  $(n+1)^{\text{th}}$  year,  $(n+2)^{\text{th}}$  year and so on.

As a result of the  $n^{\text{th}}$  data point, namely the values of  $N_n$  and  $r_n$  for the  $n^{\text{th}}$  year are given ( $r_n = n$ ), and the value of  $r_{n+1}$  for  $(n+1)^{\text{th}}$  year is also known ( $r_{n+1} = n+1$ ), if the fractal dimension  $D_{n,n+1}$  of the constant dimension fractal decided by the  $n^{\text{th}}$  data point and  $(n+1)^{\text{th}}$  data point is known, then the value for the  $(n+1)^{\text{th}}$  year can be solved from Eq.(2)

$$N_{n+1} = N_n \left( \frac{r_n}{r_{n+1}} \right)^{D_{n,n+1}} \quad (5)$$

To this analogizes, the values for the  $(n+2)^{\text{th}}$  year and the like can be solved.

As for how to decide the fractal dimension  $D_{n,n+1}$ , it needs the information given by  $D_{12}$  (decided by the given first data point and second data point),  $D_{23} \cdots D_{n-1,n}$  (decided by other given data points). But in general case, it is very difficult to discover the changing rule for these values of fractal dimension.

In this case, the above method cannot be used directly. The transformation of accumulated sum for the given values have to be carried on firstly, then the above method can be used to forecast the values of accumulated sum for the  $(n+1)^{\text{th}}$  year,  $(n+2)^{\text{th}}$  year and so on. Finally the values to be predicted are solved by the values of accumulated sum.

The advantage for using accumulated sum is that a sequence with increasing and decreasing can be changed into a monotone increasing sequence.

This method may be introduced as follows.

The first step, plotting the original data points  $(N_i, r_i)(i = 1 \sim n)$  in the double logarithmic coordinates. In the ordinary circumstances they cannot fairly agree with a constant dimension fractal model,  $N_i (i = 1, 2 \cdots n)$  may be arranged to a fundamental sequence, namely it can be written as

$$\{N_i\} = \{N_1, N_2, N_3, \dots\} \quad (i = 1, 2, \dots, n)$$

Other sequences may be constructed according to the fundamental sequence. For example, for  $S^{(1)}$ , i.e., the sequence of first order accumulated sum,  $S_1^{(1)} = N_1$ ,  $S_2^{(1)} = N_1 + N_2$ ,  $S_3^{(1)} = N_1 + N_2 + N_3$ , ...; according to analogize, the sequence of second order accumulated sum, the sequence of third order accumulated sum, and the like can be constructed, namely it can be written as

$$\{S_i^{(1)}\} = \{N_1, N_1 + N_2, N_1 + N_2 + N_3, \dots\} \quad (i = 1, 2, \dots, n) \quad (6)$$

$$\{S_i^{(2)}\} = \{S_1^{(1)}, S_1^{(1)} + S_2^{(1)}, S_1^{(1)} + S_2^{(1)} + S_3^{(1)}, \dots\} \quad (i = 1, 2, \dots, n) \quad (7)$$

$$\{S_i^{(3)}\} = \{S_1^{(2)}, S_1^{(2)} + S_2^{(2)}, S_1^{(2)} + S_2^{(2)} + S_3^{(2)}, \dots\} \quad (i = 1, 2, \dots, n)$$

$$\{S_i^{(4)}\} = \{S_1^{(3)}, S_1^{(3)} + S_2^{(3)}, S_1^{(3)} + S_2^{(3)} + S_3^{(3)}, \dots\} \quad (i = 1, 2, \dots, n)$$

It needs to point out that  $S_i^{(2)}$  denote second order accumulated sum, instead of the second power of  $S_i$ .  $S_i^{(3)}$  and the like should be comprehended similarly.

The second step, establishing the fractal models for various order accumulated sum. Taking the second order accumulated sum as an example. Plotting the data points  $(S_i^{(2)}, r_i) (i = 1 \sim n)$  in the double logarithmic coordinates, linking these points one by one, it may result in the sectioned constant dimension fractal model. For example, according to  $n$  data points, the sectioned constant dimension fractal model composed from  $n - 1$  straight lines (for different straight line, its fractal dimension is also different, this also is the simplest variable dimension fractal model), and the fractal parameters  $D_{ij}^{(2)}, (i = 1 \sim n - 1, j = i + 1)$  and  $C_{ij}^{(2)}$  for each straight line can be calculated according to Eq.(2) and Eq. (3) (in which the value of  $N_i$  is replaced by  $S_i^{(2)}$ ) Which means

$$D_{ij}^{(2)} = \ln(S_i^{(2)} / S_j^{(2)}) / \ln(r_j / r_i) \quad (8)$$

$$C_{ij}^{(2)} = S_i^{(2)} r_i^{D_{ij}^{(2)}} \quad (9)$$

The third step, choosing the best transformation and determining its corresponding fractal parameters. Separately drawing various order accumulated sum's data points in the double logarithmic coordinates, then choosing the best transformation (its values of fractal dimension are even increased or even decreased) and determining its corresponding fractal parameters. Because in the ordinary circumstances, the second order accumulated sum is the best, the case of second order accumulated sum will be

discussed only.

After choosing the fractal model, the suitable method should be used for deciding the fractal dimension  $D_{n,n+1}^{(2)}$  firstly, then uses the reconstructive Eq.(5) to carry on the forecast for accumulated sum. Because the values of fractal dimension are evenly increased or evenly decreased, using the following linear interpolation formula can solve the fractal dimension  $D_{n,n+1}^{(2)}$

$$D_{n,n+1}^{(2)} = 2D_{n-1,n}^{(2)} - D_{n-2,n-1}^{(2)} \quad (10)$$

For the second order accumulated sum, Eq.(5) can be expressed by

$$S_{n+1}^{(2)} = S_n^{(2)} \left( \frac{r_n}{r_{n+1}} \right)^{D_{n,n+1}^{(2)}} \quad (11)$$

For the reason that  $S_1^{(1)} \sim S_n^{(1)}$ ,  $S_1^{(2)} \sim S_n^{(2)}$  are already calculated, then the forecasting first order accumulated sum can be obtained from the forecasted second order accumulated sum, which means

$$S_{n+1}^{(1)} = S_{n+1}^{(2)} - S_n^{(2)} \quad (12)$$

Then the forecasting value can be obtained from the forecasted first order accumulated sum, which means

$$N_{n+1} = S_{n+1}^{(1)} - S_n^{(1)} \quad (13)$$

According to analogize similarly,  $N_{n+2}$ ,  $N_{n+3}$  and so on can be obtained.

This forecasting manner is named for multiple-value-forecasting, because according to  $n$  given data  $(N_1, N_2, N_3 \cdots N_n)$ , the next several data  $(N_{n+1}, N_{n+2}, N_{n+3}, \text{ and so on})$  are forecasted.

According to the practice, the multiple-value-forecasting is only suitable for the cases that the data are on the increase or decrease, otherwise other manners should be used.

Another forecasting manner proposed by this paper is named for single-value-forecasting, because according to  $n$  given data  $(N_1, N_2, N_3 \cdots N_n)$ , only the next one datum  $N_{n+1}$  is forecasted. After the real value of  $N_{n+1}$  is given, taking the real values of  $(N_1, N_2, N_3 \cdots N_{n+1})$  as the given data to forecast the next one datum  $N_{n+2}$ . To this analogizes,  $N_{n+3}$  and the like can be forecasted one by one.

