Two Types of Massless Fields: Dark Energy, Dark Matter and Levitation

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In the present paper we consider the massless fields described by two types of potentials with different space-time properties and different Lorentz transformations. In particular, we discuss the consequences of such approach in application to the electromagnetic field and weak gravity. The possible application for the description of dark matter and dark energy is discussed.

1. Introduction

There is some asymmetry between Lorentz transformations for potentials and field strengths in electrodynamics. The potentials are transformed as the components of four-vector while the field strengths as the component of four-tensor [1]. However, recently we proposed an alternative approach to describe the fields on the basis of sixteen component sedeonic potentials, which uses both types of Lorentz transformations [2]. In particular, it was shown that the fields having a massive photon can be described by sedeonic wave equation, which can be represented as a system of equations similar to the Maxwell's equations [3].

In the present paper we consider the description of massless fields on the basis of equations obtained as the limiting transition from the sedeonic equations for massive field.

2. Algebra of apace-time sedeons

The algebra of sedeons [2, 4] encloses four groups of values, which are differed with respect to spatial and time inversion.

- Absolute scalars (V) and absolute vectors ($\vec{V}$) are not transformed under spatial and time inversion.
- Time scalars ($V_{\mathbf{t}}$) and time vectors ($\vec{V}_{\mathbf{t}}$) are changed (in sign) under time inversion and are not transformed under spatial inversion.
- Space scalars ($V_{\mathbf{r}}$) and space vectors ($\vec{V}_{\mathbf{r}}$) are changed under spatial inversion and are not transformed under time inversion.
- Space-time scalars ($V_{\mathbf{tr}}$) and space-time vectors ($\vec{V}_{\mathbf{tr}}$) are changed under spatial and time inversion.

Here indexes $\mathbf{t}$ and $\mathbf{r}$ indicate the transformations ($\mathbf{t}$ for time inversion and $\mathbf{r}$ for spatial inversion), which change the corresponding values. All introduced values can be integrated into one space-time sedeon $\tilde{\mathbf{V}}$, which is defined by the following expression:

$$\tilde{\mathbf{V}} = V + \vec{V} + V_{\mathbf{t}} + \vec{V}_{\mathbf{t}} + V_{\mathbf{r}} + \vec{V}_{\mathbf{r}} + V_{\mathbf{tr}} + \vec{V}_{\mathbf{tr}} . \qquad (2.1)$$

Let us introduce a scalar-vector basis $\mathbf{a}_0$, $\vec{\mathbf{a}}_1$, $\vec{\mathbf{a}}_2$, $\vec{\mathbf{a}}_3$, where the element $\mathbf{a}_0$ is an absolute scalar unit ($\mathbf{a}_0 \equiv 1$), and the values $\vec{\mathbf{a}}_1$, $\vec{\mathbf{a}}_2$, $\vec{\mathbf{a}}_3$ are absolute unit vectors generating the right Cartesian basis. Further we will indicate the absolute unit vectors by symbols without arrows as $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$. We also introduce the four space-time units $\mathbf{e}_0$, $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$, where $\mathbf{e}_0$ is an absolute scalar unit ($\mathbf{e}_0 \equiv 1$); $\mathbf{e}_1$ is a time scalar unit ($\mathbf{e}_1 \equiv \mathbf{e}_{\mathbf{t}}$); $\mathbf{e}_2$ is a space scalar unit ($\mathbf{e}_2 \equiv \mathbf{e}_{\mathbf{r}}$); $\mathbf{e}_3$ is a space-time scalar unit ($\mathbf{e}_3 \equiv \mathbf{e}_{\mathbf{tr}}$). Using space-time basis $\mathbf{e}_\alpha$ and scalar-vector basis $\mathbf{a}_\beta$ (Greek indexes $\alpha, \beta = 0, 1, 2, 3$), we can introduce unified sedeonic components $V_{\alpha\beta}$ in accordance with following relations:

$$
\begin{aligned}
V &= \mathbf{e_0} V_{00} \mathbf{a_0}, \\
\vec{V} &= \mathbf{e_0} \left(V_{01}\mathbf{a_1} + V_{02}\mathbf{a_2} + V_{03}\mathbf{a_3} \right), \\
V_{\mathbf{t}} &= \mathbf{e_1} V_{10} \mathbf{a_0}, \\
\vec{V}_{\mathbf{t}} &= \mathbf{e_1} \left(V_{11}\mathbf{a_1} + V_{12}\mathbf{a_2} + V_{13}\mathbf{a_3} \right), \\
V_{\mathbf{r}} &= \mathbf{e_2} V_{20} \mathbf{a_0}, \\
\vec{V}_{\mathbf{r}} &= \mathbf{e_2} \left(V_{21}\mathbf{a_1} + V_{22}\mathbf{a_2} + V_{23}\mathbf{a_3} \right), \\
V_{\mathbf{tr}} &= \mathbf{e_3} V_{30} \mathbf{a_0}, \\
\vec{V}_{\mathbf{tr}} &= \mathbf{e_3} \left(V_{31}\mathbf{a_1} + V_{32}\mathbf{a_2} + V_{33}\mathbf{a_3} \right).
\end{aligned}
\tag{2.2}
$$

Then sedeon (2.1) can be written in the following expanded form:

$$
\begin{aligned}
\tilde{\mathbf{V}} = & \mathbf{e_0} \left(V_{00}\mathbf{a_0} + V_{01}\mathbf{a_1} + V_{02}\mathbf{a_2} + V_{03}\mathbf{a_3} \right) \\
& + \mathbf{e_1} \left(V_{10}\mathbf{a_0} + V_{11}\mathbf{a_1} + V_{12}\mathbf{a_2} + V_{13}\mathbf{a_3} \right) \\
& + \mathbf{e_2} \left(V_{20}\mathbf{a_0} + V_{21}\mathbf{a_1} + V_{22}\mathbf{a_2} + V_{23}\mathbf{a_3} \right) \\
& + \mathbf{e_3} \left(V_{30}\mathbf{a_0} + V_{31}\mathbf{a_1} + V_{32}\mathbf{a_2} + V_{33}\mathbf{a_3} \right).
\end{aligned}
\tag{2.3}
$$

The sedeonic components $V_{\alpha\beta}$ are numbers (complex in general). Further we will omit units $\mathbf{a_0}$ and $\mathbf{e_0}$ for the simplicity. The important property of sedeons is that the equality of two sedeons means the equality of all sixteen components $V_{\alpha\beta}$.

