Abstract - The holographic principle is applied to the weak interactions as a means of estimating the zero-point energy of a gas of degenerate neutrinos filling the universe.

In a nice paper entitled: “Cosmological Interpretation of the Weak Forces”, Satio Hayakawa [1], have discussed the cosmological origin of the weak forces (please see also reference [2]). In those papers [1,2], Hayakawa equaled the zero-point energy of a neutrino gas filling the universe to the product: - the Fermi interaction $G_F$ times the number density of the neutrino gas $N_\nu / V$. The equality reads

$$\frac{1}{2} \hbar \omega = G_F (N_\nu / V). \quad (1)$$

In order to estimate the zero-point energy of this system Hayakawa [1,2] took in account the balance between the attractive gravitational interaction of the gas of particles and the repulsive energy of a degenerate Fermi gas. He also considered the relation

$$N_\nu m_\nu = N_n m_n. \quad (2)$$

In (2), $N_\nu$ is the number of neutrinos in the universe and $m_\nu$ their masses, being $N_n$ the number and $m_n$ the mass of the nucleons. In his calculations, Hayakawa estimated the zero-point energy of the neutrinos oscillations, the number of these particles filling the universe and the neutrino mass. By using relation (1),
he also was able to estimate the weak Fermi constant $G_F$. Inspired in Hayakawa ideas [1,2], we have worked out a modified model [3], which results are in relative agreement with the previous ones obtained by him. Besides this, by using relation (1) we got an expression for $G_F$ as a function of $\alpha$ (the fine-structure constant and $M_W$ (the mass of the boson which intermediates the weak interactions). When compared with the usual way of to express GF as found in the literature [4], we were able to determine a value for the electroweak mixing angle.

Recently the Holographic Principle (HP) [5,6,7] has been proposed as a means to explain that the quantity of information contained in a space-volume in the presence of a gravitational field is encoded in the surface area of its event horizon. The aim of the present note is to use HP in order to deduce relation (1).

Here we quote two postulates of HP as stated by McMahon [7], but adapted to the present calculations. They are

. The total information content in a volume of space is equivalent to a theory that lives only on the surface area that encloses the region.

. The boundary of a region of this volume contains at most a single degree of freedom per Fermi area.

The modified second postulate deserves some explanation. The Fermi coupling constant $G_F$ plays an analogous role as the Newton constant $G$ in the gravitational interaction case. Therefore it is possible to define, by taking $\hbar = c = 1$,

$$L_{SF} = \sqrt{G_F}.$$  \hspace{1cm} (3)

We call $L_{SF}$, the second Fermi length (not to be confounded with the Fermi length of the free electrons in metals, for instance). Naturally, the idea behind (3), permit us to define the Fermi time $t_F$ and the Fermi mass $M_F$. We have

$$t_F = L_{SF}/c, \quad \text{and} \quad M_F = 1/\sqrt{G_F}.$$  \hspace{1cm} (4)
Fermi units of weak interactions, the analogous of the Planck units in the gravitational case, have been discussed in more details in a previous paper [8]. In reference [8] we have defined a modified Fermi coupling $G_F^*$ through the relation

$$G_F^* = G_F \frac{c^2}{\hbar^2}. \quad (5)$$

From (5) we observe that $G_F^*$ coincides with $G_F$, once we take $\hbar = c = 1$.

Let us apply the holographic principle (HP) to our problem. By using the stationary condition for the energy we write

$$\Delta F = \Delta U - T \Delta S = 0. \quad (6)$$

Now we define ($h = c = k_B = 1$)

$$\Delta U = N_0 \frac{c}{2R}, \quad \nu = 3T, \quad \Delta S = \pi R^2 / G_F. \quad (7)$$

Inserting (7) into (6) we obtain

$$N_0 \ G_F = (\nu / 2) \ (4/3) \ \pi \ R^3. \quad (8)$$

Putting $(4/3) \ \pi \ R^3$ equal to $V$ (the volume of the universe) and recovering $h$, we get

$$G_F N_0 / V = \frac{1}{2} \ h \nu - \frac{1}{2} \ h \omega, \quad (9)$$
A result quoted by Hayakawa [1,2].

It is worth to point out that, as was first proposed by Roberto Onofrio [9] in a form of a conjecture: “the weak interactions are peculiar manifestations of quantum gravity at the Fermi scale, and the Fermi constant is related to the Newtonian constant of gravitation.”

Therefore definitions (3) and (4) keep a direct relationship with Onofrio’s work [9].

References


