New Einstein gravity field equation and Dark matter problem

Sangwha-Yi

Department of Math , Taejon University 300-716

ABSTRACT

In the general relativity theory, we discover New Einstein's gravity field equation. We solve the dark matter problem of the cosmology by the New gravity field equation.

PACS Number:**04,04.90.+e,98.80,98.80.E**

Key words:General relativity theory,

New gravity field equation

Cosmology

Dark matter problem

The Hubble's constant

e-mail address:sangwha1**@nate.com**

Tel:051-624-3953

1.Introduction

We solve the dark matter problem of the cosmology by using New gravity field equation.

New gravity field equation is

$$R_{\mu\upsilon}-\frac{1}{2}g_{\mu\upsilon}R+T^{\lambda}{}_{\lambda}\frac{C_1\pi G^4h^2k_0}{c^{14}}\rho^{+}{}_{0}{}^{2}g_{\mu\upsilon}=-\frac{8\pi G}{c^4}T_{\mu\upsilon}$$

$$T^{\lambda}{}_{\lambda}=g^{\mu\upsilon}T_{\mu\upsilon},\quad C_1<0$$

$\rho^{+}{}_{0}$ is the density of charge, h is plank constant, c is light speed.

$k_0=\frac{1}{4\pi\varepsilon_0}$, ε_0 is the permittivity constant

(1)

Eq(1) is

$$(R_{\mu\upsilon}-\frac{1}{2}g_{\mu\upsilon}R)_{;\mu}+T^{\lambda}{}_{\lambda;\mu}\frac{C_1\pi G^4h^2k_0}{c^{14}}\rho^{+}{}_{0}{}^{2}g_{\mu\upsilon}=-\frac{8\pi G}{c^4}T_{\mu\upsilon;\mu}=0$$

$$g^{\mu\upsilon}(R_{\mu\upsilon}-\frac{1}{2}g_{\mu\upsilon}R)+T^{\lambda}{}_{\lambda}\frac{C_1\pi G^4h^2k_0}{c^{14}}\rho^{+}{}_{0}{}^{2}g^{\mu\upsilon}g_{\mu\upsilon}$$

$$=R-2R+4T^{\lambda}{}_{\lambda}\rho^{+}{}_{0}{}^{2}\frac{C_1\pi G^4h^2k_0}{c^{14}}=-\frac{8\pi G}{c^4}T^{\lambda}{}_{\lambda},$$

$$R=\frac{8\pi G}{c^4}T^{\lambda}{}_{\lambda}+4T^{\lambda}{}_{\lambda}\frac{C_1\pi G^4h^2k_0}{c^{14}}\rho^{+}{}_{0}{}^{2}\qquad(2)$$

Hence,

$$R_{\mu\upsilon}=-\frac{8\pi G}{c^4}(T_{\mu\upsilon}-\frac{1}{2}g_{\mu\upsilon}T^{\lambda}{}_{\lambda})+\frac{C_1\pi G^4h^2k_0}{c^{14}}T^{\lambda}{}_{\lambda}\rho^{+}{}_{0}{}^{2}g_{\mu\upsilon}\qquad(3)$$

2.Newton limitation and Weak gravity field approximation

In this theory, Newton limitation is

$$g_{\mu\upsilon}\approx\eta_{\mu\upsilon},\ |T_{ij}|<<T_{00}$$

$$R_{ij}-\frac{1}{2}g_{ij}R\approx 0\to R_{ij}\approx\frac{1}{2}\delta_{ij}R$$

$$R\approx -R_{00}+\sum_{i=1}^{3}R_{ii}=-R_{00}+\frac{3}{2}R$$

$$R\approx 2R_{00}\qquad(4)$$

Hence, Newton limitation of Eq(1)

$$R_{0000}\approx 0, R_{i0j0}\approx\frac{1}{2}\frac{\partial^2 g_{00}}{\partial x^i\partial x^j}$$

$$R_{00} - \frac{1}{2} g_{00} R + T^{\lambda}{}_{\lambda} g_{00} \frac{C_1 \pi G^4 h^2 k_0}{c^{14}} \rho^{+}{}_{0}{}^{2}$$

$$\approx R_{00} + \frac{1}{2} R \approx 2R_{00} \approx \nabla^2 g_{00} \approx -\frac{8\pi G}{c^4} T_{00} \qquad (5)$$

