

# ON NEUTROSOPHIC REFINED SETS AND THEIR APPLICATIONS IN MEDICAL DIAGNOSIS 

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#### Abstract

In this paper, we present some definitions of neutrosophic refined sets such as; union, intersection, convex and strongly convex in a new way to handle the indeterminate information and inconsistent information. Also we have examined some desired properties of neutrosophic refined sets based on these definitions. Then, we give distance measures of neutrosophic refined sets with properties. Finally, an application of neutrosophic refined set is given in medical diagnosis problem (heart disease diagnosis problem) to illustrate the advantage of the proposed approach.


Keywords - Neutrosophic sets, neutrosophic refined sets, distance measures, decision making

## 1 Introduction

Recently, several theories have been proposed to deal with uncertainty, imprecision and vagueness. Theory of probability, fuzzy set theory [46], intuitionistic fuzzy sets [7], rough set theory [27] etc. are consistently being utilized as efficient tools for dealing with diverse types of uncertainties and imprecision embedded in a system. However, all these above theories failed to deal with indeterminate and inconsistent information which exist in beliefs system. In 1995, Smarandache [39] developed a new concept called neutrosophic set (NS) which generalizes probability set, fuzzy set and intuitionistic fuzzy set. NS can be described by membership degree, indeterminacy degree and non-membership degree. This theory and their hybrid structures has proven useful in many different fields such as control theory [1], databases [3, 2],

[^0]medical diagnosis problem [4], decision making problem [5, 6, 9, 10, 11, 13, 12, 14, $17,19,20,23,25]$, physics [28], topology [24] etc.

Yager [43] firstly introduced a new theory, is called theory of bags, which is a multiset. Then, the concept of multisets were originally proposed by Blizard [8] and Calude et al. [15], as useful structures arising in many area of mathematics and computer sciences such as database queries. Several authors from time to time made a number of generalization of set theory. Since then, several researcher $[18,26,35$, 36, 37, 41, 42] discuussed more properties on fuzzy multiset. Shinoj and John [38] made an extension of the concept of fuzzy multisets by an intuitionstic fuzzy set, which called intuitionstic fuzzy multisets (IFMS). Since then in the study on IFMS , a lot of excellent results have been achieved by researcher $[22,29,30,31,32,33$, 34]. The concepts of FMS and IFMS fails to deal with indeterminacy. Therefore, Smarandache[40] give n-valued refined neutrosophic logic and its applications. Then, Ye and Ye [44] gave single valued neutrosophic sets and operations laws. Ye et al. [45] presented generalized distance measure and its similarity measures between single valued neutrosophic multi sets. Also they applied the measure to a medical diagnosis problem with incomplete, indeterminate and inconsistent information. Chatterjee et al.[16] developed single valued neutrosophic multi sets in detail.

Combining neutrosophic set models with other mathematical models has attracted the attention of many researchers. Maji et al. presented the concept of neutrosophic soft set [25] which is based on a combination of the neutrosophic set and soft set models. Broumi and Smarandache introduced the concept of the intuitionistic neutrosophic soft set [9, 12] by combining the intuitionistic neutrosophic set and soft set.

This paper is arranged in the following manner. In section 2, some definitions and notion about intuitionstic fuzzy set, intuitionstic fuzzy multisets and neutrosophic set theory. These definitions will help us in later section. In section 3 we study the concept of neutrosophic refined (multi) sets and their operations. In section 4, we present an application of neutrosophic multisets in medical diagnosis. Finally we conclude the paper.

## 2 Preliminary

In this section, we give the basic definitions and results of intuitionistic fuzzy set [7], intuitionistic fuzzy multiset [29] and neutrosophic set theory [39] that are useful for subsequent discussions.

Definition 2.1. [7] Let $E$ be a universe. An intuitionistic fuzzy set $I$ on $E$ can be defined as follows:

$$
I=\left\{<x, \mu_{I}(x), \gamma_{I}(x)>: x \in E\right\}
$$

where, $\mu_{I}: E \rightarrow[0,1]$ and $\gamma_{I}: E \rightarrow[0,1]$ such that $0 \leq \mu_{I}(x)+\gamma_{I}(x) \leq 1$ for any $x \in E$.