Let us consider the multiplication rules for the basis elements $\mathbf{a_n}$ and $\mathbf{e_k}$ (Latin indexes **n**, **k** = 1, 2, 3). The unit vectors $\mathbf{a_n}$ have the following multiplication and commutation rules:

$$\mathbf{a_n}\mathbf{a_n} = \mathbf{a_n}^2 = 1, \tag{2.4}$$

$$\mathbf{a_n}\mathbf{a_k} = -\mathbf{a_k}\mathbf{a_n} \text{ (for } \mathbf{n} \neq \mathbf{k} \text{)}, \tag{2.5}$$

$$\mathbf{a_1}\mathbf{a_2} = i\mathbf{a_3}, \quad \mathbf{a_2}\mathbf{a_3} = i\mathbf{a_1}, \quad \mathbf{a_3}\mathbf{a_1} = i\mathbf{a_2}, \tag{2.6}$$

while the space-time units $\mathbf{e_k}$ satisfy the following rules:

$$\mathbf{e_k}\mathbf{e_k} = \mathbf{e_k}^2 = 1, \tag{2.7}$$

$$\mathbf{e_n}\mathbf{e_k} = -\mathbf{e_k}\mathbf{e_n} \text{ (for } \mathbf{n} \neq \mathbf{k} \text{)}, \tag{2.8}$$

$$\mathbf{e_1}\mathbf{e_2} = i\mathbf{e_3}, \quad \mathbf{e_2}\mathbf{e_3} = i\mathbf{e_1}, \quad \mathbf{e_3}\mathbf{e_1} = i\mathbf{e_2}. \tag{2.9}$$

Here and further the value i is imaginary unit ($i^2 = -1$). The multiplication and commutation rules for sedeonic absolute unit vectors $\mathbf{a_n}$ and space-time units $\mathbf{e_k}$ can be presented for obviousness as the tables 1 and 2.

Table 1. Multiplication rules for absolute unit vectors $\mathbf{a_n}$.

	$\mathbf{a_1}$	$\mathbf{a_2}$	$\mathbf{a_3}$
$\mathbf{a_1}$	1	$i\mathbf{a_3}$	$-i\mathbf{a_2}$
$\mathbf{a_2}$	$-i\mathbf{a_3}$	1	$i\mathbf{a_1}$
$\mathbf{a_3}$	$i\mathbf{a_2}$	$-i\mathbf{a_1}$	1

Table 2. Multiplication rules for space-time units $\mathbf{e_k}$.

	$\mathbf{e_1}$	$\mathbf{e_2}$	$\mathbf{e_3}$
$\mathbf{e_1}$	1	$i\mathbf{e_3}$	$-i\mathbf{e_2}$
$\mathbf{e_2}$	$-i\mathbf{e_3}$	1	$i\mathbf{e_1}$
$\mathbf{e_3}$	$i\mathbf{e_2}$	$-i\mathbf{e_1}$	1

Note that units $\mathbf{e_k}$ commute with vectors $\mathbf{a_n}$:

$$\mathbf{a_n e_k} = \mathbf{e_k a_n} \tag{2.10}$$

for any $\mathbf{n}$ and $\mathbf{k}$.

In sedeonic algebra we assume the Clifford multiplication of vectors. The sedeonic product of two vectors $\vec{A}$ and $\vec{B}$ can be presented in the following form:

$$\vec{A}\vec{B} = \left(\vec{A}\cdot\vec{B}\right) + \left[\vec{A}\times\vec{B}\right]. \tag{2.11}$$

Here we denote the sedeonic scalar multiplication of two vectors (internal product) by symbol "$\cdot$" and round brackets

$$\left(\vec{A}\cdot\vec{B}\right) = A_1B_1 + A_2B_2 + A_3B_3 , \tag{2.12}$$

and sedeonic vector multiplication (external product) by symbol "$\times$" and square brackets

$$\left[\vec{A}\times\vec{B}\right] = i\left(A_2B_3 - A_3B_2\right) + i\left(A_3B_1 - A_1B_3\right) + i\left(A_1B_2 - A_2B_1\right). \tag{2.13}$$

Note that in sedeonic algebra the expression for the vector product differs from analogous expression in Gibbs vector algebra. For the transition from sedeons to the common used Gibbs-Heaviside vector algebra the change

$$i\left[\vec{\nabla}\times\vec{A}\right] \Rightarrow -\left[\vec{\nabla}\times\vec{A}\right] \tag{2.14}$$

should be made in all vector expressions.

3. Sedeonic equations for massive field

To begin with we shortly recall the sedeonic equations for massive field [3]. Let us consider the massive field with mass of quantum m_0. We introduce the following operators

$$\begin{aligned} \partial &= \frac{1}{c}\frac{\partial}{\partial t}, \\ \vec{\nabla} &= \frac{\partial}{\partial x}\mathbf{a_1} + \frac{\partial}{\partial y}\mathbf{a_2} + \frac{\partial}{\partial z}\mathbf{a_3}, \\ m &= \frac{m_0 c}{\hbar}, \end{aligned} \tag{3.1}$$

where c is speed of light, $\hbar$ is the Plank constant. Then the sedeonic second-order wave equation for massive field can be presented as [3]:

$$\left(i\mathbf{e_t}\partial - \mathbf{e_r}\vec{\nabla} - i\mathbf{e_{tr}}m\right)\left(i\mathbf{e_t}\partial - \mathbf{e_r}\vec{\nabla} - i\mathbf{e_{tr}}m\right)\tilde{\mathbf{W}} = \tilde{\mathbf{J}} , \tag{3.2}$$

where $\tilde{\mathbf{W}}$ is a sedeonic potential, $\tilde{\mathbf{J}}$ is a phenomenological sedeonic source of massive field. Let us choose the potential as

$$\tilde{\mathbf{W}} = ia_1\mathbf{e_t} - ia_2\mathbf{e_r} + a_3 - ia_4\mathbf{e_{tr}} + \vec{A}_1\mathbf{e_r} + \vec{A}_2\mathbf{e_t} - \vec{A}_3\mathbf{e_{tr}} + i\vec{A}_4 , \tag{3.3}$$

where components a_s and $\vec{A}_s$ are real functions of coordinates and time. Here and further the index s = 1, 2, 3, 4. Also we take the source in the following form:

$$\tilde{\mathbf{J}} = -i\rho_1\mathbf{e_t} + i\rho_2\mathbf{e_r} - \rho_3 + i\rho_4\mathbf{e_{tr}} - \vec{j}_1\mathbf{e_r} - \vec{j}_2\mathbf{e_t} + \vec{j}_3\mathbf{e_{tr}} - \vec{j}_4 i , \tag{3.4}$$

where $\rho_s = 4\pi\rho'_s$ (ρ'_k is the volume density of charges) and $\vec{j}_s = \frac{4\pi}{c}\vec{j}'_s$ ($\vec{j}'_s$ is volume density of currents).

