Is Newton's Gravitational 'Constant' Increasing?

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Abstract

The Beckenstein-Hawking formula for blackhole entropy indicates that blackholes have the highest possible entropy density in the universe. This suggests that, in addition to increasing entropy by its inexorable expansion, the universe can also increase it by creating more blackholes. And one way to accomplish this would be to have the gravitational 'constant' ('Big G') increase with time. This paper explores how the dynamic of these two competing entropic processes might play out, how 'Big G' might vary with time, and where to look for evidence.

Keywords: cosmology; gravity; entropy; Schwarzschild radius; event horizon; Big G; changing gravitational constant

1. Introduction

1.1 Blackhole Entropy

The formula for blackhole entropy (S_{BH}) developed by Jacob Bekenstein [1] and Stephen Hawking [2] in 1973-74 is:

$S_{BH}= kA/4l^2$ *Equation 1*

where k = Boltzmann's constant, A = the surface area of the event horizon and l = the Planck length.

The Planck length is the hypothetical smallest possible distance in the universe and arises from an exercise in dimensional analysis, whereby a number of measured physical constants are brought together in a way to give the unit of metres.

$l= \sqrt{\hbar G/c^3} = 1.616 \times 10^{-35} m$

The existence of a smallest possible unit of distance, carries the clear implication that space is quantized, and that we must also have a smallest possible unit of area and volume. The smallest possible area ('Planck area' = $1.73 \times 10^{-70} m^2$) is represented by the l^2 term in Equation 1.

The holographic principle asserts that higher dimensional information can be stored on a lower dimensional surface, as is the case when we store 3D images (holograms) as interference patterns on flat pieces of film. As it goes, a number of physicists including Gerard t'Hooft [3,4], Leonard Susskind [5], Charles Thorn [6], Raphael Bousso [7] and Juan Maldecena [8] have utilized a variety of approaches to introduce the idea that holographic information storage is possible within space itself. Without attempting to go into the complexities of how this is supposed to work, I merely point out that the idea of lower dimensional phenomena giving rise to higher dimensional phenomena is not a foreign one to modern physics. In the case of blackholes, Susskind and t'Hooft have made the case that the event horizon must be an information storage surface, that keeps track of everything that went into the blackhole.

1.2 Maximum Storage Capacity of Space

Accepting the legitimacy of the 'storage surface' concept, the reader is to imagine the event horizon as a region of space in which the maximum storage capacity has been reached, in regard of the quantity of matter to be kept track of: similar to a CD or DVD that has maxed out its storage. To calculate the amount of information storage needed for a given quantity of mass, we turn to Karl Schwarzschild's formula for the radius of a spherical, non-rotating, neutrally charged blackhole:

$$R_S = 2GM/c^2 \qquad \textit{Equation 2}$$

where R_S represents the Schwarzschild radius, M = mass, c = the speed of light and G is Newton's gravitational constant (aka 'Big G'). For a given amount of mass, Equation 2 gives the radius of a sphere for which the spatial information storage capacity would be maxed out for that mass; and the surface area of the sphere in Planck areas also represents the entropy of that mass in its most spatially compressed form.

In terms of calculating the gravitational field for more complicated mass distributions than a sphere, Isaac Newton discovered a symmetry principle that greatly simplifies the math. Essentially, we are allowed to pretend that all of the matter within the complicated object, be it an asteroid or the observable universe itself, is concentrated at one point: its centre of mass. Following this approach, if we want to find the total amount of information/entropy within the observable universe, we can do so by estimating the total mass present, pretending it is concentrated at a single point and calculating the surface area of the resultant blackhole.

The most widely accepted number for the mass of ordinary (non-dark) matter is 1.46×10^{53} kg [9]. This estimate is based on the amount of normal matter needed to achieve critical density, whereby the universe's gravitational potential energy is to exactly match its kinetic energy of expansion. The mass (M) of normal matter can also be estimated on the basis of dimensional analysis, by combining the measured parameters, c, T and G via $M=c^3T/G$ to give a value of 1.8×10^{53}kg [10]. Here T is the age of the universe. Whatever of the validity of these models, they do generate numbers that appear to be in the right ballpark, in terms of the amount of normal matter we can detect through our various telescopes. Running both these mass estimates through equation 2 we get R_S = 23 billion light years for the smaller number, and 28 billion light years for the larger. It is curious that the higher estimate is roughly twice our Hubble radius (Figure 1), which is estimated at about 14 billion light years, based on the current rate of expansion.