Definition 2.2. [29] Let $E$ be a universe. An intuitionistic fuzzy multiset $K$ on $E$ can be defined as follows:

$$
K=\left\{<x,\left(\mu_{K}^{1}(x), \mu_{K}^{2}(x), \ldots, \mu_{K}^{P}(x)\right),\left(\gamma_{K}^{1}(x), \gamma_{K}^{2}(x), \ldots, \gamma_{K}^{P}(x)\right)>: x \in E\right\}
$$

where, $\mu_{K}^{1}(x), \mu_{K}^{2}(x), \ldots, \mu_{K}^{P}(x): E \rightarrow[0,1]$ and $\gamma_{K}^{1}(x), \gamma_{K}^{2}(x), \ldots, \gamma_{K}^{P}(x): E \rightarrow[0,1]$ such that $0 \leq \mu_{K}^{i}(x)+\gamma_{K}^{i}(x) \leq 1(i=1,2, \ldots, P)$ and $\mu_{K}^{1}(x) \leq \mu_{K}^{2}(x) \leq \ldots \leq \mu_{K}^{P}(x)$ for any $x \in E$.

Here, $\left(\mu_{K}^{1}(x), \mu_{K}^{2}(x), \ldots, \mu_{K}^{P}(x)\right)$ and $\left(\gamma_{K}^{1}(x), \gamma_{K}^{2}(x), \ldots, \gamma_{K}^{P}(x)\right)$ is the membership sequence and non-membership sequence of the element $x$, respectively.

We arrange the membership sequence in decreasing order but the corresponding non membership sequence may not be in decreasing or increasing order.

Definition 2.3. [39] Let $U$ be a space of points (objects), with a generic element in U denoted by $u$. A neutrosophic set ( N -set) A in U is characterized by a truthmembership function $T_{A}$, an indeterminacy-membership function $I_{A}$ and a falsitymembership function $F_{A} . T_{A}(x), I_{A}(x)$ and $F_{A}(x)$ are real standard or nonstandard subsets of $[0,1]$. It can be written as

$$
A=\left\{<u,\left(T_{A}(x), I_{A}(x), F_{A}(x)\right)>: x \in E, T_{A}(x), I_{A}(x), F_{A}(x) \in[0,1]\right\}
$$

There is no restriction on the sum of $T_{A}(x) ; I_{A}(x)$ and $F_{A}(x)$, so $0 \leq T_{A}(x)+$ $I_{A}(x)+F_{A}(x) \leq 3$.

Definition 2.4. [21] $t$-norms are associative, monotonic and commutative two valued functions $t$ that map from $[0,1] \times[0,1]$ into $[0,1]$. These properties are formulated with the following conditions: $\forall a, b, c, d \in[0,1]$,

1. $t(0,0)=0$ and $t(a, 1)=t(1, a)=a$,
2. If $a \leq c$ and $b \leq d$, then $t(a, b) \leq t(c, d)$
3. $t(a, b)=t(b, a)$
4. $t(a, t(b, c))=t(t(a, b), c)$

Definition 2.5. [21] $t$-conorms ( $s$-norm) are associative, monotonic and commutative two placed functions $s$ which map from $[0,1] \times[0,1]$ into $[0,1]$. These properties are formulated with the following conditions: $\forall a, b, c, d \in[0,1]$,

1. $s(1,1)=1$ and $s(a, 0)=s(0, a)=a$,
2. if $a \leq c$ and $b \leq d$, then $s(a, b) \leq s(c, d)$
3. $s(a, b)=s(b, a)$
4. $s(a, s(b, c))=s(s(a, b), c)$
$t$-norm and $t$-conorm are related in a sense of lojical duality. Typical dual pairs of non parametrized $t$-norm and $t$-conorm are complied below:
5. Drastic product:

$$
t_{w}(a, b)= \begin{cases}\min \{a, b\}, & \max \{a b\}=1 \\ 0, & \text { otherwise }\end{cases}
$$

2. Drastic sum:

$$
s_{w}(a, b)= \begin{cases}\max \{a, b\}, & \min \{a b\}=0 \\ 1, & \text { otherwise }\end{cases}
$$