In order to compare with the fractal method, the following linear forecasting method is used.

$$N_{n+1} = 2N_n - N_{n-1} \quad (14)$$

#### 4.2 Confidence limit

Suppose that, according to single-value-forecasting or multiple-value-forecasting,  $n$  data  $(Y_1', Y_2', Y_3' \cdots Y_n')$  are forecasted already, the corresponding real values are  $(Y_1, Y_2, Y_3 \cdots Y_n)$ , then for the  $(n+1)^{\text{th}}$  forecasting value  $Y_{n+1}'$ , the confidence limit corresponding to  $(1-\alpha) \times 100\%$  is as follows

$$Y_{n+1}' \pm t_{\alpha} S_Y \quad (15)$$

where:  $S_Y = \sqrt{\frac{\sum (Y - Y')^2}{n-2}}$

In this paper, the value of  $\alpha$  will be taken as  $\alpha = 0.05$ .

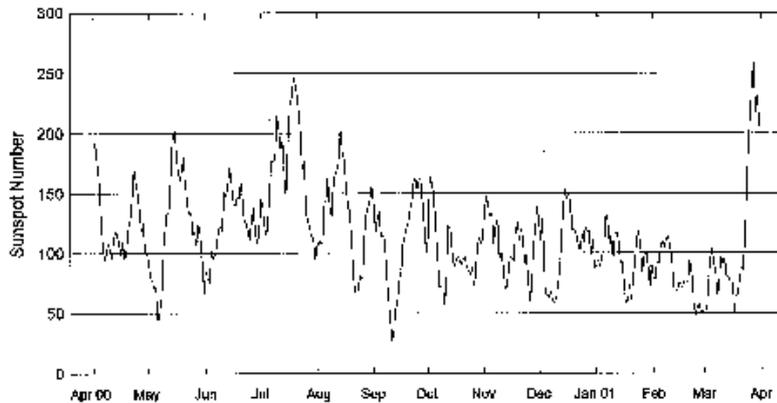
#### 4.3 Examples of single-value-forecasting

Example 1, according to the sunspot numbers from April 1 to 20, 2000, to forecast the sunspot numbers from April 21 to June 9, 2000 with the method of single-value-forecasting.

The given data are shown in the following table (downloaded from : <http://www.ngdc.noaa.gov/ngdc.html>, <http://sgd.ngdc.noaa.gov/sgd/jsp/solarfront.jsp>).

International Relative Sunspot Numbers  
Apr 2000 - Mar 2001

27  
Mar 01



Day	Apr 00	May	Jun	Jul	Aug	Sep	Oct <sup>a</sup>	Nov <sup>a</sup>	Dec <sup>a</sup>	Jan 01 <sup>a</sup>	Feb <sup>a</sup>	Mar <sup>a</sup>
1	187	81	65	145	106	142	115	135	124	89	78	52
2	193	80	79	141	110	118	164	147	109	94	78	53
3	177	76	75	124	107	128	159	141	128	88	92	75
4	164	71	101	114	110	134	150	130	66	98	91	92
5	129	71	95	127	144	114	128	133	65	101	105	104
6	108	42	99	154	143	114	97	103	63	130	110	91
7	84	52	105	177	164	110	71	122	68	131	105	85
8	100	84	120	177	140	85	72	127	61	105	111	63
9	108	99	122	179	128	55	71	95	58	115	114	79
10	102	120	119	215	154	42	57	101	62	95	105	97
11	96	133	151	202	165	26	82	90	72	115	100	90
12	113	133	147	186	170	35	122	72	89	117	71	95
13	118	181	156	194	176	63	121	70	114	111	71	80
14	114	193	171	164	204	60	104	84	135	92	68	80
15	105	205	158	148	183	77	89	98	153	92	75	75
16	98	189	142	197	178	85	92	95	145	75	73	75
17	110	170	139	224	152	108	97	94	151	59	71	51
18	94	161	147	228	140	112	95	116	138	60	76	65
19	103	187	145	246	133	121	90	125	118	73	75	60
20	121	180	159	241	106	124	94	110	120	61	76	80
21	128	163	147	231	77	137	97	120	116	81	94	88
22	145	143	127	216	67	142	89	113	107	93	81	85
23	170	132	124	199	67	160	85	91	102	112	59	113
24	160	134	119	171	77	163	82	98	115	118	48	148
25	151	115	111	177	81	153	88	74	108	106	58	188
26	138	117	129	133	79	161	73	58	121	84	58	218
27	118	106	138	126	113	162	80	79	118	97	50	241
28	124	124	115	120	132	142	106	106	118	102	51	258
29	100	117	109	113	138	119	113	123	99	90		218
30	100	93	114	112	144	100	108	138	111	70		231
31		67		93	157		111		87	93		205
Mean	125.5	121.8	124.9	170.1	130.5	109.7	100.1	106.5	104.5	95.1	80.1	114.2

<sup>a</sup> = Provisional.

Take April 1, 2000 as the first day and let  $r_1 = 1$ , that day's sunspot number is  $N_1$ , and  $N_1 = 187$ . According to analogize similarly, for April 20, 2000,  $r_{20} = 20$ ,  $N_{20} = 121$ .

According to the given sunspot numbers from April 1 to 20, 2000, with the Eqs. (10) - (13) and single-value-forecasting, all the forecasting results are shown in Table 1.

Table 1 Forecasting results for the sunspot numbers from April 21 to June 9, 2000

No.	Month	day	real value	FM*	FM error	LM**	LMerror
1	4	21	128	124	-4	139	11
2		22	145	132	-13	135	-10
3		23	170	151	-19	162	-8
4		24	160	180	20	195	35
5		25	151	167	16	150	-1
6		26	136	151	20	142	6
7		27	118	138	20	121	3
8		28	124	117	-7	100	-24
9		29	100	124	24	130	30
10		30	100	97	-3	76	-24
11	5	1	91	97	6	100	9
12		2	80	88	8	82	2
13		3	76	76	0	69	-7
14		4	71	72	1	72	1
15		5	71	67	-4	66	-5
16		6	42	67	25	71	29
17		7	52	36	-16	13	-39
18		8	64	48	-16	62	2
19		9	99	61	-38	76	-23
20		10	120	99	-21	134	14
21		11	133	122	-11	141	8
22		12	133	136	3	146	13
23		13	161	136	-25	133	-28
24		14	193	160	-27	189	-4
25		15	205	200	-5	225	20
26		16	189	213	24	217	28
27		17	170	195	25	179	3
28		18	161	174	13	151	-10
29		19	167	164	-3	152	-15
30		20	180	171	-9	173	-7
31		21	163	185	22	193	30
32		22	143	166	23	146	3
33		23	132	144	12	123	-9
34		24	134	132	-2	121	-13
35		25	115	134	19	136	21
36		26	117	114	-3	96	-21
37		27	106	116	10	119	13
38		28	124	104	-20	95	-29
39		29	117	124	7	142	25
40		30	93	116	23	110	17
41		31	67	91	24	69	2
42	6	1	85	63	-22	41	-44

43	2	79	83	4	103	24
44	3	75	76	1	73	-2
45	4	101	72	-29	71	-30
46	5	95	100	5	127	30
47	6	99	94	-5	89	-10
48	7	105	98	-7	103	-2
49	8	120	104	-16	111	-9
50	9	122	120	-2	135	13

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\*FM = Fractal forecasting method

\*\*LM = Linear forecasting method

From the results of Table 1 it is known that in the above continual 50 days' predictions given by fractal forecasting method, there are 22 days that the error less than 10, the proportion is 44%; 14 days the error is greater than or equal to 11, and less than or equal to 20, the proportion is 28%; 13 days the error is greater than or equal to 21, and less than or equal to 30, the proportion is 26%; only 1 day the error greater than 31, the proportion is 2%. The maximum error is equal to 38.