Let us introduce the scalar ε_s and vector $\vec{A}_s$ field strengths according the following definitions:

$$
\begin{aligned}
&\varepsilon_1 = \partial a_1 + \left(\vec{\nabla}\cdot\vec{A}_1\right) + m a_4,\\
&\varepsilon_2 = \partial a_2 + \left(\vec{\nabla}\cdot\vec{A}_2\right) - m a_3,\\
&\varepsilon_3 = \partial a_3 + \left(\vec{\nabla}\cdot\vec{A}_3\right) + m a_2,\\
&\varepsilon_4 = \partial a_4 + \left(\vec{\nabla}\cdot\vec{A}_4\right) - m a_1,\\
&\vec{E}_1 = -\partial\vec{A}_1 - \vec{\nabla}a_1 + i\left[\vec{\nabla}\times\vec{A}_2\right] + m\vec{A}_4,\\
&\vec{E}_2 = -\partial\vec{A}_2 - \vec{\nabla}a_2 - i\left[\vec{\nabla}\times\vec{A}_1\right] - m\vec{A}_3,\\
&\vec{E}_3 = -\partial\vec{A}_3 - \vec{\nabla}a_3 - i\left[\vec{\nabla}\times\vec{A}_4\right] + m\vec{A}_2,\\
&\vec{E}_4 = -\partial\vec{A}_4 - \vec{\nabla}a_4 + i\left[\vec{\nabla}\times\vec{A}_3\right] - m\vec{A}_1.
\end{aligned}
\tag{3.5}
$$

Taking into account (3.5) we get that

$$
\begin{aligned}
&\left(i\mathbf{e_t}\partial - \mathbf{e_r}\vec{\nabla} - i\mathbf{e_{tr}}m\right)\left(ia_1\mathbf{e_t} - ia_2\mathbf{e_r} + a_3 - ia_4\mathbf{e_{tr}} + \vec{A}_1\mathbf{e_r} + \vec{A}_2\mathbf{e_t} - \vec{A}_3\mathbf{e_{tr}} + i\vec{A}_4\right)\\
&= -\varepsilon_1 + i\varepsilon_2\mathbf{e_{tr}} + i\varepsilon_3\mathbf{e_t} - i\varepsilon_4\mathbf{e_r} + \vec{E}_1\mathbf{e_{tr}} - i\vec{E}_2 + \vec{E}_3\mathbf{e_r} + \vec{E}_4\mathbf{e_t},
\end{aligned}
\tag{3.6}
$$

and the initial wave equation (3.2) is reduced to the following equation:

$$
\begin{aligned}
&\left(i\mathbf{e_t}\partial - \mathbf{e_r}\vec{\nabla} - i\mathbf{e_{tr}}m\right)\left(-\varepsilon_1 + i\varepsilon_2\mathbf{e_{tr}} + i\varepsilon_3\mathbf{e_t} - i\varepsilon_4\mathbf{e_r} + \vec{E}_1\mathbf{e_{tr}} - i\vec{E}_2 + \vec{E}_3\mathbf{e_r} + \vec{E}_4\mathbf{e_t}\right)\\
&= -i\rho_1\mathbf{e_t} + i\rho_2\mathbf{e_r} - \rho_3 + i\rho_4\mathbf{e_{tr}} - \vec{j}_1\mathbf{e_r} - \vec{j}_2\mathbf{e_t} + \vec{j}_3\mathbf{e_{tr}} - \vec{j}_4 i.
\end{aligned}
\tag{3.7}
$$

Producing the action of the operator on the left side of equation (3.7) and separating the values with different space-time properties, we obtain a system of equations for the field strengths, similar to the system of Maxwell equations in electrodynamics:

$$
\begin{aligned}
&\partial\varepsilon_1 + \left(\vec{\nabla}\cdot\vec{E}_1\right) - m\varepsilon_4 = \rho_1,\\
&\partial\varepsilon_2 + \left(\vec{\nabla}\cdot\vec{E}_2\right) + m\varepsilon_3 = \rho_2,\\
&\partial\varepsilon_3 + \left(\vec{\nabla}\cdot\vec{E}_3\right) - m\varepsilon_2 = \rho_3,\\
&\partial\varepsilon_4 + \left(\vec{\nabla}\cdot\vec{E}_4\right) + m\varepsilon_1 = \rho_4,\\
&\partial\vec{E}_1 + \vec{\nabla}\varepsilon_1 + i\left[\vec{\nabla}\times\vec{E}_2\right] + m\vec{E}_4 = -\vec{j}_1,\\
&\partial\vec{E}_2 + \vec{\nabla}\varepsilon_2 - i\left[\vec{\nabla}\times\vec{E}_1\right] - m\vec{E}_3 = -\vec{j}_2,\\
&\partial\vec{E}_3 + \vec{\nabla}\varepsilon_3 - i\left[\vec{\nabla}\times\vec{E}_4\right] + m\vec{E}_2 = -\vec{j}_3,\\
&\partial\vec{E}_4 + \vec{\nabla}\varepsilon_4 + i\left[\vec{\nabla}\times\vec{E}_3\right] - m\vec{E}_1 = -\vec{j}_4.
\end{aligned}
\tag{3.8}
$$

All these equations are coupled by the mass terms. From the system of equations (3.8) we can get some relations for the energy and momentum of the massive field. First the Pointing theorem is written as

$$
\begin{aligned}
&\frac{1}{2}\partial\left(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2 + \vec{E}_1^2 + \vec{E}_2^2 + \vec{E}_3^2 + \vec{E}_4^2\right) + \left(\vec{\nabla}\cdot\left(\varepsilon_1\vec{E}_1 + \varepsilon_2\vec{E}_2 + \varepsilon_3\vec{E}_3 + \varepsilon_4\vec{E}_4 - i\left[\vec{E}_1\times\vec{E}_2\right] + i\left[\vec{E}_3\times\vec{E}_4\right]\right)\right)\\
&= \varepsilon_1\rho_1 + \varepsilon_2\rho_2 + \varepsilon_3\rho_3 + \varepsilon_4\rho_4 - \left(\vec{E}_1\cdot\vec{j}_1\right) - \left(\vec{E}_2\cdot\vec{j}_2\right) - \left(\vec{E}_3\cdot\vec{j}_3\right) - \left(\vec{E}_4\cdot\vec{j}_4\right).
\end{aligned}
\tag{3.9}
$$

Here the volume density of energy is

$$
w = \frac{1}{8\pi}\left(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2 + \vec{E}_1^2 + \vec{E}_2^2 + \vec{E}_3^2 + \vec{E}_4^2\right),
\tag{3.10}
$$

and the volume density of energy flux is

$$
\vec{p} = \frac{c}{4\pi}\left(\varepsilon_1\vec{E}_1 + \varepsilon_2\vec{E}_2 + \varepsilon_3\vec{E}_3 + \varepsilon_4\vec{E}_4 - i\left[\vec{E}_1\times\vec{E}_2\right] + i\left[\vec{E}_3\times\vec{E}_4\right]\right).
\tag{3.11}
$$