Weak gravity field approximation is

$$g_{\mu\upsilon} = \eta_{\mu\upsilon} + h_{\mu\upsilon}$$

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4}(T_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} T^{\lambda}{}_{\lambda}) + \frac{C_1 \pi G^4 h^2 k_0}{c^{14}} T^{\lambda}{}_{\lambda} \rho^{+}{}_{0}{}^{2} g_{\mu\upsilon}$$

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4} S_{\mu\upsilon}$$

$$S_{\mu\upsilon} = T_{\mu\upsilon} - \frac{1}{2} \eta_{\mu\upsilon} T^{\lambda}{}_{\lambda} + \frac{1}{8} \frac{C_1 G^3 h^2 k_0}{c^{10}} T^{\lambda}{}_{\lambda} \eta_{\mu\upsilon} \rho^{+}{}_{0}{}^{2}$$

$$h_{\mu\upsilon}(t, \vec{x}) = \frac{4G}{c^2} \int d^4 x' \frac{S_{\mu\upsilon}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|}$$

$$h_{00}(\vec{x}) = \frac{4G}{rc^2} \int d^3 x' [T_{00} - \frac{1}{2} T_{00} + \frac{C_1 G^3 h^2 k_0}{8c^{10}} T_{00} \rho^{+}{}_{0}{}^{2}]$$

$$\approx \frac{4G}{rc^2} \int d^3 x' [\frac{1}{2} T_{00}] = \frac{2GM}{rc^2}$$

$$h_{ij}(\vec{x}) = \frac{4G}{rc^2} \int d^3 x' [T_{ij} + \frac{1}{2} \delta_{ij} T_{00} - \frac{C_1 G^3 h^2 k_0}{8c^{10}} T_{00} \delta_{ij} \rho^{+}{}_{0}{}^{2}]$$

$$\approx \frac{4G}{rc^2} \int d^3 x' [\frac{1}{2} \delta_{ij} T_{00}] = \frac{2GM}{rc^2} \delta_{ij}$$

$$c^2 d\tau^2 = -g_{\mu\upsilon} dx^{\mu} dx^{\upsilon} \approx (1 - \frac{2GM}{rc^2}) c^2 dt^2 - (1 + \frac{2GM}{rc^2}) \delta_{ij} dx^i dx^j \qquad (6)$$

In Eq(3), if $T_{\mu\upsilon} = 0$,

$$R_{\mu\upsilon} = 0 \qquad (7)$$

The solution of Eq(7) is Schwarzschild solution.

$$c^2 d\tau^2 = -g_{\mu\upsilon} dx^{\mu} dx^{\upsilon} = (1 - \frac{2GM}{rc^2}) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GM}{rc^2}} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 \qquad (8)$$

3.Solving Dark matter problem

The cosmologic theory is Robertson-Walker soloution.

$$c^2 d\tau^2 = c^2 dt^2 - \Omega^2(t)[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] \quad (9)$$

$$T^{\mu\upsilon} = pg^{\mu\upsilon} + (p/c^2 + \rho)U^{\mu}U^{\upsilon}$$

$$U^{\mu} = (c,0,0,0), T_{00} = \rho(t)c^2, \; T_{ij} = p(t)g_{ij}$$

$$T_{00} = \rho(t)c^2 \;\; , \;\; T_{0i} = 0, \; T_{ij} = p(t)g_{ij}$$

$$T^{\lambda}{}_{\lambda} = -\rho(t)c^2 + 3p(t) -$$

$$R_{00} = 3\frac{\ddot{\Omega}}{\Omega} \;\; , \;\; R_{0i} = 0,$$

$$R_{ij} = -(\Omega\ddot{\Omega} + 2\dot{\Omega}^2 + 2k)\frac{g_{ij}}{\Omega^2} \quad (10)$$