While for the results given by linear forecasting method, there are 23 days that the error less than 10, the proportion is 46%; 9 days the error is greater than or equal to 11, and less than or equal to 20, the proportion is 18%; 15 days the error is greater than or equal to 21, and less than or equal to 30, the proportion is 30%; 3 days the error greater than 31, the proportion is 6%. The maximum error is equal to 44.

The results of confidence limit of fractal forecasting method are as follows.

For April 30, the forecasting value is equal to 97 (the real value is also equal to 97),

$S_Y = 19.26$  ,  $t_{0.05}(7) = 2.365$  , according to Eq.(15), it gives

Confidence limit:  $97 \pm 45.5$

For May 10, the forecasting value is equal to 99 (the real value is equal to 120),

$S_Y = 17.71$  ,  $t_{0.05}(17) = 2.110$  , according to Eq.(15), it gives

Confidence limit:  $99 \pm 37.0$

For May 20, the forecasting value is equal to 171 (the real value is equal to 180),

$S_Y = 17.94$  ,  $t_{0.05}(27) = 2.052$  , according to Eq.(15), it gives

Confidence limit:  $171 \pm 36.8$

For May 30, the forecasting value is equal to 116 (the real value is equal to 93),

$S_Y = 17.12$  ,  $t_{0.05}(37) = 2.028$  , according to Eq.(15), it gives

Confidence limit:  $116 \pm 34.7$

For June 9, the forecasting value is equal to 120 (the real value is equal to 122),

$S_Y = 17.01$  ,  $t_{0.05}(47) = 2.014$  , according to Eq.(15), it gives

Confidence limit:  $120 \pm 34.3$

It should be noted that, in this example, the time interval is equal to one day, therefore the sunspot number of the next day can be forecasted only. If the time interval is equal to one week, one month, one year and the like, then the sunspot number of the next week, next month, next year and the like can be forecasted.

For example, the time interval is equal to one month, if the given data are as follows  
total number in Jan., total number in Feb., ... total number in Nov.

Then the total number in Dec. can be forecasted.

While, for other case, if the given data are selected as follows  
number of Jan. 15, number of Feb. 15, ... number of Nov. 15

Then the number of Dec. 15 can be forecasted one month ahead of time.

Example 2, according to the Penticton 2800MHz solar flux from April 1 to 20, 2000, to forecast the Penticton 2800MHz solar flux from April 21 to June 9, 2000 with single-value-forecasting.

The given data are also downloaded from: <http://www.ngdc.noaa.gov/ngdc.html>, <http://sgd.ngdc.noaa.gov/sgd/jsp/solarfront.jsp> .

Similar to Example 1, the forecasting results are shown in Table 2.

Table 2 Forecasting results for the 2800MHz solar flux from April 21 to June 9, 2000

No.	Month	day	real value	FM*	FM error
1	4	21	189.2	185.7	-3.5
2		22	204.1	193.3	-10.8
3		23	208.4	210.1	1.7
4		24	208.1	214.4	-6.3
5		25	205.1	213.3	-8.2
6		26	192.4	209.3	16.9
7		27	186.0	194.2	8.2
8		28	186.0	186.7	0.7
9		29	177.5	186.6	9.1
10		30	172.0	177.0	5.0
11	5	1	160.1	170.8	10.7
12		2	155.3	157.6	2.3
13		3	139.6	152.5	12.9
14		4	136.8	135.4	-1.4
15		5	132.1	132.7	0.6
16		6	129.1	128.6	-1.1
17		7	133.4	125.1	-8.3
18		8	139.6	130.3	-9.3
19		9	152.5	137.3	-15.1
20		10	182.8	151.6	-31.2
21		11	181.3	184.8	3.5
22		12	194.4	183.1	-11.3
23		13	222.0	197.1	-24.9
24		14	237.6	227.0	-10.6

25	15	249.9	243.7	-6.2	
26	16	264.5	256.7	-7.8	
27	17	268.1	272.1	4.0	
28	18	258.8	275.6	16.8	
29	19	260.4	265.0	4.6	
30	20	251.6	266.3	14.7	
31	21	238.0	256.5	18.5	
32	22	220.3	241.5	21.2	
33	23	209.5	222.1	12.6	
34	24	194.3	210.4	16.1	
35	25	177.4	193.9	16.5	
36	26	172.4	175.7	3.3	
37	27	166.2	170.6	4.4	
38	28	160.2	163.8	3.6	
39	29	153.1	157.8	4.7	
40	30	150.5	150.3	-0.2	
41	31	158.7	147.7	-11.0	
42	6	1	152.3	156.6	4.3
43	2	192.7	150.0	-42.7	
44	3	170.7	193.0	22.3	
45	4	174.7	169.7	-5.0	
46	5	176.1	174.1	-2.0	
47	6	192.0	175.4	-16.6	
48	7	185.8	193.3	6.5	
49	8	179.9	185.8	5.9	
50	9	174.1	179.6	5.5	

---

\*FM = Fractal forecasting method

From the results of Table 2 it is known that in the above continual 50 days' predictions given by fractal forecasting method, there are 30 days that the error less than 10, the proportion is 60%; 15 days the error is greater than 10, and less than or equal to 20, the proportion is 30%; 3 days the error is greater than 20, and less than or equal to 30, the proportion is 6%; only 2 days the error greater than 30, the proportion is 4%. The maximum error is equal to 42.7.

While for the results given by linear forecasting method, the maximum error is equal to 62.4.

The results of confidence limit of fractal forecasting method are as follows.

For April 30, the forecasting value is equal to 177.0 (the real value is equal to 172.0),

$S_y = 9.82$ ,  $t_{0.05}(7) = 2.365$ , according to Eq.(15), it gives

$$\text{Confidence limit: } 177.0 \pm 23.2$$

For May 10, the forecasting value is equal to 151.6 (the real value is equal to 182.8),

$S_Y = 9.00$  ,  $t_{0.05}(17) = 2.110$  , according to Eq.(15), it gives

Confidence limit:  $151.6 \pm 19.0$  .

For May 20, the forecasting value is equal to 266.3 (the real value is equal to 251.6),

$S_Y = 11.60$  ,  $t_{0.05}(27) = 2.052$  , according to Eq.(15), it gives

Confidence limit:  $266.3 \pm 23.8$  .

For May 30, the forecasting value is equal to 150.3 (the real value is equal to 150.5),

$S_Y = 12.08$  ,  $t_{0.05}(37) = 2.028$  , according to Eq.(15), it gives

Confidence limit:  $150.3 \pm 24.5$  .

For June 9, the forecasting value is equal to 179.6 (the real value is equal to 174.1),

$S_Y = 13.24$  ,  $t_{0.05}(47) = 2.014$  , according to Eq.(15), it gives

Confidence limit:  $179.6 \pm 26.7$  .

The concept of “warning interval” presented in this paper is as follows.

As well-known, the concept of “confidence limit” has already obtained the widespread application. However it has a minor defect in something otherwise perfect that, the existing biggest forecasting error isn’t emphasized. For this reason, the concept of “warning interval” is used to cooperate with the concept of “confidence limit”, and its upper limit or lower limit may be closer to the real value.