Corresponding expression for the energy gradient is

$$
\begin{aligned}
&4\pi\vec{\nabla}w+2im\left[\vec{E}_2\times\vec{E}_4\right]+2im\left[\vec{E}_1\times\vec{E}_3\right]\\
&-i\partial\left[\vec{E}_1\times\vec{E}_2\right]+\varepsilon_1\partial\vec{E}_1+\varepsilon_2\partial\vec{E}_2-\vec{E}_1\partial\varepsilon_1-\vec{E}_2\partial\varepsilon_2\\
&-\left(\vec{E}_1\cdot\vec{\nabla}\right)\vec{E}_1-\left(\vec{E}_2\cdot\vec{\nabla}\right)\vec{E}_2-\vec{E}_1\left(\vec{\nabla}\cdot\vec{E}_1\right)-\vec{E}_2\left(\vec{\nabla}\cdot\vec{E}_2\right)\\
&+i\varepsilon_1\left[\vec{\nabla}\times\vec{E}_2\right]-i\varepsilon_2\left[\vec{\nabla}\times\vec{E}_1\right]+i\left[\vec{E}_2\times\vec{\nabla}\varepsilon_1\right]-i\left[\vec{E}_1\times\vec{\nabla}\varepsilon_2\right]\\
&+i\partial\left[\vec{E}_3\times\vec{E}_4\right]+\varepsilon_3\partial\vec{E}_3+\varepsilon_4\partial\vec{E}_4-\vec{E}_3\partial\varepsilon_3-\vec{E}_4\partial\varepsilon_4\\
&-\left(\vec{E}_3\cdot\vec{\nabla}\right)\vec{E}_3-\left(\vec{E}_4\cdot\vec{\nabla}\right)\vec{E}_4-\vec{E}_3\left(\vec{\nabla}\cdot\vec{E}_3\right)-\vec{E}_4\left(\vec{\nabla}\cdot\vec{E}_4\right)\\
&-i\varepsilon_3\left[\vec{\nabla}\times\vec{E}_4\right]+i\varepsilon_4\left[\vec{\nabla}\times\vec{E}_3\right]-i\left[\vec{E}_4\times\vec{\nabla}\varepsilon_3\right]+i\left[\vec{E}_3\times\vec{\nabla}\varepsilon_4\right]\\
&=-\vec{E}_1\rho_1-\vec{E}_2\rho_2-\varepsilon_1\vec{j}_1-\varepsilon_2\vec{j}_2-i\left[\vec{E}_2\times\vec{j}_1\right]+i\left[\vec{E}_1\times\vec{j}_2\right]\\
&-\vec{E}_3\rho_3-\vec{E}_4\rho_4-\varepsilon_3\vec{j}_3-\varepsilon_4\vec{j}_4+i\left[\vec{E}_4\times\vec{j}_3\right]-i\left[\vec{E}_3\times\vec{j}_4\right].
\end{aligned}
\tag{3.12}
$$

Note that this expression contains two terms with masses.

4. Two types of Lorentz transformations

In the frames of sedeonic algebra the transformation of values from one inertial coordinate system to another are carried out with the following sedeons [3]:

$$
\begin{aligned}
\tilde{\mathbf{L}}&=\cosh\vartheta-\mathbf{e}_{\mathbf{tr}}\vec{n}\sinh\vartheta,\\
\tilde{\mathbf{L}}^{*}&=\cosh\vartheta+\mathbf{e}_{\mathbf{tr}}\vec{n}\sinh\vartheta,
\end{aligned}
\tag{4.1}
$$

where $\tanh(2\vartheta)=v/c$; v is velocity of motion along the vector $\vec{n}$. The transformed sedeonic potential can be presented as

$$
\tilde{\mathbf{W}}'=\tilde{\mathbf{L}}^{*}\tilde{\mathbf{W}}\,\tilde{\mathbf{L}}.
\tag{4.2}
$$

In the transition from one inertial system to another the components of potential are transformed in different ways. The components of the first group (Group I), which comprises $a_1, a_2, \vec{A}_1, \vec{A}_2$ transformed as follows:

$$
\begin{aligned}
a_1'&=a_1\cosh(2\vartheta)-\left(\vec{n}\cdot\vec{A}_1\right)\sinh(2\vartheta),\\
a_2'&=a_2\cosh(2\vartheta)-\left(\vec{n}\cdot\vec{A}_2\right)\sinh(2\vartheta),\\
\vec{A}_1'&=\vec{A}_1+\left(\vec{n}\cdot\vec{A}_1\right)\vec{n}\left(\cosh(2\vartheta)-1\right)-a_1\vec{n}\sinh(2\vartheta),\\
\vec{A}_2'&=\vec{A}_2+\left(\vec{n}\cdot\vec{A}_2\right)\vec{n}\left(\cosh(2\vartheta)-1\right)-a_2\vec{n}\sinh(2\vartheta).
\end{aligned}
,
\tag{4.3}
$$

If we take the x axis directed along the vector $\vec{n}$, then we get

$$
\begin{aligned}
A_{1y}'&=A_{1y},\\
A_{1z}'&=A_{1z},\\
A_{2y}'&=A_{2y},\\
A_{2z}'&=A_{2z},\\
a_1'&=a_1\frac{1}{\sqrt{1-(v/c)^2}}-A_{1x}\frac{v/c}{\sqrt{1-(v/c)^2}},\\
a_2'&=a_2\frac{1}{\sqrt{1-(v/c)^2}}-A_{2x}\frac{v/c}{\sqrt{1-(v/c)^2}},\\
A_{1x}'&=A_{1x}\frac{1}{\sqrt{1-(v/c)^2}}-a_1\frac{v/c}{\sqrt{1-(v/c)^2}},\\
A_{2x}'&=A_{1x}\frac{1}{\sqrt{1-(v/c)^2}}-a_2\frac{v/c}{\sqrt{1-(v/c)^2}}.
\end{aligned}
\tag{4.4}
$$

where $\gamma = v/c$. The components of the second group (Group II), which comprises $a_3, a_4, \vec{A}_3, \vec{A}_4$ transformed as follows:

$$
\begin{aligned}
& a_3' = a_3, \\
& a_4' = a_4, \\
& \vec{A}_3' = \vec{A}_3 \cosh(2\vartheta) - (\vec{n}\cdot\vec{A}_3)\vec{n}(\cosh(2\vartheta)-1) - i\left[\vec{n}\times\vec{A}_4\right]\sinh(2\vartheta), \\
& \vec{A}_4' = \vec{A}_4 \cosh(2\vartheta) - (\vec{n}\cdot\vec{A}_4)\vec{n}(\cosh(2\vartheta)-1) + i\left[\vec{n}\times\vec{A}_3\right]\sinh(2\vartheta).
\end{aligned}
\tag{4.5}
$$