3. Bounded product:

$$
t_{1}(a, b)=\max \{0, a+b-1\}
$$

4. Bounded sum:

$$
s_{1}(a, b)=\min \{1, a+b\}
$$

5. Einstein product:

$$
t_{1.5}(a, b)=\frac{a . b}{2-[a+b-a . b]}
$$

6. Einstein sum:

$$
s_{1.5}(a, b)=\frac{a+b}{1+a . b}
$$

7. Algebraic product:

$$
t_{2}(a, b)=a . b
$$

8. Algebraic sum:

$$
s_{2}(a, b)=a+b-a . b
$$

9. Hamacher product:

$$
t_{2.5}(a, b)=\frac{a . b}{a+b-a . b}
$$

10. Hamacher sum:

$$
s_{2.5}(a, b)=\frac{a+b-2 . a . b}{1-a . b}
$$

11. Minumum:

$$
t_{3}(a, b)=\min \{a, b\}
$$

12. Maximum:

$$
s_{3}(a, b)=\max \{a, b\}
$$

## 3 Neutrosophic Refined Sets

In this section, we present some definitions of neutrosophic refined sets with operations. Also we have examined some desired properties of neutrosophic refined sets based on these definitions and operations. Some of it is quoted from [29, 32, 38, 39, 40].

In the following, some definition and operatios on intuitionistic fuzzy multiset defined in [18, 29], we extend this definition to NRS by using [20, 40].

Definition 3.1. [40, 44] Let $E$ be a universe. A neutrosophic refined set (NRS) $A$ on $E$ can be defined as follows:

$$
\begin{array}{r}
A=\left\{<x,\left(T_{A}^{1}(x), T_{A}^{2}(x), \ldots, T_{A}^{P}(x)\right),\left(I_{A}^{1}(x), I_{A}^{2}(x), \ldots, I_{A}^{P}(x)\right),\right. \\
\left.\left(F_{A}^{1}(x), F_{A}^{2}(x), \ldots, F_{A}^{P}(x)\right)>: x \in E\right\}
\end{array}
$$

where, $T_{A}^{1}(x), T_{A}^{2}(x), \ldots, T_{A}^{P}(x): E \rightarrow[0,1], I_{A}^{1}(x), I_{A}^{2}(x), \ldots, I_{A}^{P}(x): E \rightarrow[0,1]$ and $F_{A}^{1}(x), F_{A}^{2}(x), \ldots, F_{A}^{P}(x): E \rightarrow[0,1]$ such that $0 \leq T_{A}^{i}(x)+I_{A}^{i}(x)+F_{A}^{i}(x) \leq 3(i=$ $1,2, \ldots, P)$ and $T_{A}^{1}(x) \leq T_{A}^{2}(x) \leq \ldots \leq T_{A}^{P}(x)$ for any $x \in E .\left(T_{A}^{1}(x), T_{A}^{2}(x), \ldots, T_{A}^{P}(x)\right)$,
$\left(I_{A}^{1}(x), I_{A}^{2}(x), \ldots, I_{A}^{P}(x)\right)$ and $\left(F_{A}^{1}(x), F_{A}^{2}(x), \ldots, F_{A}^{P}(x)\right)$ is the truth membership sequence, indeterminacy membership sequence and falsity membership sequence of the element $x$, respectively. Also, P is called the dimension of NRS A.

In [44] truth membership sequences are increase and other sequences (indeterminacy membership, falsity membership) are not increase or decrease. But throughout this paper the truth membership sequences, indeterminacy membership sequences , falsity membership sequences are not increase or decrease. The set of all Neutrosophic refined sets on E is denoted by $\operatorname{NRS}(\mathrm{E})$.

Definition 3.2. [44] Let $A, B \in N R S(E)$. Then,

1. $A$ is said to be NM subset of $B$ is denoted by $A \widetilde{\subseteq} B$ if $T_{A}^{i}(x) \leq T_{B}^{i}(x), I_{A}^{i}(x) \geq$ $I_{B}^{i}(x), F_{A}^{i}(x) \geq F_{B}^{i}(x), \forall x \in E$.
2. $A$ is said to be neutrosophic equal of $B$ is denoted by $A=B$ if $T_{A}^{i}(x)=T_{B}^{i}(x)$, $I_{A}^{i}(x)=I_{B}^{i}(x), F_{A}^{i}(x)=F_{B}^{i}(x), \forall x \in E$.
3. the complement of A denoted by $A^{\widetilde{c}}$ and is defined by

$$
\begin{array}{r}
A^{\widetilde{c}}=\left\{<x,\left(F_{A}^{1}(x), F_{A}^{2}(x), \ldots, F_{A}^{P}(x)\right),\left(1-I_{A}^{1}(x), 1-I_{A}^{2}(x), \ldots, 1-I_{A}^{P}(x)\right),\right. \\
\\
\left.\left(T_{A}^{1}(x), T_{A}^{2}(x), \ldots, T_{A}^{P}(x)\right)>: x \in E\right\}
\end{array}
$$

In the following, some definitions and operations with properties on neutrosophic multi set defined in $[16,44,45]$, we generalized these definitions.