Supposing the  $(n+1)^{\text{th}}$  forecasting value is  $N_{n+1}$ , while within the  $n$  forecasting values before  $N_{n+1}$ , the biggest forecasting error is  $E_{\text{max}}$ , then the “warning interval” for the  $(n+1)^{\text{th}}$  forecasting value is as follows

$$N_{n+1} \pm E_{\text{max}}$$

Example 3, according to the THULE cosmic ray indices from March 1 to 19, 2001, to forecast the THULE cosmic ray indices from March 20 to 31, 2001 with the method of single-value-forecasting.

The given data are also downloaded from: <http://www.ngdc.noaa.gov/ngdc.html>, <http://sgd.ngdc.noaa.gov/sgd/jsp/solarfront.jsp> .

All the forecasting results are shown in Table 3.

Table 3 Forecasting results for the cosmic ray indices from March 20 to 31, 2001

No.	Month day	real value	FM*	FM error	warning interval	confidence limit
1	3 20	4079.7	4172.9	93.2		
2	21	4094.7	4069.0	-25.7	$4069.0 \pm 93.2$	
3	22	4136.1	4089.4	-46.7	$4089.4 \pm 93.2$	
4	23	4146.5	4139.3	-7.2	$4139.3 \pm 93.2$	$4139.3 \pm 1304.9$
5	24	4140.0	4151.7	1,7	$4151.7 \pm 93.2$	$4151.7 \pm 314.6$

6	25	4147.1	4143.8	-3.3	4143.8 ± 93.2	4143.8 ± 191.8
7	26	4156.5	4152.5	-4.0	4152.5 ± 93.2	4152.5 ± 145.4
8	27	4058.4	4162.7	104.3	4162.7 ± 93.2	4162.7 ± 120.5
9	28	4001.3	4050.0	48.7	4050.0 ± 104.3	4050.0 ± 148.8
10	29	4003.7	3986.9	-16.8	3986.9 ± 104.3	3986.9 ± 133.2
11	30	4066.0	3991.3	-74.7	3991.3 ± 104.3	3991.3 ± 125.4
12	31	3975.2	4063.7	88.5	4063.7 ± 104.3	4063.7 ± 126.6

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\*FM = Fractal forecasting method

From the results of Table 3 it is known that in the above continual 12 days' predictions given by fractal forecasting method, there are 4 days that the error less than 10, the proportion is 33%; 1 day the error is greater than 10, and less than or equal to 20, the proportion is 8%; 1 day the error is greater than 20, and less than or equal to 30, the proportion is 8%; 5 days the error greater than 30, the proportion is 42%. The maximum error is equal to 104.3.

While for the results given by linear forecasting method, the maximum error is equal to 153.1.

From the results of Table 3 it is also known that for this example, all the upper limits or lower limits of "warning interval" are closer to the real values than that of "confidence limit".

## 5 Certainty-uncertainty principles

In quantum mechanics, the uncertainty principle refers to the position and momentum of a particle cannot be determined simultaneously, the uncertainty of position ( $\Delta x$ ) and uncertainty of momentum ( $\Delta p$ ) obey the following inequality

$$\Delta x \Delta p \geq h / 4\pi \quad (16)$$

where,  $h$  is the Planck constant.

One of the reasons for 1932 Nobel Prize for physics is "for the creation of quantum mechanics". As well-known, the most famous contribution of Heisenberg is uncertainty principle, therefore one of the most important reasons for Heisenberg was awarded the Nobel Prize is the creation of uncertainty principle. While, this paper points out that the original uncertainty principle is improper.

### 5.1 Heisenberg inequality, Ozawa inequality and their forms of equality

Heisenberg inequality (Eq.16) can be changed into the following form of equality

$$\Delta x \Delta p = kh / 4\pi \quad (17)$$

where,  $k$  is a real number and  $k \geq 1$ .

Ozawa inequality<sup>[11]</sup> can be written as follows

$$\Delta Q \Delta P + \Delta Q \sigma(P) + \sigma(Q) \Delta P \geq h / 4\pi \quad (18)$$

It can be changed into the following form of equality

$$\Delta Q \Delta P + \Delta Q \sigma(P) + \sigma(Q) \Delta P = kh / 4\pi \quad (19)$$

where,  $k$  is a real number and  $k \geq 1$ .

## 5.2 "Certainty-uncertainty principles" with general form

Neutrosophy is proposed by Prof. Florentin Smarandache in 1995.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea  $\langle A \rangle$  together with its opposite or negation  $\langle \text{Anti-}A \rangle$  and the spectrum of "neutralities"  $\langle \text{Neut-}A \rangle$  (i.e. notions or ideas located between the two extremes, supporting neither  $\langle A \rangle$  nor  $\langle \text{Anti-}A \rangle$ ). The  $\langle \text{Neut-}A \rangle$  and  $\langle \text{Anti-}A \rangle$  ideas together are referred to as  $\langle \text{Non-}A \rangle$ .

Neutrosophy is the base of neutrosophic logic, neutrosophic set, neutrosophic probability and statistics used in engineering applications (especially for software and information fusion), medicine, military, cybernetics, and physics.

Neutrosophic Logic is a general framework for unification of many existing logics, such as fuzzy logic (especially intuitionistic fuzzy logic), paraconsistent logic, intuitionistic logic, etc. The main idea of NL is to characterize each logical statement in a 3D Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of  $]0, 1+[$  without necessarily connection between them.

More information about Neutrosophy may be found in references [12, 13].

According to Neutrosophy, the original uncertainty principle can be extended into the following "certainty-uncertainty principles" with general form

$$\Delta x \Delta p = Kh \quad (20)$$

where,  $K$  is a real number and  $K > 0$ .

Eq.(20) can be divided into three principles:

The first one is the "uncertainty principle" ( $K \geq K_1$ ): a particle's position and momentum cannot be known simultaneously.

Obviously, if  $K_1 = 1/4\pi$ , then it is the original uncertainty principle.

The second one is the "certainty principle" ( $K \leq K_2$ ): a particle's position and momentum can be known simultaneously.

Referring to the experiments for establishing Ozawa inequality, the value of  $K_2$  can be decided by related experiments.

The third one is the neutral (fuzzy) "indeterminacy principle" ( $K_2 < K < K_1$ ): whether or not a particle's position and momentum can be known simultaneously is undetermined.

Similarly, the original Ozawa inequality can be extended into the following Ozawa type's "certainty-uncertainty principles" with general form

$$\Delta Q \Delta P + \Delta Q \sigma(P) + \sigma(Q) \Delta P = Kh \quad (21)$$

where,  $K$  is a real number and  $K > 0$ .

Eq.(21) can be divided into three principles:

The first one is the "certainty principle" ( $K \geq K_1$ ): a particle's position and momentum can be known (namely can be measured with zero-error) simultaneously (here  $\sigma(P)$  or  $\sigma(Q)$  is equal to infinity).

Obviously, if  $K_1 = 1/4\pi$ , then it is the original Ozawa inequality (with equality form).

It should be noted that here the first one is not the uncertainty principle, but certainty principle.

The second one is the "uncertainty principle" ( $K \leq K_2$ ): a particle's position and momentum cannot be known simultaneously.

The third one is the neutral (fuzzy) "indeterminacy principle" ( $K_2 < K < K_1$ ): whether or not a particle's position and momentum can be known simultaneously is undetermined.