For the x axis directed along the vector $\vec{n}$ we get

$$
\begin{aligned}
& a_3' = a_3, \\
& a_4' = a_4, \\
& A_{3x}' = A_{3x}, \\
& A_{4x}' = A_{4x}, \\
& A_{3y}' = A_{3y}\frac{1}{\sqrt{1-(v/c)^2}} - A_{4z}\frac{v/c}{\sqrt{1-(v/c)^2}}, \\
& A_{3z}' = A_{3z}\frac{1}{\sqrt{1-(v/c)^2}} + A_{4y}\frac{v/c}{\sqrt{1-(v/c)^2}}, \\
& A_{4y}' = A_{4y}\frac{1}{\sqrt{1-(v/c)^2}} + A_{3z}\frac{v/c}{\sqrt{1-(v/c)^2}}, \\
& A_{4z}' = A_{4z}\frac{1}{\sqrt{1-(v/c)^2}} - A_{3y}\frac{v/c}{\sqrt{1-(v/c)^2}}.
\end{aligned}
\tag{4.6}
$$

Thus, these two groups are differed by their space-time properties and by Lorentz transformations. Similarly, epy field sources are also divided into two groups differing by Lorentz transformations:

$$
\begin{aligned}
& \rho_1' = \rho_1 \cosh(2\vartheta) - (\vec{n}\cdot\vec{j}_1)\sinh(2\vartheta), \\
& \rho_2' = \rho_2 \cosh(2\vartheta) - (\vec{n}\cdot\vec{j}_2)\sinh(2\vartheta), \\
& \vec{j}_1' = \vec{j}_1 + (\vec{n}\cdot\vec{j}_1)\vec{n}(\cosh(2\vartheta)-1) - \rho_1\vec{n}\sinh(2\vartheta), \\
& \vec{j}_2' = \vec{j}_2 + (\vec{n}\cdot\vec{j}_2)\vec{n}(\cosh(2\vartheta)-1) - \rho_2\vec{n}\sinh(2\vartheta),
\end{aligned}
\tag{4.7}
$$

and

$$
\begin{aligned}
& \rho_3' = \rho_3, \\
& \rho_4' = \rho_4, \\
& \vec{j}_3' = \vec{j}_3 \cosh(2\vartheta) - (\vec{n}\cdot\vec{j}_3)\vec{n}(\cosh(2\vartheta)-1) - i\left[\vec{n}\times\vec{j}_4\right]\sinh(2\vartheta), \\
& \vec{j}_4' = \vec{j}_4 \cosh(2\vartheta) - (\vec{n}\cdot\vec{j}_4)\vec{n}(\cosh(2\vartheta)-1) + i\left[\vec{n}\times\vec{j}_3\right]\sinh(2\vartheta).
\end{aligned}
\tag{4.8}
$$

Also we have the following Lorentz transformations for the field strengths:

$$
\begin{aligned}
& \varepsilon_1' = \varepsilon_1, \\
& \varepsilon_2' = \varepsilon_2, \\
& \vec{E}_1' = \vec{E}_1 \cosh(2\vartheta) - (\vec{m}\cdot\vec{E}_1)\vec{m}(\cosh 2\vartheta - 1) - i\left[\vec{m}\times\vec{E}_2\right]\sinh(2\vartheta), \\
& \vec{E}_2' = \vec{E}_2 \cosh(2\vartheta) - (\vec{m}\cdot\vec{E}_2)\vec{m}(\cosh 2\vartheta - 1) + i\left[\vec{m}\times\vec{E}_1\right]\sinh(2\vartheta),
\end{aligned}
\tag{4.9}
$$

and

$$
\begin{aligned}
& \varepsilon_3' = \varepsilon_3 \cosh(2\vartheta) - (\vec{m}\cdot\vec{E}_3)\sinh(2\vartheta), \\
& \varepsilon_4' = \varepsilon_4 \cosh(2\vartheta) - (\vec{m}\cdot\vec{E}_4)\sinh(2\vartheta), \\
& \vec{E}_3' = \vec{E}_3 + (\vec{m}\cdot\vec{E}_3)\vec{m}(\cosh 2\vartheta - 1) - \varepsilon_3\vec{m}\sinh(2\vartheta), \\
& \vec{E}_4' = \vec{E}_4 + (\vec{m}\cdot\vec{E}_4)\vec{m}(\cosh 2\vartheta - 1) - \varepsilon_4\vec{m}\sinh(2\vartheta).
\end{aligned}
\tag{4.10}
$$

If we take the x axis directed along the vector $\vec{n}$, then we get following Lorentz transformations for the components of the field strengths:

$$
\begin{aligned}
&\varepsilon_1' = \varepsilon_1, \\
&\varepsilon_2' = \varepsilon_2, \\
&E_{1x}' = E_{1x}, \\
&E_{2x}' = E_{2x}, \\
&E_{1y}' = E_{1y}\frac{1}{\sqrt{1-(v/c)^2}} - E_{2z}\frac{v/c}{\sqrt{1-(v/c)^2}}, \\
&E_{1z}' = E_{1z}\frac{1}{\sqrt{1-(v/c)^2}} + E_{2y}\frac{v/c}{\sqrt{1-(v/c)^2}}, \\
&E_{2y}' = E_{2y}\frac{1}{\sqrt{1-(v/c)^2}} + E_{1z}\frac{v/c}{\sqrt{1-(v/c)^2}}, \\
&E_{2z}' = E_{2z}\frac{1}{\sqrt{1-(v/c)^2}} - E_{1y}\frac{v/c}{\sqrt{1-(v/c)^2}},
\end{aligned}
\tag{4.11}
$$

and

$$
\begin{aligned}
&E_{3y}' = E_{3y}, \\
&E_{3z}' = E_{3z}, \\
&E_{4y}' = E_{4y}, \\
&E_{4z}' = E_{4z}, \\
&\varepsilon_4' = \varepsilon_4\frac{1}{\sqrt{1-(v/c)^2}} - E_{4x}\frac{v/c}{\sqrt{1-(v/c)^2}}, \\
&\varepsilon_3' = \varepsilon_3\frac{1}{\sqrt{1-(v/c)^2}} - E_{3x}\frac{v/c}{\sqrt{1-(v/c)^2}}, \\
&E_{3x}' = E_{3x}\frac{1}{\sqrt{1-(v/c)^2}} - \varepsilon_3\frac{v/c}{\sqrt{1-(v/c)^2}}, \\
&E_{4x}' = E_{4x}\frac{1}{\sqrt{1-(v/c)^2}} - \varepsilon_4\frac{v/c}{\sqrt{1-(v/c)^2}}.
\end{aligned}
\tag{4.12}
$$

5. Sedeonic equations for massless electromagnetic fields

If the mass of field quantum m_0 is zero, then the equation (3.2) describes the massless field. In this case we have

$$\left(i\mathbf{e_t}\partial - \mathbf{e_r}\vec{\nabla}\right)\left(i\mathbf{e_t}\partial - \mathbf{e_r}\vec{\nabla}\right)\tilde{\mathbf{W}} = \tilde{\mathbf{J}}. \tag{5.1}$$