Definition 3.3. Let $A, B \in N R S(E)$. Then,

1. If $T_{A}^{i}(x)=0$ and $I_{A}^{i}(x)=F_{A}^{i}(x)=1$ for all $x \in E$ and $i=1,2, \ldots, P$ then $A$ is called null $n s$-set and denoted by $\tilde{\Phi}$.
2. If $T_{A}^{i}(x)=1$ and $I_{A}^{i}(x)=F_{A}^{i}(x)=0$ for all $x \in E$ and $i=1,2, \ldots, P$, then $A$ is called universal $n s$-set and denoted by $\tilde{E}$.

Definition 3.4. Let $A, B \in N R S(E)$. Then,

1. the union of $A$ and $B$ is denoted by $A \widetilde{\cup} B=C_{1}$ and is defined by

$$
\begin{array}{r}
C=\left\{<x,\left(T_{C}^{1}(x), T_{C}^{2}(x), \ldots, T_{C}^{P}(x)\right),\left(I_{C}^{1}(x), I_{C}^{2}(x), \ldots, I_{C}^{P}(x)\right),\right. \\
\left.\left(F_{C}^{1}(x), F_{C}^{2}(x), \ldots, F_{C}^{P}(x)\right)>: x \in E\right\}
\end{array}
$$

where $T_{C}^{i}=s\left\{T_{A}^{i}(x), T_{B}^{i}(x)\right\}, I_{C}^{i}=t\left\{I_{A}^{i}(x), I_{B}^{i}(x)\right\}, F_{C}^{i}=t\left\{F_{A}^{i}(x), F_{B}^{i}(x)\right\}$, $\forall x \in E$ and $i=1,2, \ldots, P$.
2. the intersection of $A$ and $B$ is denoted by $A \widetilde{\cap} B=D$ and is defined by

$$
\begin{array}{r}
D=\left\{<x,\left(T_{D}^{1}(x), T_{D}^{2}(x), \ldots, T_{D}^{P}(x)\right),\left(I_{D}^{1}(x), I_{D}^{2}(x), \ldots, I_{D}^{P}(x)\right),\right. \\
\left.\left(F_{D}^{1}(x), F_{D}^{2}(x), \ldots, F_{D}^{P}(x)\right)>: x \in E\right\}
\end{array}
$$

where $T_{D}^{i}=t\left\{T_{A}^{i}(x), T_{B}^{i}(x)\right\}, I_{D}^{i}=s\left\{I_{A}^{i}(x), I_{B}^{i}(x)\right\}, F_{D}^{i}=s\left\{F_{A}^{i}(x), F_{B}^{i}(x)\right\}$, $\forall x \in E$ and $i=1,2, \ldots, P$.

Proposition 3.5. Let $A, B, C \in N R S(E)$. Then,

1. $A \widetilde{\cup} B=B \widetilde{\cup} A$ and $A \widetilde{\cap} B=B \widetilde{\cap} A$
2. $A \widetilde{\cup}(B \widetilde{\cup} C)=(A \widetilde{\cup} B) \widetilde{\cup} C$ and $A \widetilde{\cap}(B \widetilde{\cap} C)=(A \widetilde{\cap} B) \widetilde{\cap} C$

Proof: The proofs can be easily made.
Proposition 3.6. Let $A, B, C \in N R S(E)$. Then,

1. $A \widetilde{\cup} A=A$ and $A \widetilde{\cap} A=A$
2. $A \widetilde{\cap} \Phi=\tilde{\Phi}$ and $A \widetilde{\cap} E=A$
3. $A \widetilde{\cup} \Phi=A$ and $A \widetilde{\cup} E=\tilde{E}$
4. $A \widetilde{\cap}(B \widetilde{\cup} C)=(A \widetilde{\cap} B) \widetilde{\cup}(A \widetilde{\cap} C)$ and $A \widetilde{\cup}(B \widetilde{\cap} C)=(A \widetilde{\cup} B) \widetilde{\cap}(A \widetilde{\cup} C)$
5. $\left(A^{\widetilde{c}}\right)^{\tilde{c}}=A$.