### 5.3 "Certainty-uncertainty principles" with variable dimension fractal form

In order to process Eq. (20) and Eq.(21), as well as other equalities and inequalities that may arise in the future with unified manner, we will discuss the "certainty-uncertainty principles" with variable dimension fractal form.

The general form of variable dimension fractal is as follows

$$N = \frac{C}{r^D} \quad (22)$$

where,  $D = f(r)$ , instead of a constant.

For the sake of convenience, we only discuss the situation of  $C = 1$ , that is

$$N = \frac{1}{r^D} \quad (23)$$

Thus, Eq.(20) can be written as the following variable dimension fractal form

$$\Delta x \Delta p = \frac{1}{h^D} \quad (24)$$

Solving this equation, it gives

$$D = -\frac{\ln(Kh)}{\ln h} \quad (25)$$

Then, the values of  $D_1$  and  $D_2$  corresponding to  $K_1$  and  $K_2$  can be calculated by Eq.(25), for example

$$D_1 = -\frac{\ln(K_1 h)}{\ln h} \quad (26)$$

Similarly, Eq.(21) can be written as the following variable dimension fractal form

$$\Delta Q \Delta P = \frac{1}{h^D} \quad (27)$$

Solving this equation, it gives

$$D = -\frac{\ln(Kh - \Delta Q \sigma(P) - \sigma(Q) \Delta P)}{\ln h} \quad (28)$$

Then, the values of  $D_1$  and  $D_2$  corresponding to  $K_1$  and  $K_2$  can be calculated by Eq.(28), for example

$$D_1 = -\frac{\ln(K_1 h - \Delta Q \sigma(P) - \sigma(Q) \Delta P)}{\ln h} \quad (29)$$

#### 5.4 Solving the problem of light speed with Newton mechanics

In accordance with the original uncertainty principle, discussing high-speed particle's speed and track with Newton mechanics is unreasonable; but according to "certainty-uncertainty principles", Newton mechanics can be used to discuss the problem of gravitational deflection of a photon orbit around the Sun (it presents the same result of deflection angle as given by general relativity). The solving method can be found in reference [14]; in which, for problem of gravitational deflection of a photon orbit around the Sun, the improved formula of gravitation between Sun and photon is as follows:

$$F = -\frac{GMm}{r^2} - \frac{1.5GMm_0^2}{r^4} \quad (30)$$

where:  $r_0$  is the shortest distance between the light and the Sun, if the light and the Sun are tangent, it is equal to the radius of the Sun.

The funny thing is that, for this problem, the maximum gravitational force given by the improved formula is 2.5 times of that given by the original Newton's law of gravity.

#### 5.5 To be restricted (or constrained) by principle (law) of conservation of energy

For the reason that in physics the principles, laws and the like that are regardless of the principle (law) of conservation of energy may be invalid; therefore "certainty-uncertainty principles" should be restricted (or constrained) by principle (law) of conservation of energy, and thus it can satisfy the principle (law) of conservation of energy.

The general form of the principle (law) of conservation of energy is as follows

$$E(t) = E(0) = \text{const}$$

or

$$1 - \frac{E(t)}{E(0)} = 0$$

Thus, referring to reference [13] for applying least square method to establish "partial and temporary unified theory of natural science so far" including all the equations of

natural science so far (in which, the theory of everything to express all of natural laws, described by Hawking that a single equation could be written on a T-shirt, is partially and temporarily realized in the form of "partial and temporary unified variational principle of natural science so far"), Eq.(20) (one kind of "certainty-uncertainty principles" with general form) can be restricted (or constrained) by principle (law) of conservation of energy as follows

$$(\Delta x \Delta p - Kh)^2 + w(1 - \frac{E(t)}{E(0)})^2 = 0 \quad (31)$$

where,  $K$  is a real number and  $K > 0$ ,  $w$  is a suitable positive weighted number.

Similarly, Eq.(21) (one kind of Ozawa type's "certainty-uncertainty principles" with general form) can be restricted (or constrained) by principle (law) of conservation of energy as follows

$$(\Delta Q \Delta P + \Delta Q \sigma(P) + \sigma(Q) \Delta P - Kh)^2 + w(1 - \frac{E(t)}{E(0)})^2 = 0 \quad (32)$$

For Eq.(24) (the variable dimension fractal form of Eq.(20)), it can be restricted (or constrained) by principle (law) of conservation of energy as follows

$$(\Delta x \Delta p - \frac{1}{h^D})^2 + w(1 - \frac{E(t)}{E(0)})^2 = 0 \quad (33)$$

For Eq.(27) (the variable dimension fractal form of Eq.(21)), it can be restricted (or constrained) by principle (law) of conservation of energy as follows

$$(\Delta Q \Delta P - \frac{1}{h^D})^2 + w(1 - \frac{E(t)}{E(0)})^2 = 0 \quad (34)$$

As the cases that "certainty-uncertainty principles" should be restricted (or constrained) by other principles (laws) and the like, similar method can be used.

According to certainty-uncertainty principles, we can present the conclusion as follows: for some properties (such as )

"certainty principle", namely a particle's position and momentum can be known simultaneously; "uncertainty principle", namely a particle's position and momentum cannot be known simultaneously; and neutral (fuzzy) "indeterminacy principle", namely whether or not a particle's position and momentum can be known simultaneously is undetermined.

According to "determine-uncertainty principle", can obtained conclusion: for some properties of strong interaction wave and weak interaction wave (such as shape, energy and the like), there may be three situations: the shape, energy and the like of strong interaction wave and weak interaction wave can be known simultaneously; cannot be known simultaneously; and the neutral (fuzzy) situation that whether or not these properties can be known simultaneously will be undetermined.

## 6 Conservation of parity and nonconservation of parity

In reference [15], we reconsider the problems about conservation of parity and nonconservation of parity, and the experiment of Chien-Shiung Wu et al in 1957, with a new viewpoint. It should also point out that, besides law of conservation of energy, all the physical laws that cannot be derived from law of conservation of energy are probably not correct (or their probabilities of correctness are less than 100%). So far, because only the law of conservation of energy can be qualified to become the unique truth in physics, therefore for all the theories and laws in physics (including the relativity theory, law of gravity, Newton's second law, law of conservation of momentum, law of conservation angular momentum, so-called parity laws, and so on), we should re-examine their relations with law of conservation of energy. In fact, many theories and laws (such as improved Newton's second law and improved law of gravity) can be derived according to law of conservation of energy, which show the great potentiality of law of conservation of energy, and giving full play to the role of law of conservation of energy will completely change the situation of physics. In addition, in a more wide range, the law of conservation of energy can be used to deal with all the problems related to energy in physics, astronomy, mechanics, engineering, chemistry, biology, and the like with a unified way.

In the experiment of Chien-Shiung Wu et al in 1957, they found that the number of the electrons that exiting angle  $\theta > 90^\circ$  is 40% more than that of  $\theta < 90^\circ$ . For this result, we cannot simply say that parity is conservation or nonconservation. The correct way of saying should be that the probability of conservation of parity is 71% and the probability of nonconservation of parity is only 29%.

Similarly, the probabilities of correctness for other laws of conservation should be determined by law of conservation of energy or experiment (currently for the most cases the correctness can only be determined by experiment).

## 7 A Revision to Gödel's Incompleteness Theorem

According to Gödel's incompleteness theorem, some scholars concluded: there is no unified theory. In other words, they should think that the unified theory of nuclear (force) waves cannot exist certainly.