Hence,

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4}(T_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}T^{\lambda}{}_{\lambda}) + \frac{C_1\pi G^4 h^2 k_0}{c^{14}}T^{\lambda}{}_{\lambda}{\rho^{+}{}_0}^2 g_{\mu\upsilon}$$

$$3\frac{\ddot{\Omega}}{\Omega} = -\frac{4\pi G}{c^4}(\rho c^2 + 3p) - \frac{C_1\pi G^4 h^2 k_0}{c^{14}}(-\rho c^2 + 3p){\rho^{+}{}_0}^2 \quad (11)$$

$$-(\Omega\ddot{\Omega} + 2\dot{\Omega}^2 + 2k)\frac{1}{\Omega^2} = -\frac{4\pi G}{c^4}(\rho c^2 - p) + \frac{C_1\pi G^4 h^2 k_0}{c^{14}}(-\rho c^2 + 3p){\rho^{+}{}_0}^2 \quad (12)$$

Eq(11)+ $3\times$ Eq(12) is

$$-6\frac{(\dot{\Omega}^2 + k)}{\Omega^2} = -16\frac{\pi G}{c^4}\rho c^2 + 2\frac{C_1\pi G^4 h^2 k_0}{c^{14}}(-\rho c^2 + 3p){\rho^{+}{}_0}^2$$

$$\rightarrow \quad \frac{(\dot{\Omega}^2 + k)}{\Omega^2} = \frac{8}{3}\frac{\pi G}{c^4}\rho c^2 - \frac{1}{3}\frac{C_1\pi G^4 h^2 k_0}{c^{14}}(-\rho c^2 + 3p){\rho^{+}{}_0}^2 \quad (13)$$

Hence,

$$(\frac{8\pi G}{3c^2} + \frac{C_1\pi G^4 h^2 k_0}{c^{12}}{\rho^{+}{}_0}^2)\rho(t) \approx \frac{8\pi G}{3c^2}\rho(t)$$

$$= \frac{(\dot{\Omega}^2 + k)}{\Omega^2} + p(t)\frac{C_1\pi G^4 h^2 k_0}{c^{14}}{\rho^{+}{}_0}^2$$

$$\rightarrow \quad \rho(t) \approx \frac{3c^2}{8\pi G}[(\frac{\dot{\Omega}}{\Omega})^2 + \frac{k}{\Omega^2} + p(t)\frac{C_1\pi G^4 h^2 k_0}{c^{14}}{\rho^{+}{}_0}^2] \quad (14)$$

In this time,

The present time of Universe $t_0 \approx \frac{\Omega(t_0)}{\dot{\Omega}(t_0)} = {H_0}^{-1}$ (15)

Therefore, Eq(14) is

$$\rho(t_0) \approx \frac{3c^2}{8\pi G}[(\frac{\dot{\Omega}(t_0)}{\Omega(t_0)})^2 + \frac{k}{\Omega(t_0)^2} + p(t_0)\frac{C_1 \pi G^4 h^2 k_0}{c^{14}}\rho^{+}{}_0{}^2]$$

$$\approx \frac{3c^2}{8\pi G}[H_0{}^2 + \frac{k}{\Omega(t_0)^2} + p(t_0)\frac{C_1 \pi G^4 h^2 k_0}{c^{14}}\rho^{+}{}_0{}^2] \ ,$$

$$C_1 < 0 \qquad (16)$$

4. Conclusion

Therefore,

$$\rho_c = \frac{3c^2}{8\pi G}H_0{}^2 \approx 5\times 10^{-30} gm/cm^3 , \ \ \rho(t_0) \approx 2\times 10^{-31} gm/cm^3$$

According to Eq(16), the present universe's pressure $p(t_0)$ has to be huge. Physically, the meaning of $\rho^{+}{}_0$ is the density of charges in stars or galaxies.

Reference

[1]S.Weinberg,Gravitation and Cosmology(John wiley & Sons,Inc,1972)

[2]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V

[3]C.Misner, K,Thorne and J. Wheeler, Gravitation(W.H.Freedman & Co.,1973)

[4]S.Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)

[5]R.Adler,M.Bazin and M.Schiffer,Introduction to General Relativity(McGraw-Hill,Inc.,1965)

[6]E.Hubble,Proc. Nat.Acad. Sci. U. S. 15, 169(1929)

[7]A.Sandage,"Distances to Galaxies:the Hubble Constant, the Friedmann Time and the Edge of World"' in Proceedings of the Symposium on the Galaxy and the Distant Scale, Essex, England(1972)

[8]A.Sandage, Astrophys, J. 178, 1(1972)

[9]E.Kasner, Am. J. Math. 43, 217(1921)

[10]G.Birkoff,Relativity and Modern Physics(Harvard University Press,1923),p.253