Proof. It is clear from Definition 3.3-3.4.
Theorem 3.7. Let $A, B \in N R S(E)$. Then, De Morgan's law is valid.

1. $(A \widetilde{\cup} B)^{\widetilde{c}}=A^{\widetilde{c}} \widetilde{\cap} B^{\tilde{c}}$
2. $(A \widetilde{\cap} B)^{\widetilde{c}}=A^{\widetilde{c}} \widetilde{\cup} B^{\widetilde{c}}$

Proof. $A, B \in N R S(E)$ is given. From Definition 3.2 and Definition 3.4, we have 1.

$$
\begin{aligned}
(A \widetilde{\cup} B)^{\widetilde{c}}= & \left\{<x,\left(s\left\{T_{A}^{1}(x), T_{B}^{1}(x)\right\}, s\left\{T_{A}^{2}(x), T_{B}^{2}(x)\right\}, \ldots, s\left\{T_{A}^{P}(x), T_{B}^{P}(x)\right\}\right),\right. \\
& \left(t\left\{I_{A}^{1}(x), I_{B}^{1}(x)\right\}, t\left\{I_{A}^{2}(x), I_{B}^{2}(x)\right\}, \ldots, t\left\{I_{A}^{P}(x), I_{B}^{P}(x)\right\}\right), \\
& \left.\left(t\left\{F_{A}^{1}(x), F_{B}^{1}(x)\right\}, t\left\{F_{A}^{2}(x), F_{B}^{2}(x)\right\}, \ldots, t\left\{F_{A}^{P}(x), F_{B}^{P}(x)\right\}\right)>: x \in E\right\}^{\widetilde{c}} \\
= & \left\{<x,\left(, t\left\{F_{A}^{1}(x), F_{B}^{1}(x)\right\}, t\left\{F_{A}^{2}(x), F_{B}^{2}(x)\right\}, \ldots, t\left\{F_{A}^{P}(x), F_{B}^{P}(x)\right\}\right)\right. \\
& \left(1-t\left\{I_{A}^{1}(x), I_{B}^{1}(x)\right\}, 1-t\left\{I_{A}^{2}(x), I_{B}^{2}(x)\right\}, \ldots, 1-t\left\{I_{A}^{P}(x), I_{B}^{P}(x)\right\}\right), \\
& \left.\left(s\left\{T_{A}^{1}(x), T_{B}^{1}(x)\right\}, s\left\{T_{A}^{2}(x), T_{B}^{2}(x)\right\}, \ldots, s\left\{T_{A}^{P}(x), T_{B}^{P}(x)\right\}\right)>: x \in E\right\} \\
= & \left\{<x,\left(, t\left\{F_{A}^{1}(x), F_{B}^{1}(x)\right\}, t\left\{F_{A}^{2}(x), F_{B}^{2}(x)\right\}, \ldots, t\left\{F_{A}^{P}(x), F_{B}^{P}(x)\right\}\right)\right. \\
& \left(s\left\{1-I_{A}^{1}(x), 1-I_{B}^{1}(x)\right\}, s\left\{1-I_{A}^{2}(x), 1-I_{B}^{2}(x)\right\}, \ldots,\right. \\
& \left.s\left\{1-I_{A}^{P}(x), 1-I_{B}^{P}(x)\right\}\right), \\
& \left.\left(s\left\{T_{A}^{1}(x), T_{B}^{1}(x)\right\}, s\left\{T_{A}^{2}(x), T_{B}^{2}(x)\right\}, \ldots, s\left\{T_{A}^{P}(x), T_{B}^{P}(x)\right\}\right)>: x \in E\right\} \\
= & A^{\widetilde{c} \widetilde{\cap} B^{\widetilde{c}} .}
\end{aligned}
$$

2. 