Nevertheless, a revision to Gödel's incompleteness theorem is presented in reference [16].

As well-known, the most celebrated results of Gödel are as follows.

Gödel's First Incompleteness Theorem: Any adequate axiomatizable theory is incomplete.

Gödel's Second Incompleteness Theorem: In any consistent axiomatizable theory which can encode sequences of numbers, the consistency of the system is not provable in the system.

In literature, the Gödel's incompleteness theorem is usually briefly stated as follows: Any formal mathematical axiom system is "incomplete", because it always has one proposition that can neither be proved, nor disproved.

Gödel's incompleteness theorem is a significant result in history of mathematical logic and has greatly influenced mathematics, physics and philosophy among others.

But, any theory cannot be the ultimate truth. With the progress of science, new theories will be proposed to replace the old ones. Gödel's incompleteness theorem

cannot be an exception. In fact, Gödel's Incompleteness theorem can be revised into the incompleteness axiom with the Smarandache's neutrosophy.

It should be noted that, there are some errors in the proof of Gödel's incompleteness theorem.

It will be shown that in the proof of Gödel's incompleteness theorem, all possible situations are not considered.

First, in the proof, the following situation is not considered: wrong results can be deduced from some axioms. For example, from the axiom of choice a paradox, the doubling ball theorem, can be deduced, which says that a ball of volume 1 can be decomposed into pieces and reassembled into two balls both of volume 1. It follows that in certain cases, the proof of Gödel's incompleteness theorem may be faulty.

Second, in the proof of Gödel's incompleteness theorem, only four situations are considered, that is, one proposition can be proved to be true, cannot be proved to be true, can be proved to be false, cannot be proved to be false and their combinations such as one proposition can neither be proved to be true nor be proved to be false. But those are not all possible situations. In fact, there may be many kinds of indeterminate situations, including it can be proved to be true in some cases and cannot be proved to be true in other cases; it can be proved to be false in some cases and cannot be proved to be false in other cases; it can be proved to be true in some cases and can be proved to be false in other cases; it cannot be proved to be true in some cases and cannot be proved to be false in other cases; it can be proved to be true in some cases and can neither be proved to be true, nor be proved to be false in other cases; and so on.

Because so many situations are not considered, we may say that the proof of Gödel's incompleteness theorem is faulty, at least, is not one with all sided considerations.

In order to better understand the case, we consider an extreme situation, where one proposition as shown in Gödel's incompleteness theorem can neither be proved, nor disproved. It may be assumed that this proposition can be proved in 9999 cases, only in 1 case it can neither be proved, nor disproved. We will see whether or not this situation has been considered in the proof of Gödel's incompleteness theorem.

Some people may argue that, this situation is equivalent to the one where the proposition can neither be proved, nor disproved. But the difference lies in the distinction between the part and the whole. If one case may represent the whole situation, many important theories cannot be applied. For example the general theory of relativity involves singular points, the law of universal gravitation does not allow the case where the distance  $r$  is equal to zero. Accordingly, whether or not one may say that the general theory of relativity and the law of universal gravitation cannot be applied as a whole? Similarly, the situation also cannot be considered as the one that can be proved. But, this problem may be easily solved with the neutrosophic method.

Moreover, if we apply the Gödel's incompleteness theorem to itself, we may obtain the following possibility: in one of all formal mathematical axiom systems, the Gödel's incompleteness theorem can neither be proved, nor disproved.

If all possible situations can be considered, the Gödel's incompleteness theorem can be improved in principle. But, with our boundless universe being ever changing and being extremely complex, it is impossible "considering all possible situations". As far as

"considering all possible situations" is concerned, the Smarandache's neutrosophy is quite good, possibly, the best. Therefore we propose to revise the Gödel's incompleteness theorem into the incomplete axiom with Smarandache's neutrosophy.

### 7.1 The Incompleteness Axiom

Considering all possible situations with Smarandache's neutrosophy, one may revise the Gödel's Incompleteness theorem into the incompleteness axiom: Any proposition in any formal mathematical axiom system will represent, respectively, the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of  $] -0, 1+[$ .

### 7.2 Several Famous Paradoxes in History

The proof of Gödel's incompleteness theorem has a close relation with some paradoxes.

However, after considering all possible situations, any paradox may no longer be a paradox.

Now we discuss several famous paradoxes in history.

Example 1. The "Barber paradox", one of Russell's paradoxes.

Consider all men in a small town as members of a set. Now imagine that a barber puts up a sign in his shop that reads I shave all those men, and only those men, who do not shave themselves. Obviously, we may divide the set of men in this town into two subsets, those who shave themselves, and those who are shaved by the barber. To which subset does the barber himself belong? The barber cannot belong to the first subset, because if he shave himself, he will not be shaved by the barber, or by himself; he cannot not belong to the second subset as well, because if he is shaved by the barber, or by himself, he will not be shaved by the barber.

Now we will see from where comes the contradiction.

The contradiction comes from the fact that the barber's rule does not take all possible situations into consideration.

First, we should divide the set of men in this town into three subsets, those who shave themselves, those who are shaved by the barber, and those who neither shave themselves, nor are shaved by the barber. If the barber belongs to the third subset, no contradiction will appear. For this purpose, the barber should declare himself that he will be the third kind of person, and from now on, he will not be shaved by anyone; otherwise, if the barber's mother is not a barber, he can be shaved by his mother.

Second, the barber cannot shave all men in this town. For example, the barber cannot shave those who refuse to be shaved by the barber. Therefore, if the barber is the one who cannot shave himself and "who refuse to be shaved by the barber" , no contradiction will occur.

There also exist indeterminate situations to avoid the contradiction. The barber may say: "If I meet men from another universe, I will shave myself, otherwise I will not shave myself."

Example 2. "Liar's paradox", another Russell's paradox.

Epimenides was a Cretan who said that, "All Cretans are liars". Is this statement true

or false? If this statement is true, he (a Cretan) is a liar, therefore, this statement is false; if this statement is false, that means that he is not a liar, this statement will be true. Therefore, we always come across a contradiction.

Now we will see from where comes the contradiction.

First, here the term "liar" should be defined. Considering all possible situations, a "liar" can be one of the following categories: those whose statements are all lies; those whose statements are partly lies, and partly truths; those whose statements are partly lies, partly truths and sometimes it is not possible to judge whether they are truths or lies. For the sake of convenience, we will not consider the situation where it is not possible to judge whether the statements are true or false.

Next, the first kind of liar is impossible, i.e., a Cretan could not be a liar whose statements are all lies. This conclusion can not be reached by deduction, instead, it is obtained through experience and general knowledge. With the situation where a liar's statements are partly truths, and partly lies, Epimenides' statement "All Cretans are liars", will not cause any contradiction. According to the definitions of "liar" of the second category and the fact that Epimenides' statements could not be all lies, this particular statement of Epimenides' can be true and with his other statements being possibly lies, Epimenides may still be a liar.

Example 3. Dialogue paradox. Considering the following dialogue between two persons A and B.

A: What B says is true. B: What A says is false.

If the statement of A is true, it follows that the statement of B is true, that is, the statement "What A says is false" is true, which implies that the statement of A must be false. We come to a contradiction..

On the other hand, if the statement of A is false, it follows that the statement of B must be false, that is, the statement "What A says is false" is false, which implies that the statement of A must be true. We also come to a contradiction.

So the statement of A could neither be true nor false.

Now we will see how that contradiction is solved.