The point of calculating the Schwarzschild radius of a blackhole based on the matter present in our observable universe is that: if the density is too great, it implies we should find ourselves inside a blackhole. As it turns out, currently accepted physics indicates that all the matter with which we can have had any information exchange is now dispersed within a sphere of radius 47 billion light years; which is considerably greater than the 28-billion-light-year radius calculated above. In other words, we are fortunate that the observable matter is currently thought to be dispersed within this larger sphere, or else we would be inside a blackhole. However, I will challenge the reasoning behind the currently accepted 47-billion-light-year radius below.

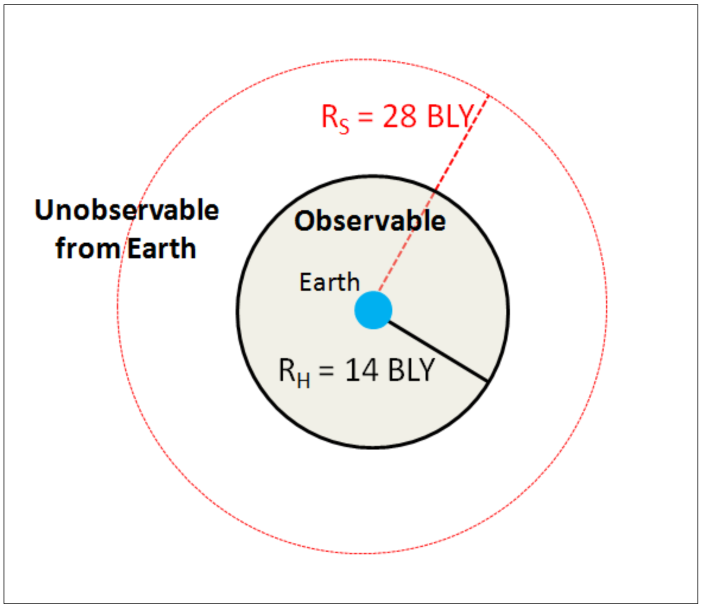

Figure 1

Schematic showing the Hubble radius as well as Schwarzschild radius of the universe based on a total mass of 1.8 x 10^{53}kg The Hubble radius is the distance at which space is moving away from us at the speed of light because of the cosmological expansion occurring between us and it. It is effectively the boundary of our 'observable universe'. As the maximum speed at which information can approach us (the speed of light) cannot advance against the outward movement of the space it is passing through, we can never receive information from beyond this boundary unless the expansion rate slows down with respect to the rate at which the universe is aging. R_H depends upon the rate of expansion, and R_S depends upon the average density of matter/energy — which is assumed to be roughly isotropic within and beyond our cosmic horizon.

1.3 Spatial Expansion and the Passage of Time

Before proceeding further I must make reference to a relevant point raised in a paper (Fagan, 2017) [11], that I posted to viXra.org (http://vixra.org/abs/1602.0324?ref=9201741), regarding the nature of time and its implications for our Hubble radius. The main focus of that article, was to re-interpret the meaning of spacetime curvature, as a variation in the dark-energy density of space, and to explore the implications. Inherent to that argument was the assertion that the rate at which time passes is simply a manifestation of the rate at which space expands. This is not an entirely new idea, as the correlation between expanding space and the forward passage of time is widely recognized as the 'cosmological arrow of time'. In other words, we always see time march forward and we always observe that space expands; which, although indicating a strong correlation between the two phenomena, does not prove cause and effect. And, when it comes to exploring the nature of time, spatial expansion is not the only 'one-way' phenomena that we observe in nature. For example, heat always flows from hotter to colder regions, giving us a thermodynamic arrow of time. Indeed, the thermodynamic arrow can be more generally understood as an offshoot of an idea, originally articulated in Arthur Eddington's 1928 book 'The Nature of the Physical World', and known as the 'entropic arrow of time': which essentially says that time flows forward as entropy increases. In this regard, if we accept the quantization of space, then adding more space quanta via expansion, like pulling back a piston, means that there are more ways in which the spatial quanta can be re-arranged, and more ways in which the matter and radiation within space can be re-arranged: i.e. higher entropy.