$$
\begin{aligned}
(A \widetilde{\cap} B)^{\widetilde{c}}= & \left\{<x,\left(t\left\{T_{A}^{1}(x), T_{B}^{1}(x)\right\}, t\left\{T_{A}^{2}(x), T_{B}^{2}(x)\right\}, \ldots, t\left\{T_{A}^{P}(x), T_{B}^{P}(x)\right\}\right),\right. \\
& \left(s\left\{I_{A}^{1}(x), I_{B}^{1}(x)\right\}, s\left\{I_{A}^{2}(x), I_{B}^{2}(x)\right\}, \ldots, s\left\{I_{A}^{P}(x), I_{B}^{P}(x)\right\}\right), \\
& \left.\left(s\left\{F_{A}^{1}(x), F_{B}^{1}(x)\right\}, s\left\{F_{A}^{2}(x), F_{B}^{2}(x)\right\}, \ldots, s\left\{F_{A}^{P}(x), F_{B}^{P}(x)\right\}\right)>: x \in E\right\}^{\widetilde{c}} \\
= & \left\{<x,\left(, s\left\{F_{A}^{1}(x), F_{B}^{1}(x)\right\}, s\left\{F_{A}^{2}(x), F_{B}^{2}(x)\right\}, \ldots, s\left\{F_{A}^{P}(x), F_{B}^{P}(x)\right\}\right)\right. \\
& \left(1-s\left\{I_{A}^{1}(x), I_{B}^{1}(x)\right\}, 1-s\left\{I_{A}^{2}(x), I_{B}^{2}(x)\right\}, \ldots, 1-s\left\{I_{A}^{P}(x), I_{B}^{P}(x)\right\}\right), \\
& \left.\left(t\left\{T_{A}^{1}(x), T_{B}^{1}(x)\right\}, t\left\{T_{A}^{2}(x), T_{B}^{2}(x)\right\}, \ldots, t\left\{T_{A}^{P}(x), T_{B}^{P}(x)\right\}\right)>: x \in E\right\} \\
= & \left\{<x,\left(, s\left\{F_{A}^{1}(x), F_{B}^{1}(x)\right\}, s\left\{F_{A}^{2}(x), F_{B}^{2}(x)\right\}, \ldots, s\left\{F_{A}^{P}(x), F_{B}^{P}(x)\right\}\right)\right. \\
& \left(t\left\{1-I_{A}^{1}(x), 1-I_{B}^{1}(x)\right\}, t\left\{1-I_{A}^{2}(x), 1-I_{B}^{2}(x)\right\}, \ldots,\right. \\
& \left.t\left\{1-I_{A}^{P}(x), 1-I_{B}^{P}(x)\right\}\right), \\
& \left.\left(t\left\{T_{A}^{1}(x), T_{B}^{1}(x)\right\}, t\left\{T_{A}^{2}(x), T_{B}^{2}(x)\right\}, \ldots, t\left\{T_{A}^{P}(x), T_{B}^{P}(x)\right\}\right)>: x \in E\right\} \\
= & A^{\widetilde{c} \widetilde{\cap} B^{\widetilde{c}} .}
\end{aligned}
$$

Theorem 3.8. Let $P$ be the power set of all NRS defined in the universe E. Then $(P, \widetilde{\cap}, \widetilde{\cup})$ is a distributive lattice.

Proof: The proofs can be easily made by showing properties; idempotency, commutativity, associativity and distributivity
Definition 3.9. Let $E$ is a real Euclidean space $E^{n}$. Then, a NRS A is convex if and only if

$$
\begin{gathered}
T_{A}^{i}(a x+(1-a) y) \geq T_{A}^{i}(x) \wedge T_{A}(y), I_{A}^{i}(a x+(1-a) y) \leq I_{A}^{i}(x) \vee I_{A}^{i}(y) \\
F_{A}^{i}(a x+(1-a) y) \leq F_{A}^{i}(x) \vee F_{A}^{i}(y)
\end{gathered}
$$

for every $x, y \in E, a \in I$ and $i=1,2, \ldots, P$.
Definition 3.10. Let E is a real Euclidean space $E^{n}$. Then, a NRS A is strongly convex if and only if

$$
\begin{gathered}
T_{A}^{i}(a x+(1-a) y)>T_{A}^{i}(x) \wedge T_{A}(y), I_{A}^{i}(a x+(1-a) y)<I_{A}^{i}(x) \vee I_{A}^{i}(y) \\
F_{A}^{i}(a x+(1-a) y)<F_{A}^{i}(x) \vee F_{A}^{i}(y)
\end{gathered}
$$

for every $x, y \in E, a \in I$ and $i=1,2, \ldots, P$.
Theorem 3.11. Let $A, B \in N R S(E)$. Then, $A \widetilde{\cap} B$ is a convex(strongly convex) when both A and B are convex(strongly convex).