The sedeonic potential $\tilde{\mathbf{W}}$ and field source $\tilde{\mathbf{J}}$ have the same space-time structure (3.3)-(3.4) and the same Lorentz transformations (4.3)-(4.6). In massless case we can define two groups of field strengths

$$
\begin{aligned}
&\varepsilon_1 = \partial a_1 + \left(\vec{\nabla}\cdot\vec{A}_1\right), \\
&\varepsilon_2 = \partial a_2 + \left(\vec{\nabla}\cdot\vec{A}_2\right), \\
&\vec{E}_1 = -\partial\vec{A}_1 - \vec{\nabla}a_1 + i\left[\vec{\nabla}\times\vec{A}_2\right], \\
&\vec{E}_2 = -\partial\vec{A}_2 - \vec{\nabla}a_2 - i\left[\vec{\nabla}\times\vec{A}_1\right],
\end{aligned}
\tag{5.2}
$$

and

$$
\begin{aligned}
&\varepsilon_3 = \partial a_3 + \left(\vec{\nabla}\cdot\vec{A}_3\right),\\
&\varepsilon_4 = \partial a_4 + \left(\vec{\nabla}\cdot\vec{A}_4\right),\\
&\vec{E}_3 = -\partial\vec{A}_3 - \vec{\nabla}a_3 - i\left[\vec{\nabla}\times\vec{A}_4\right],\\
&\vec{E}_4 = -\partial\vec{A}_4 - \vec{\nabla}a_4 + i\left[\vec{\nabla}\times\vec{A}_3\right],
\end{aligned}
\tag{5.3}
$$

which are satisfy the two independent systems of Maxwell equations:

$$
\begin{aligned}
&\partial\varepsilon_1 + \left(\vec{\nabla}\cdot\vec{E}_1\right) = \rho_1\,,\\
&\partial\varepsilon_2 + \left(\vec{\nabla}\cdot\vec{E}_2\right) = \rho_2,\\
&\partial\vec{E}_1 + \vec{\nabla}\varepsilon_1 + i\left[\vec{\nabla}\times\vec{E}_2\right] = -\vec{j}_1,\\
&\partial\vec{E}_2 + \vec{\nabla}\varepsilon_2 - i\left[\vec{\nabla}\times\vec{E}_1\right] = -\vec{j}_2,
\end{aligned}
\tag{5.4}
$$

and

$$
\begin{aligned}
&\partial\varepsilon_3 + \left(\vec{\nabla}\cdot\vec{E}_3\right) = \rho_3,\\
&\partial\varepsilon_4 + \left(\vec{\nabla}\cdot\vec{E}_4\right) = \rho_4,\\
&\partial\vec{E}_3 + \vec{\nabla}\varepsilon_3 - i\left[\vec{\nabla}\times\vec{E}_4\right] = -\vec{j}_3,\\
&\partial\vec{E}_4 + \vec{\nabla}\varepsilon_4 + i\left[\vec{\nabla}\times\vec{E}_3\right] = -\vec{j}_4.
\end{aligned}
\tag{5.5}
$$

For simplicity let us consider the equations without magnetic charges and magnetic currents ($\rho_2 = 0,\ \vec{j}_2 = 0,\ \rho_4 = 0,\ \vec{j}_4 = 0$). Taking into account the Lorentz gauge

$$
\begin{aligned}
&\varepsilon_1 = \partial a_1 + \left(\vec{\nabla}\cdot\vec{A}_1\right) = 0,\\
&\varepsilon_2 = \partial a_2 + \left(\vec{\nabla}\cdot\vec{A}_2\right) = 0,\\
&\varepsilon_3 = \partial a_3 + \left(\vec{\nabla}\cdot\vec{A}_3\right) = 0,\\
&\varepsilon_4 = \partial a_4 + \left(\vec{\nabla}\cdot\vec{A}_4\right) = 0,
\end{aligned}
\tag{5.6}
$$

the equations (5.4) and (5.5) can be rewritten as

$$
\begin{aligned}
&\left(\vec{\nabla}\cdot\vec{E}_1\right) = \rho_1\,,\\
&\left(\vec{\nabla}\cdot\vec{E}_2\right) = 0,\\
&\partial\vec{E}_1 + i\left[\vec{\nabla}\times\vec{E}_2\right] = -\vec{j}_1,\\
&\partial\vec{E}_2 - i\left[\vec{\nabla}\times\vec{E}_1\right] = 0,
\end{aligned}
\tag{5.7}
$$

and

$$
\begin{aligned}
&\left(\vec{\nabla}\cdot\vec{E}_3\right) = \rho_3,\\
&\left(\vec{\nabla}\cdot\vec{E}_4\right) = 0,\\
&\partial\vec{E}_3 - i\left[\vec{\nabla}\times\vec{E}_4\right] = -\vec{j}_3,\\
&\partial\vec{E}_4 + i\left[\vec{\nabla}\times\vec{E}_3\right] = 0.
\end{aligned}
\tag{5.8}
$$

Then the relation for the gradient of volume density of energy (3.12) takes the following form:

$$
\begin{aligned}
&\frac{1}{2}\vec{\nabla}\left(\vec{E}_1^2 + \vec{E}_2^2 + \vec{E}_3^2 + \vec{E}_4^2\right) - i\partial\left[\vec{E}_1\times\vec{E}_2\right] + i\partial\left[\vec{E}_3\times\vec{E}_4\right]\\
&-\left(\vec{E}_1\cdot\vec{\nabla}\right)\vec{E}_1 - \left(\vec{E}_2\cdot\vec{\nabla}\right)\vec{E}_2 - \vec{E}_1\left(\vec{\nabla}\cdot\vec{E}_1\right) - \vec{E}_2\left(\vec{\nabla}\cdot\vec{E}_2\right)\\
&-\left(\vec{E}_3\cdot\vec{\nabla}\right)\vec{E}_3 - \left(\vec{E}_4\cdot\vec{\nabla}\right)\vec{E}_4 - \vec{E}_3\left(\vec{\nabla}\cdot\vec{E}_3\right) - \vec{E}_4\left(\vec{\nabla}\cdot\vec{E}_4\right)\\
&= -\vec{E}_1\rho_1 - i\left[\vec{E}_2\times\vec{j}_1\right] - \vec{E}_3\rho_3 + i\left[\vec{E}_4\times\vec{j}_3\right].
\end{aligned}
\tag{5.9}
$$

It can be clearly seen that in this expression the field strengths and charges of first group are not mixed with the field strengths and charges of second group. So the expressions for the Lorentz forces have the following form:

$$\vec{F}_{\mathrm{eI}} = \vec{E}_1\rho_1 + i\left[\vec{E}_2 \times \vec{j}_1\right], \tag{5.10}$$

$$\vec{F}_{\mathrm{eII}} = \vec{E}_3\rho_3 - i\left[\vec{E}_4 \times \vec{j}_3\right]. \tag{5.11}$$