In the earlier referenced paper [11], I made the case that the expansion of space and forward passage of time was much more than a simple correlation, but was in fact a matter of definition. In other words, if every quanta of space inserted between two points that were once in direct contact adds exactly one quanta of time; then the increased radius of the observable universe must equate exactly to increased age of the observable universe. In short, expansion of the universe = aging of the universe. And from this, as illustrated in Figure 2, it is clear why the Hubble radius (R_H) is currently equal to the speed of light (c) multiplied by age of the universe (T): R_H=cT. Rather than pure coincidence, I propose this to be a law of nature: because the distance to any surface, including the surface of last scattering or the Big Bang itself, equals the number of space and time quanta that have emerged along any line between us and that surface. Thus, if more spatial quanta are added between us and some point in space within our Hubble radius, then more time is added to the age of the universe in exactly the amount needed for a signal from that point to reach us, as illustrated in Figure 2. This is at odds with accepted theory, which can have the rate at which time passes varying at different stages of the universe's evolution, as a function of the expansion rate — as elucidated in solutions to the Friedman equation. By analogy, in the model being proposed here, if we imagine space being represented by an elastic band, the faster the elastic stretches, the faster time passes; and always as a one to one correlation.

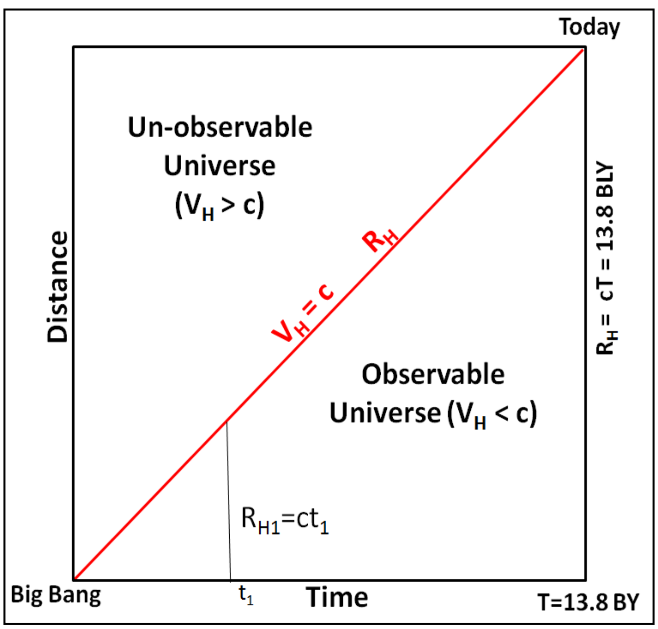

Figure 2

Illustration of why the Hubble Radius should always be equal to the speed of light multiplied by the age of the universe, if our distance in space quanta from any point in space that were in contact with at the Big Bang is equal to our distance in time quanta from that object. This being the case, the distance to the Big Bang in time must also equal its distance in space, with the speed of light as the proportionality constant for converting time to distance and vice versa. Galaxies receding at less than c will always remain inside our cosmic horizon, while galaxies receding at c or greater than c will remain invisible to us.

1.4 Implications for the Cosmic Horizon

The conjecture also implies that the matter that was originally within our Hubble radius after the time of cosmic inflation (assuming it occurred), will always remain there; because again, even if the rate of expansion is accelerating, the rate at which time passes accelerates by exactly the same amount; thereby providing more time for light and gravitational effects from the matter within the horizon to reach us. This means that we should always see the same galaxies even as they become more redshifted via continued expansion. This prediction provides an additional means to check the conjecture over the long haul, as we ultimately develop the technology to view the most distant galaxies and keep track of them. However, if the rate at which the universe ages is different than the rate of expansion, then we should observe new galaxies coming into view if the expansion rate is less than the rate of aging; or, alternatively, galaxies disappearing across the horizon if the rate of expansion exceeds the rate of aging (which is the currently accepted model).

In astrophysical jargon the idea that the rate of expansion and aging are always equal means that we always have an expansional scale-factor of one. Using this math, and running the tape in reverse then implies that the surface of last scattering (when all of the plasma in the embryonic universe cooled below 3000^0K and turned to atoms) occurred at an age of 12.5 million years instead of the accepted 378,000 years. [i.e. temperature now (2.725^0K)/temperature then (3000^0K) = age of universe then (12.5MY)/age of universe now (13.8 BY)]. It also implies that the most distant matter from which we can ever receive information, lies at a current (co-moving) distance of 13.8 billion light years instead of the accepted number of 47 billion light years; and, that the cosmic microwave background photons we are currently detecting were emitted from a co-moving distance of 12.5 million light years.

This being the case, then if the Schwarzschild radius of the mass within our cosmic horizon is 28 billion light years, and the density of matter in the areas beyond our cosmic horizon is about the same as within it, then we should find ourselves inside a blackhole! And this calculation does not even consider the potential effects of the hypothetical dark-matter which is supposed to be about 5.5 times as abundant as normal matter, and would generate a Schwarzschild radius of 175 billion light years; which would put us into a blackhole even if the normal matter was spread over a radius of 47 billion light years as predicted by accepted physics.