Proof. It is clear from Definition 3.9-3.10.
Definition 3.12. [16] Let $A, B \in N R S(E)$. Then,

1. Hamming distance $d_{H D}(A, B)$ between A and B , defined by;

$$
\begin{aligned}
d_{H D}(A, B)=\sum_{j=1}^{P} \sum_{i=1}^{n}\left(\left|T_{A}^{j}\left(x_{i}\right)-T_{B}^{j}\left(x_{i}\right)\right|+\right. & \left|I_{A}^{j}\left(x_{i}\right)-I_{B}^{j}\left(x_{i}\right)\right|+ \\
& \left.\left|F_{A}^{j}\left(x_{i}\right)-F_{B}^{j}\left(x_{i}\right)\right|\right)
\end{aligned}
$$

2. Normalized hamming distance $d_{N H D}(A, B)$ between A and B , defined by;

$$
\begin{aligned}
d_{N H D}(A, B)=\frac{1}{3 n P} \sum_{j=1}^{P} \sum_{i=1}^{n}( & \left|T_{A}^{j}\left(x_{i}\right)-T_{B}^{j}\left(x_{i}\right)\right|+\left|I_{A}^{j}\left(x_{i}\right)-I_{B}^{j}\left(x_{i}\right)\right|+ \\
& \left.\left|F_{A}^{j}\left(x_{i}\right)-F_{B}^{j}\left(x_{i}\right)\right|\right)
\end{aligned}
$$

3. Euclidean distance $d_{E D}(A, B)$ between A and B , defined by;

$$
d_{E D}(A, B)=\sum_{j=1}^{P} \sum_{i=1}^{n} \sqrt{\begin{array}{r}
\left(T_{A}^{j}\left(x_{i}\right)-T_{B}^{j}\left(x_{i}\right)\right)^{2}+\left(I_{A}^{j}\left(x_{i}\right)-I_{B}^{j}\left(x_{i}\right)\right)^{2}+ \\
\left(F_{A}^{j}\left(x_{i}\right)-F_{B}^{j}\left(x_{i}\right)\right)^{2}
\end{array}}
$$

4. Normalized euclidean distance $d_{N E D}(A, B)$ between A and B , defined by;

$$
d_{N E D}(A, B)=\frac{1}{3 n . P} \sum_{j=1}^{P} \sum_{i=1}^{n} \sqrt{\begin{array}{r}
\left(T_{A}^{j}\left(x_{i}\right)-T_{B}^{j}\left(x_{i}\right)\right)^{2}+\left(I_{A}^{j}\left(x_{i}\right)-I_{B}^{j}\left(x_{i}\right)\right)^{2}+ \\
\left(F_{A}^{j}\left(x_{i}\right)-F_{B}^{j}\left(x_{i}\right)\right)^{2}
\end{array}}
$$

## 4 Medical Diagnosis Via NRS Theory

In the following, the example on intuitionistic fuzzy multiset given in $[18,31,33,38]$, we extend this definition to NRS.

Let $\mathrm{P}=\left\{\mathrm{P}_{1}, \mathrm{P}_{2}, \mathrm{P}_{3}, \mathrm{P}_{4}\right\}$ be a set of patients, $\mathrm{D}=\{$ Viral Fever, Tuberculosis, Typhoid, Throat disease $\}$ be a set of diseases and $\mathrm{S}=\{$ Temperature, cough, throat pain, headache, body pain\} be a set of symptoms. In Table I each symptom $S_{i}$ is described by three numbers: Membership T, non-membership F and indeterminacy I.

|  | Viral Fever | Tuberculosis | Typhoid | Throat disease |
| :---: | :---: | :---: | :---: | :---: |
| Temperature | $(0.8,0.2,0.1)$ | $(0.3,0.4,0.2)$ | $(0.4,0.6,0.3)$ | $(0.5,0.7,0.1)$ |
| Cough | $(0.2,0.3,0.7)$ | $(0.2,0.5,0.3)$ | $(0.4,0.5,0.4)$ | $(0.8,0.3,0.2)$ |
| Throat Pain | $(0.3,0.4,0.5)$ | $(0.4,0.4,0.3)$ | $(0.3,0.6,0.4)$ | $(0.6,0.5,0.4)$ |
| Headache | $(0.5,0.3,0.3)$ | $(0.5,0.2,0.3)$ | $(0.5,0.6,0.2)$ | $(0.4,0.3,0.5)$ |
| Body Pain | $(0.5,0.2,0.4)$ | $(0.4,0.5,0.3)$ | $(0.6,0.5,0.3)$ | $(0.2,0.6,0.4)$ |