6. Sedeonic equations for weak gravitational fields

The weak gravitational field can be described by the following sedeonic equation [5]:

$$\left(i\mathbf{e}_{\mathbf{t}}\partial - \mathbf{e}_{\mathbf{r}}\vec{\nabla}\right)\left(i\mathbf{e}_{\mathbf{t}}\partial - \mathbf{e}_{\mathbf{r}}\vec{\nabla}\right)\tilde{\mathbf{V}} = -\tilde{\mathbf{I}}, \tag{6.1}$$

where $\tilde{\mathbf{V}}$ is a sedeonic potential, $\tilde{\mathbf{I}}$ is a phenomenological sedeonic source of gravitational field. Let us choose the potential as

$$\tilde{\mathbf{V}} = ib_1\mathbf{e}_{\mathbf{t}} - ib_2\mathbf{e}_{\mathbf{r}} + b_3 - ib_4\mathbf{e}_{\mathbf{tr}} + \vec{B}_1\mathbf{e}_{\mathbf{r}} + \vec{B}_2\mathbf{e}_{\mathbf{t}} - \vec{B}_3\mathbf{e}_{\mathbf{tr}} + i\vec{B}_4, \tag{6.2}$$

where components b_{s} and $\vec{B}_{\mathrm{s}}$ are real functions of coordinates and time (index s = 1, 2, 3, 4). Also we take the source in the following form:

$$\tilde{\mathbf{I}} = -i\beta_1\mathbf{e}_{\mathbf{t}} + i\beta_2\mathbf{e}_{\mathbf{r}} - \beta_3 + i\beta_4\mathbf{e}_{\mathbf{tr}} - \vec{l}_1\mathbf{e}_{\mathbf{r}} - \vec{l}_2\mathbf{e}_{\mathbf{t}} + \vec{l}_3\mathbf{e}_{\mathbf{tr}} - \vec{l}_4 i, \tag{6.3}$$

where $\beta_{\mathrm{s}} = 4\pi\beta'_{\mathrm{s}}$ (β'_{k} is the volume density of gravitational charges) and $\vec{l}_{\mathrm{s}} = \frac{4\pi}{c}\vec{l}'_{\mathrm{s}}$ ($\vec{l}'_{\mathrm{s}}$ is volume density of gravitational currents). The sedeonic potential $\tilde{\mathbf{V}}$ and field source $\tilde{\mathbf{I}}$ have the same space-time structure (3.3)-(3.4) and the same Lorentz transformations (4.3)-(4.8). Let us introduce two groups of scalar g_{s} and vector $\vec{G}_{\mathrm{s}}$ field strengths according to the following definitions:

$$\begin{aligned}
g_1 &= \partial b_1 + \left(\vec{\nabla}\cdot\vec{B}_1\right),\\
g_2 &= \partial b_2 + \left(\vec{\nabla}\cdot\vec{B}_2\right),\\
\vec{G}_1 &= -\partial\vec{B}_1 - \vec{\nabla}b_1 + i\left[\vec{\nabla}\times\vec{B}_2\right],\\
\vec{G}_2 &= -\partial\vec{B}_2 - \vec{\nabla}b_2 - i\left[\vec{\nabla}\times\vec{B}_1\right],
\end{aligned} \tag{6.4}$$

and

$$\begin{aligned}
g_3 &= \partial b_3 + \left(\vec{\nabla}\cdot\vec{B}_3\right),\\
g_4 &= \partial b_4 + \left(\vec{\nabla}\cdot\vec{B}_4\right),\\
\vec{G}_3 &= -\partial\vec{B}_3 - \vec{\nabla}b_3 - i\left[\vec{\nabla}\times\vec{B}_4\right],\\
\vec{G}_4 &= -\partial\vec{B}_4 - \vec{\nabla}b_4 + i\left[\vec{\nabla}\times\vec{B}_3\right].
\end{aligned} \tag{6.5}$$

These field strengths satisfy the two independent systems of Maxwell equations:

$$\begin{aligned}
&\partial g_1 + \left(\vec{\nabla}\cdot\vec{G}_1\right) = -\beta_1,\\
&\partial g_2 + \left(\vec{\nabla}\cdot\vec{G}_2\right) = -\beta_2,\\
&\partial\vec{G}_1 + \vec{\nabla}g_1 + i\left[\vec{\nabla}\times\vec{G}_2\right] = \vec{l}_1,\\
&\partial\vec{G}_2 + \vec{\nabla}g_2 - i\left[\vec{\nabla}\times\vec{G}_1\right] = \vec{l}_2,
\end{aligned} \tag{6.6}$$

and

$$\begin{aligned}
&\partial g_3 + \left(\vec{\nabla}\cdot\vec{G}_3\right) = -\beta_3,\\
&\partial g_4 + \left(\vec{\nabla}\cdot\vec{G}_4\right) = -\beta_4,\\
&\partial\vec{G}_3 + \vec{\nabla}g_3 - i\left[\vec{\nabla}\times\vec{G}_4\right] = \vec{l}_3,\\
&\partial\vec{G}_4 + \vec{\nabla}g_4 + i\left[\vec{\nabla}\times\vec{G}_3\right] = \vec{l}_4.
\end{aligned} \tag{6.7}$$

For simplicity let us consider the equations without gravitomagnetic charges and currents ($\beta_2 = 0, \vec{l}_2 = 0, \beta_4 = 0, \vec{l}_4 = 0$). Taking into account the Lorentz gauge

$$\begin{aligned}
&g_1 = \partial b_1 + \left(\vec{\nabla}\cdot\vec{B}_1\right) = 0,\\
&g_2 = \partial b_2 + \left(\vec{\nabla}\cdot\vec{B}_2\right) = 0,\\
&g_3 = \partial b_3 + \left(\vec{\nabla}\cdot\vec{B}_3\right) = 0,\\
&g_4 = \partial b_4 + \left(\vec{\nabla}\cdot\vec{B}_4\right) = 0,
\end{aligned} \tag{6.8}$$

the equations (6.6) and (6.7) can be rewritten as

$$\begin{aligned}
&\left(\vec{\nabla}\cdot\vec{G}_1\right) = -\beta_1\,,\\
&\left(\vec{\nabla}\cdot\vec{G}_2\right) = 0,\\
&\partial\vec{G}_1 + i\left[\vec{\nabla}\times\vec{G}_2\right] = \vec{l}_1,\\
&\partial\vec{G}_2 - i\left[\vec{\nabla}\times\vec{G}_1\right] = 0,
\end{aligned} \tag{6.9}$$

and

$$\begin{aligned}
&\left(\vec{\nabla}\cdot\vec{G}_3\right) = -\beta_3,\\
&\left(\vec{\nabla}\cdot\vec{G}_4\right) = 0,\\
&\partial\vec{G}_3 - i\left[\vec{\nabla}\times\vec{G}_4\right] = \vec{l}_3,\\
&\partial\vec{G}_4 + i\left[\vec{\nabla}\times\vec{G}_3\right] = 0.
\end{aligned} \tag{6.10}$$