It should be noted that, this dialogue poses a serious problem. If A speaks first, before B says anything, how can A know whether or not what B says is true? Otherwise, if B speaks first, B would not know whether what A says is true or false. If A and B speak at the same time, they would not know whether the other's statement is true or false.

For solving this problem, we must define the meaning of "lie". In general situations the "lie" may be defined as follows: with the knowledge of the facts of a case, a statement does not tally with the facts. But in order to consider all possible situations, especially those in this dialogue, another definition of "lie" must be given. For the situation when one does not know the facts of the case, and one makes a statement irresponsibly, can this statement be defined as a "lie"? There exist two possibilities: It is a lie, and it is not a lie. For either possibility, the contradiction can be avoided.

Consider the first possibility (It is a lie).

If A speaks first, before B makes his statement, it follows that A does not know the facts of the case, and makes the statement irresponsibly, it is a lie. Therefore the statement of A is false. B certainly also knows this point, therefore B's statement: "What A

says is false", is a truth.

Whereas, if B speaks first before A makes his statement, it follows that B does not know the facts of the case, and makes the statement irresponsibly, it is a lie. Therefore the statement of B is false. A certainly also knows this point, therefore A's statement: "What B says is true", is false.

If A and B speak at the same time, it follows that A and B do not know the facts of the case, and make their statements irresponsibly, these statements are all lies. Therefore, the statements of A and B are all false.

Similarly, consider the second possibility (It is not a lie), the contradiction also can be avoided.

If we do not consider all the above situations, what can we do? With a lie detector! The results of the lie detector can be used to judge whose statement is true, whose statement is false.

According to incompleteness axiom, partial and temporary unified theory so far can exist. In this paper, we will discuss partial and temporary unified theory of nuclear (force) waves so far, and the like.

## 8 Several partial and temporary unified theories so far

### 8.1 Applying least square method to establish "partial and temporary unified gravitational theory so far"

Firstly, it should be noted that, for different gravitational problems, the different formulas or different gravitational theories should be applied. The "universal gravitational formulas or equations" actually cannot exist. For this conclusion, many scholars do not realize it. In addition, all of the different gravitational formulas can be written as the form of that the right side of the expression is equal to zero.

The first formula should be mentioned is Newton's universal gravitational formula

$$F = -\frac{GMm}{r^2} \quad (35)$$

It can be written as the following form

$$F_1 = 0 \quad (35')$$

where:  $F_1 = F + \frac{GMm}{r^2}$

Prof. Hu Ning derived an equation according to general relativity, with the help of Hu's equation and Binet's formula, in reference [17] we derived the following improved Newton's formula of universal gravitation

$$F = -\frac{GMm}{r^2} - \frac{3G^2M^2mp}{c^2r^4} \quad (36)$$

where: G is gravitational constant, M and m are the masses of the two objects, r is the distance between the two objects, c is the speed of light, p is the half normal chord for the

object  $m$  moving around the object  $M$  along with a curve, and the value of  $p$  is given by:  $p = a(1-e^2)$  (for ellipse),  $p = a(e^2-1)$  (for hyperbola),  $p = y^2/2x$  (for parabola).

This formula can give the same results as given by general relativity for the problem of planetary advance of perihelion and the problem of gravitational deflection of a photon orbit around the Sun.

It can be written as the following form

$$F_2 = 0 \quad (36')$$

where: 
$$F_2 = F + \frac{GMm}{r^2} + \frac{3G^2M^2mp}{c^2r^4}$$

It should be noted that, according to Eq.(35) and Eq.(36) the FTL (faster than light) can exist.

In some cases, we should also consider the following gravitational formula including three terms

$$F = -\frac{GMm}{r^2} \left( 1 + \frac{3GMp}{c^2r^2} + \frac{wG^2M^2p^2}{c^4r^4} \right) \quad (37)$$

where:  $w$  is a constant to be determined.

It can be written as the following form

$$F_3 = 0 \quad (37')$$

where: 
$$F_3 = F + \frac{GMm}{r^2} \left( 1 + \frac{3GMp}{c^2r^2} + \frac{wG^2M^2p^2}{c^4r^4} \right)$$

But for the example that a small ball rolls along the inclined plane in the gravitational field of the Earth, all of the above mentioned formulas cannot be applied. In reference [18], we present the following gravitational formula with the variable dimension fractal form (the fractal dimension is variable, instead of constant).

$$F = -GMm r^{2-\delta} \quad (38)$$

where:  $\delta = 1.206 \times 10^{-12} u$ ,  $u$  is the horizon distance that the small ball rolls.

It can be written as the following form

$$F_4 = 0 \quad (38')$$

where: 
$$F_4 = F + GMm / r^{2-\delta}$$

In addition, the gravitational field equations of Einstein's theory of general relativity, and the gravitational formula and gravitational equations derived by other scholars, can also be written as the form of that the right side of the expression is equal to zero.

In some cases, when dealing with gravitational problem, we should also consider some principle of conservation, such as the principle of conservation of energy. Here we write the principle of conservation of energy as the form of that the right side of the expression is equal to zero. So do the other principles of conservation.

In references [13,14], we discussed two cases to apply the principle of conservation of energy directly and indirectly.

To apply the principle of conservation of energy directly is as follows.

Supposing that the initial total energy of a closed system is equal to  $W(0)$ , and for time  $t$  the total energy is equal to  $W(t)$ , then according to the principle of conservation of energy, it gives

$$W(0) = W(t) \quad (39)$$

It can be written as the following form

$$F_5 = \frac{W(t)}{W(0)} - 1 = 0 \quad (40)$$

To apply the principle of conservation of energy indirectly is as follows.

Supposing that we are interested in a special physical quantity  $Q$ , not only it can be calculated by using the principle of conservation of energy, but also can be calculated by using other gravitational formula. For distinguishing the values, let's denote the value given by other laws as  $Q$ , while denote the value given by the principle of conservation of energy as  $Q'$ , then the equation to apply the principle of conservation of energy indirectly is as follows

$$F_6 = \frac{Q}{Q'} - 1 = 0 \quad (41)$$

Now we discuss some solitary equations established only for the solitary points or special cases.

The first one is the solitary equation about the gravitational constant.

$$S_1 = G - 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2 = 0 \quad (42)$$

The second one is considering the deflection angle for the problem of gravitational deflection of a photon orbit around the Sun.

By using general relativity or improved Newton's formula of universal gravitation (namely Eq.(36)), the deflection angle  $\phi_0$  reads

$$\phi_0 = 1.75''$$

However, according to the experiment, we should have  $\phi = 1.77 \pm 0.20$ , taking the average, it gives

$$\phi = 1.77''$$

According to this expression, the corresponding solitary equation is as follows

$$S_2 = \phi - 1.77'' = 0 \quad (43)$$

Other solitary equations include: the solitary equations established by the values of

planetary advance of perihelion, the solitary equations established by the unusual values of gravity at different times during total solar eclipse, and the like. Due to the limited space, they are no longer listed.

Applying least square method, "partial and temporary unified gravitational theory so far" can be expressed in the following form of "partial and temporary unified gravitational variational principle so far"

$$\Pi_{\text{GRAVITY}} = \sum_1^n W_i \int_{\Omega_i} F_i^2 d\Omega_i + \sum_1^m W_j' S_j^2 = \min_0 \quad (44)$$

where: the subscript GRAVITY denotes that the suitable scope is the gravity, all of the equations  $F_i = 0$  denote so far discovered (derived) all of the equations related to gravity, all of the equations  $S_i = 0$  denote so far discovered (derived) all of the solitary equations related to gravity, and  $W_i$  and  $W_j'$  are suitable positive weighted constants.