Table I -NRS R: The relation among Symptoms and Diseases
The results obtained different time intervals such as: 8:00 am 12:00 am and 4:00 pm in a day as Table II;

|  | Temparature | Cough | Throat pain | Headache | Body Pain |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}_{1}$ | $(0.1,0.3,0.7)$ | $(0.3,0.2,0.6)$ | $(0.8,0.5,0)$ | $(0.3,0.3,0.6)$ | $(0.4,0.4,0.4)$ |
|  | $(0.2,0.4,0.6)$ | $(0.2,0.4,0)$ | $(0.7,0.6,0.1)$ | $(0.2,0.4,0.7)$ | $(0.3,0.2,0.7)$ |
|  | $(0.1,0.1,0.9)$ | $(0.1,0.3,0.7)$ | $(0.8,0.3,0.1)$ | $(0.2,0.3,0.6)$ | $(0.2,0.3,0.7)$ |
| $\mathrm{P}_{2}$ | $(0.5,0.3,0.3)$ | $(0.7,0.3,0.6)$ | $(0.8,0.6,0.1)$ | $(0.4,0.2,0.6)$ | $(0.6,0.2,0.4)$ |
|  | $(0.3,0.4,0.5)$ | $(0.6,0.4,0.3)$ | $(0.6,0.3,0.1)$ | $(0.5,0.4,0.7)$ | $(0.5,0.4,0.6)$ |
|  | $(0.4,0.2,0.6)$ | $(0.4,0.1,0.7)$ | $(0.7,0.5,0.1)$ | $(0.4,0.3,0.6)$ | $(0.6,0.3,0.6)$ |
| $\mathrm{P}_{3}$ | $(0.7,0.4,0.6)$ | $(0.7,0.2,0.5)$ | $(0.5,0.8,0.4)$ | $(0.6,0.3,0.4)$ | $(0.6,0.3,0.3)$ |
|  | $(0.4,0.5,0.3)$ | $(0.6,0.5,0.1)$ | $(0.6,0.4,0.4)$ | $(0.5,0.3,0.4)$ | $(0.6,0.5,0.4)$ |
|  | $(0.3,0.3,0.5)$ | $(0.4,0.2,0.2)$ | $(0.7,0.6,0.3)$ | $(0.4,0.4,0.5)$ | $(0.6,0.2,0.8)$ |
| $\mathrm{P}_{4}$ | $(0.3,0.4,0.6)$ | $(0.5,0.4,0.4)$ | $(0.5,0.6,0.31)$ | $(0.7,0.4,0.2)$ | $(0.3,0.3,0.5)$ |
|  | $(0.6,0.3,0.3)$ | $(0.6,0.5,0.3)$ | $(0.7,0.5,0.6)$ | $(0.4,0.3,0.4)$ | $(0.7,0.5,0.2)$ |
|  | $(0.4,0.2,0.5)$ | $(0.4,0.2,0.2)$ | $(0.8,0.5,0.3)$ | $(0.3,0.6,0.5)$ | $(0.3,0.5,0.4)$ |

Table II -NRS Q: the relation Beween Patient and Symptoms.

The normalized Hamming distance between Q and R is computed as;

|  | Viral Fever | Tuberculosis | Typhoid | Throat disease |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}_{1}$ | 0.266 | $\mathbf{0 . 2 3}$ | 0.28 | 0.25 |
| $\mathrm{P}_{2}$ | 0.213 | 0.202 | 0.206 | $\mathbf{0 . 1 9}$ |
| $\mathrm{P}_{3}$ | 0.206 | 0.173 | $\mathbf{0 . 1 6}$ | 0.166 |
| $\mathrm{P}_{4}$ | 0.22 | 0.155 | $\mathbf{0 . 1 4 6}$ | 0.157 |

Table III :The normalized Hamming distance between Q and R
The lowest distance from the table III gives the proper medical diagnosis. Patient $P_{1}$ suffers from Tuberculosis, Patient $P_{2}$ suffers from Throat diseas, Patient $P_{3}$ suffers from Typhoid disease and Patient $P_{4}$ suffers from Typhoid

## 5 Conclusion

In this paper, we firstly defined some definitions on neutrosophic refined sets and investigated some of their basic properties. The concept of neutrosophic refined (NRS) generalizes the fuzzy multisets and intuitionstic fuzzy multisets. Then, an application of NRS in medical diagnosis is discussed. In the proposed method, we measured the distances of each patient from each diagnosis by considering the symptoms of that particular disease.

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