Then the gradient of gravitational energy is

$$\begin{aligned}
&\frac{1}{2}\vec{\nabla}\left(\vec{G}_1^2 + \vec{G}_2^2 + \vec{G}_3^2 + \vec{G}_4^2\right) - i\partial\left[\vec{G}_1\times\vec{G}_2\right] + i\partial\left[\vec{G}_3\times\vec{G}_4\right]\\
&-\left(\vec{G}_1\cdot\vec{\nabla}\right)\vec{G}_1 - \left(\vec{G}_2\cdot\vec{\nabla}\right)\vec{G}_2 - \vec{G}_1\left(\vec{\nabla}\cdot\vec{G}_1\right) - \vec{G}_2\left(\vec{\nabla}\cdot\vec{G}_2\right)\\
&-\left(\vec{G}_3\cdot\vec{\nabla}\right)\vec{G}_3 - \left(\vec{G}_4\cdot\vec{\nabla}\right)\vec{G}_4 - \vec{G}_3\left(\vec{\nabla}\cdot\vec{G}_3\right) - \vec{G}_4\left(\vec{\nabla}\cdot\vec{G}_4\right)\\
&= \vec{G}_1\rho_1 + i\left[\vec{G}_2\times\vec{j}_1\right] + \vec{G}_3\rho_3 - i\left[\vec{G}_4\times\vec{j}_3\right].
\end{aligned} \tag{6.11}$$

It is clearly seen that in this expression the fields and charges of first group are not mixed with the fields and charges of second group. So the expressions for the Lorentz forces have the form:

$$\vec{F}_{\mathrm{gI}} = \vec{G}_1\beta_1 + i\left[\vec{G}_2\times\vec{l}_1\right], \tag{6.12}$$

$$\vec{F}_{\mathrm{gII}} = \vec{G}_3\beta_3 - i\left[\vec{G}_4\times\vec{l}_3\right]. \tag{6.13}$$

7. Two types of photino and gravitino

The neutrino field has two component connected with electromagnetic field (photino) and gravitational field (gravitino). The free photino and gravitino are described by the sedeonic first-order equations [5]:

$$\left(i\mathbf{e}_{\mathbf{t}}\partial - \mathbf{e}_{\mathbf{r}}\vec{\nabla}\right)\tilde{\mathbf{W}} = 0\,, \tag{7.1}$$

$$\left(i\mathbf{e}_{\mathbf{t}}\partial - \mathbf{e}_{\mathbf{r}}\vec{\nabla}\right)\tilde{\mathbf{V}} = 0\,, \tag{7.2}$$

which are equivalent to the following systems:

$$\begin{aligned}
&\partial a_1 + \left(\vec{\nabla}\cdot\vec{A}_1\right) = 0,\\
&\partial a_2 + \left(\vec{\nabla}\cdot\vec{A}_2\right),\\
&-\partial\vec{A}_1 - \vec{\nabla}a_1 + i\left[\vec{\nabla}\times\vec{A}_2\right] = 0,\\
&-\partial\vec{A}_2 - \vec{\nabla}a_2 - i\left[\vec{\nabla}\times\vec{A}_1\right] = 0,
\end{aligned} \tag{7.3}$$

$$\begin{aligned}
&\partial a_3 + \left(\vec{\nabla}\cdot\vec{A}_3\right) = 0,\\
&\partial a_4 + \left(\vec{\nabla}\cdot\vec{A}_4\right) = 0,\\
&-\partial\vec{A}_3 - \vec{\nabla}a_3 - i\left[\vec{\nabla}\times\vec{A}_4\right] = 0,\\
&-\partial\vec{A}_4 - \vec{\nabla}a_4 + i\left[\vec{\nabla}\times\vec{A}_3\right] = 0,
\end{aligned} \tag{7.4}$$

$$\begin{aligned}
&\partial b_1 + \left(\vec{\nabla}\cdot\vec{B}_1\right) = 0,\\
&\partial b_2 + \left(\vec{\nabla}\cdot\vec{B}_2\right) = 0,\\
&-\partial\vec{B}_1 - \vec{\nabla}b_1 + i\left[\vec{\nabla}\times\vec{B}_2\right] = 0,\\
&-\partial\vec{B}_2 - \vec{\nabla}b_2 - i\left[\vec{\nabla}\times\vec{B}_1\right] = 0,
\end{aligned} \tag{7.5}$$

$$\begin{aligned}
&\partial b_3 + \left(\vec{\nabla}\cdot\vec{B}_3\right) = 0,\\
&\partial b_4 + \left(\vec{\nabla}\cdot\vec{B}_4\right) = 0,\\
&-\partial\vec{B}_3 - \vec{\nabla}b_3 - i\left[\vec{\nabla}\times\vec{B}_4\right] = 0,\\
&-\partial\vec{B}_4 - \vec{\nabla}b_4 + i\left[\vec{\nabla}\times\vec{B}_3\right] = 0.
\end{aligned} \tag{7.6}$$

As seen there are two types of photinos and two types of gravitinos, which are differed in Lorentz transformations.

8. Dark energy and dark matter

We can suppose the existence of two types of electrical charges ρ_1, ρ_3 (and corresponding currents $\vec{j}_1, \vec{j}_3$) and two types of gravitational charges β_1, β_3 (and corresponding currents $\vec{l}_1, \vec{l}_3$). The electrical charges ρ_1 and ρ_3 do not interact neither by electrostatic Coulomb forces nor by means of electromagnetic waves exchanging. Similarly, the gravitational charges β_1 and β_3 do not interact neither by gravitational forces nor by means of gravitational waves exchanging. Thus, we can suppose the existence of four types of substances with different sets of electrical and gravitational charges

$$\text{I - }(\rho_1,\beta_1)\text{; II - }(\rho_1,\beta_3)\text{; III - }(\rho_3,\beta_1)\text{; IV - }(\rho_3,\beta_3).$$

Assuming for definiteness that the first set (ρ_1,β_1) is realized for the matter in terrestrial conditions, we can expect that there are few more types of substances.

1. The matter (ρ_1,β_3) interacts with Earth matter by means of electromagnetic fields but does not interact by gravitational fields. This matter is visible and levitates in the Earth gravitational field.
2. The matter (ρ_3,β_1) interacts with Earth matter by means of gravitational fields but does not interact by electromagnetic fields. This matter is gravitationally attracted to the Earth and is invisible.
3. The matter (ρ_3,β_3) does not interact with Earth matter neither by means of electromagnetic fields, nor by gravitational fields. This substance is invisible and indifferent to the Earth gravitational field.

To explain the astronomically observed effects associated with dark matter [6, 7] we can accept that in the universe there is the (ρ_3,β_1) substance. The dark energy can be considered as the electromagnetic energy connected with $\vec{E}_3$ and $\vec{E}_4$ fields and corresponding neutrinos.

7. Conclusion

Thus, we have shown that in the frames of sedeonic approach the massless fields can be described by two types of potentials with different space-time properties and different Lorentz transformations. It allows us to suppose the existence of four types of hypothetic matter with different electromagnetic and gravitational properties. In particular, this model can be applied to the explanation of the dark matter and dark energy properties.

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