It should be noted that, as we establish "partial and temporary unified theory so far" and the corresponding "partial and temporary unified variational principle so far", the including phenomenon is allowed. For example, the three terms gravitational formula Eq.(37) includes Eq.(36), while Eq.(36) includes Eq.(35). But we still consider these three equations simultaneously. This is because that, in some cases Eq.(36) is more convenient; as for Eq.(35), it is enough in most cases, moreover, putting Eq.(35) at the most prominent position, express our respect to Newton who is the greatest scientist in the history. In addition, the coexisting phenomenon is also allowed. For example, the gravitational formulas of classical mechanics, the gravitational field equations of Einstein's theory of general relativity, and the equations of other gravitational theories are coexisting. For the solution that is satisfying two or more than two theories simultaneously, or solving the problems in different fields simultaneously, and the like, we will discuss them in other papers (such solutions may only be reached with the method of variational principle).

Now we discuss the applications of variational principle Eq.(44).

Example 1. Setting  $W_2 = 1$  and  $W_1' = 1$  in variational principle Eq.(44), and other weighted constants are all equal to 0, namely applying Eq.(36) and Eq.(42) to derive the changing rule for the gravitational coefficient  $G'$  (instead of the gravitational constant  $G$ ) and make the gravitational formula in accordance with the inverse square law.

In reference [19], changing Eq.(36) into the following form in accordance with the inverse square law

$$F = -\frac{G'Mm}{r^2}$$

It gives

$$-\frac{G'Mm}{r^2} = -\frac{GMm}{r^2} - \frac{3G^2M^2mp}{c^2r^4}$$

Then we have the changing rule for the gravitational coefficient  $G'$  as follows

$$G' = G \left( 1 + \frac{3GM}{c^2 r^2} \right) \quad (45)$$

For problem of Mercury's advance of perihelion, we have

$$(1 + 5.038109 \times 10^{-8})G \leq G' \leq (1 + 1.162308 \times 10^{-7})G$$

For problem of gravitational deflection of a photon orbit around the Sun, we have

$$G \leq G' \leq 2.5G$$

Example 2. Setting  $W_4 = 1$  and  $W_6 = 1$  in variational principle Eq.(44), and other weighted constants are all equal to 0, namely applying Eq.(35) and Eq.(41) to determine the unknown  $\delta$  in Eq.(36).

According to Eq.(41), variational principle Eq.(44) can be simplified into the following form applied the law of conservation of energy indirectly

$$\Pi = \int_{x_1}^{x_2} \left( \frac{Q}{Q'} - 1 \right)^2 dx = \min \quad (46)$$

The solution procedure can be found in references [13,14]. For the final optimum approximate solution, the value of  $\Pi$  calculated by the improved universal gravitational formula and improved Newton's second law is equal to 0.1906446, it is only 0.033% of the value of  $\Pi_0$  calculated by the original universal gravitational formula and original Newton's second law.

Example 3. Setting  $W_3 = 1$  and  $W_2' = 1$  in variational principle Eq.(44), and other weighted constants are all equal to 0, namely applying Eq.(37) and Eq.(43) to determine the unknown  $w$  in Eq.(37).

The solution procedure can be found in references [13,14], the final result is as follows.

The range of value of  $w$  is as follows

$$0.08571 \leq w \leq 0.42857$$

Taking the average, it gives

$$w = 0.25714$$

For the problem of gravitational deflection of a photon orbit around the Sun, the general relativity cannot give the solution that is exactly equal to the experimental value, while the method presented in this paper can do so.

It should be noted that, for variation principle Eq.(44), if there is an exact solution, then its right side can be equal to 0, here the variational principle Eq.(44) is exactly equivalent to  $F_i = 0$  and  $S_i = 0$  (see example 1 and example 3). If there is only an approximate solution, the right side of variational principles Eq.(44) can only be approximately equal to 0, at this moment we can apply the appropriate optimization method to seek the best approximate solution, and the effect of the solution can be judged

according to the extent that the value of  $\Pi$  is close to 0 (see example 2).

## 8.2 Partial and temporary unified theory of nuclear (force) waves so far, partial and temporary unified theory of natural science so far, and the like

To extend the above mentioned method, we can get various "partial and temporary unified theories so far".

For unified dealing with the problems of four fundamental interactions, applying least square method, "partial and temporary unified theory of four fundamental interactions so far" can be expressed in the following form of "partial and temporary unified variational principle of four fundamental interactions so far"

$$\Pi_{\text{G.E.S.W}} = \sum_1^n W_i \int_{\Omega_i} F_i^2 d\Omega_i + \sum_1^m W_j' S_j^2 = \min_0 \quad (47)$$

where: the subscript G.E.S.W denotes that the suitable scope is the four fundamental interactions, all of the equations  $F_i = 0$  denote so far discovered (derived) all of the equations related to four fundamental interactions, all of the equations  $S_j = 0$  denote so far discovered (derived) all of the solitary equations related to four fundamental interactions, and  $W_i$  and  $W_j'$  are suitable positive weighted constants.

It should be noted that, in "partial and temporary unified variational principle of four fundamental interactions so far", if only the contents related to nuclear (force) waves (strong interaction wave and weak interaction wave) are held, and all other contents are cancelled, then it gives the partial and temporary unified theory of nuclear (force) waves so far.

For unified dealing with the problems of natural science, applying least square method, "partial and temporary unified theory of natural science so far" can be expressed in the following form of "partial and temporary unified variational principle of natural science so far"

$$\Pi_{\text{NATURE}} = \sum_1^n W_i \int_{\Omega_i} F_i^2 d\Omega_i + \sum_1^m W_j' S_j^2 = \min_0 \quad (48)$$

where: the subscript NATURE denotes that the suitable scope is all of the problems of natural science, all of the equations  $F_i = 0$  denote so far discovered (derived) all of the equations related to natural science, all of the equations  $S_j = 0$  denote so far discovered (derived) all of the solitary equations related to natural science, and  $W_i$  and  $W_j'$  are suitable positive weighted constants.

In this way, the theory of everything to express all of natural laws, described by Hawking that a single equation could be written on a T-shirt, is partially and temporarily realized in the form of "partial and temporary unified variational principle of natural science

so far".

As already noted, for "partial and temporary unified theory so far" and the corresponding "partial and temporary unified variational principle so far", the including phenomenon and coexisting phenomenon are allowed. Here we would like to point out that, besides the including process and coexisting process, the simplifying process is also allowed. For example, the first simplifying result of "partial and temporary unified theory of natural science so far" is "theory of conservation of energy", it can be expressed in the following form of "first simplifying variational principle for partial and temporary unified theory of natural science so far" (it is shorted as "variational principle of conservation of energy").

$$\Pi_{\text{NATURE}}^{\text{SIMPLE-1}} = \int_{t_1}^{t_2} (W(t)/W(0) - 1)^2 dt = \min_0 \quad (49)$$

This "variational principle of conservation of energy" can be applied for unified dealing with many problems in physics, mechanics, astronomy, biology, engineering, and even many issues in social science. For example, in reference [20], based on "theory of conservation of energy", for some cases we derived Newton's second law, the law of universal gravitation, and the like.

Further topics are finding more simplifying processes (simplifying variational principles) and their combinations. These will make "partial and temporary unified theory of natural science so far" simpler, clearer, more perfect, and more practical.

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