A Note on The Dimensionless Gravitational Coupling Constant

April 21, 2016

Abstract

In this paper we are rewriting the gravitational coupling constant in a slightly different form than has been shown before (without changing its value). This makes it simpler to understand what is meant and what is not meant by a dimensionless gravitational coupling constant.

Key words: Dimensionless gravitational coupling constant, electron mass, Planck mass, Proton mass.

The Dimensionless Gravitational Coupling Constant

The gravitational coupling constant, often described as the dimensionless gravitational constant, has been discussed in a series of papers in theoretical physics, see for example Silk [6], Rozental [5], Neto [4] and Burrows and Ostriker [1]. The gravitational coupling constant α_G is defined as the gravitational attraction between pair of electrons and is normally given by

$$\alpha_G = \frac{Gm_e^2}{\hbar c} = \left(\frac{m_e}{m_p}\right)^2 \approx 1.7518 \times 10^{-45} \tag{1}$$

where $\hbar$ is Planck's reduced constant, m_e is the electron mass, and m_p is the Planck mass. Haug [2, 3] suggest that the Newton gravitational constant should be written as a function of Planck's reduced constant[1]

$$G_p = \frac{l_p^2 c^3}{\hbar} \tag{2}$$

This way of writing Newton's gravitational constant does not change the value of the constant. If one knows the Planck length, then the gravitational constant is known, or alternatively and more practically one can calibrate the Planck length based on empirical measurements of the gravitational constant. Based on this the gravitational coupling constant is given by:

$$\begin{aligned}
\alpha_G &= \frac{Gm_em_e}{\hbar c} \\
\alpha_G &= \frac{\frac{l_p^2c^3}{\hbar}m_e^2}{\hbar c} \\
\alpha_G &= \frac{m_e^2}{\frac{\hbar^2}{l_p^2}\frac{1}{c^2}} \\
\alpha_G &= \frac{m_e^2}{m_p^2} \\
\alpha_G &= \frac{\frac{\hbar^2}{\bar{\lambda}_e^2}\frac{1}{c^2}}{\frac{\hbar^2}{l_p^2}\frac{1}{c^2}} \\
\alpha_G &= \frac{\frac{\hbar^2}{\bar{\lambda}_e^2}}{\frac{\hbar^2}{l_p^2}} \\
\alpha_G &= \frac{\frac{1}{\bar{\lambda}_e^2}}{\frac{1}{l_p^2}} \\
\alpha_G &= \frac{l_p^2}{\bar{\lambda}_e^2}
\end{aligned} \tag{3}$$

[1] Here we use a notation more familiar to modern physics.

where $\bar{\lambda}_e$ is the reduced Compton wavelength of the electron. Further we naturally get the same value as before by using this new expression for the dimensionless gravitational coupling constant:

$$\alpha_G = \frac{l_p^2}{\bar{\lambda}_e^2} = \frac{(1.61623 \times 10^{-35})^2}{(2.4263102389 \times 10^{-12})^2} =\approx 1.7518 \times 10^{-45} \tag{4}$$

The Compton wavelength of the electron is the rest-mass wavelength of the electron and the reduced Compton wavelength is this length divided by 2π, that is $\frac{\lambda_e}{2\pi}$. Further l_p is the Planck length, which actually is the Compton wavelength of the Planck mass. The gravitational coupling constant is simply the reduced Compton wavelength of the Planck mass squared, divided by the square of the reduced Compton wavelength of the electron.

$$\alpha_G = \frac{(\text{Reduced Compton length Planck mass})^2}{(\text{Reduced Compton length mass of interest})^2} \tag{5}$$

The meaning of the term dimensionless in the gravitational coupling constant simply infers that its value will not change even if we change our unit systems for length, time, and kg and thereby change the units for mass, the speed of light, and the Planck constant. However, it is important to be aware that the gravitational coupling constant is not the same for different masses; it only holds between two electrons and shows the gravitational force between those two elections relative to the gravitational force between two Planck masses. For example, the gravitational coupling constant between two protons is

$$\alpha_{G,Proton} = \frac{l_p^2}{\bar{\lambda}_{proton}^2} = \frac{(1.61623 \times 10^{-35})^2}{(2.10309 \times 10^{-16})^2} \approx 5.90595 \times 10^{-39} \tag{6}$$

Further, and more importantly, the gravitational coupling constant between two Planck masses must be

$$\begin{aligned}
\alpha_{G,Planck} &= \frac{Gm_pm_p}{\hbar c} \\
\alpha_{G,Planck} &= \frac{\frac{l_p^2c^3}{\hbar}m_p^2}{\hbar c} \\
\alpha_{G,Planck} &= \frac{m_p^2}{\frac{\hbar^2}{l_p^2}\frac{1}{c^2}} \\
\alpha_{G,Planck} &= \frac{m_p^2}{m_p^2} \\
\alpha_{G,Planck} &= \frac{l_p^2}{l_p^2} = 1
\end{aligned} \tag{7}$$

That the gravitational coupling constant normally is assigned such a low value 1.7518×10^{-45} simply has to do with the low masses of electrons relative to the Planck mass. It would make more sense to say that the fundamental gravitational coupling constant is linked to two Planck masses and that its value is 1. It basically shows that that the gravitational force is very strong between two dense bodies that are very close to each other.

References

[1] Burrows, A. S., and J. P. Ostriker (2013): "Astronomical Reach of Fundamental Physics," *Proceedings of the National Academy of Sciences of the United States of America*, 111(7), 31–36.

[2] Haug, E. G. (2016a): "The Gravitational Constant and the Planck Units. A Deeper Understanding of the Quantum Realm," *www.ViXra.org March 13 2016.*

[3] ——— (2016b): "Planck Quantization of Newton and Einstein Gravitation," *www.ViXra.org March 19 2016.*

[4] Neto, M. d. O. (2005): "Using the Dimensionless Newton Gravity Constant $\bar{\alpha}_G$ to Estimate Planetary Orbits," *Chaos, Solitons and Fractals*, 24(1), 19–27.

[5] Rozental, I. L. (1980): "On the Numerical Values of the Fine-Structure Constant and the Gravitational Constant," *Soviet Journal of Experimental and Theoretical Physics Letters*, 31(9), 19–27.

[6] Silk, J. (1977): "Cosmogony and the Magnitude of the Dimensionless Gravitational Coupling Constant," *Nature*, 265, 710–711.