2. Counter-Balancing Gravity

2.1 No Dark-Matter

So how is it that we do not find ourselves inside of or on the event horizon of a blackhole? As for the thus-far undetected dark-matter, I made the case in my earlier paper [11], that it simply does not exist; and that the gravitational potential we currently attribute to it arises partly from congenital variations in the dark-energy density of space (structural gravity), which equate to spatial curvature. And given that such statistical variations are as likely to create negative as well as positive curvature over large scales they will essentially average to about zero over the volume of the observable universe.

But we are still left with conundrum that the amount of normal matter present is enough to put us in a blackhole with a radius of roughly twice our Hubble radius. One way the puzzle might be resolved is if about half of what see through our telescopes, and believe to be normal matter, is actually antimatter. Here, when I speak of what we 'see', I am including the dispersed gas between the galaxies, which is where I have proposed [11] that most of the antimatter would be found. The proviso is that antimatter has anti-mass, which introduces negative curvature and repels matter; and, that its presence, when averaged over the large scale, cancels out the gravitational potential of an equal amount of matter located elsewhere. Therefore, if there is roughly an equal amount of matter and antimatter, the net gravitational potential of the observable universe would be close to zero. In this way we would have always been protected from blackhole status, even in the universe's very early days when everything we see was concentrated into a much smaller space.

In terms of cosmological models based on Einstein's field equation, the situation where antimatter exactly balanced matter would be equivalent to Willem de Sitter's zero-matter solution, in which the rate of expansion is always equal to the rate at which time passes. However, the model that I propose is rooted in the principle that this must be the case by definition, whether the net amount of matter present is zero or not.

2.2 Implications for 'Big G'

We must also consider the situation where the amount of positive gravitational potential from matter significantly exceeded that of antimatter, which would still put us at some risk of being relegated to blackhole status — depending on how great the imbalance was in favour of matter. Figure 3 illustrates the point that wherever one is located, if the density of matter is too great within our cosmic horizon or adjacent to it, we can still lie within a Schwarzschild radius (R_S) of the centre of mass of all that matter and thereby become overwhelmed by 'gravitational information'.

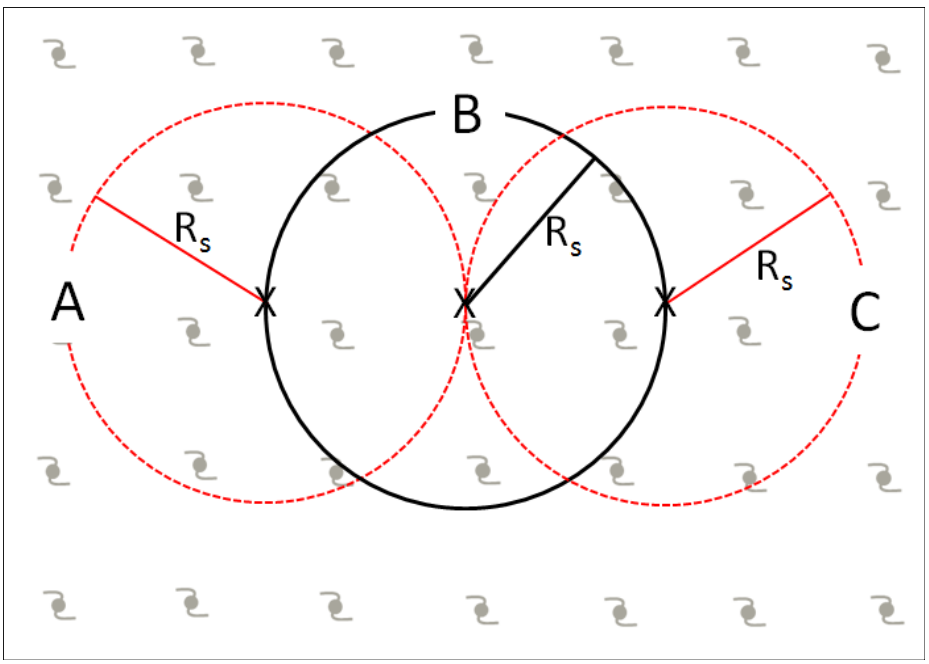

Figure 3

Illustrates that no matter where we place the centre (X) in a system with the same average density we still end up on an event horizon if the density of matter/energy is too high

What protects us from blackhole status in this case is the rate of spatial expansion. In short, the rate of expansion has to be fast enough to outrun and shield us from overwhelming 'gravitational creep' that would arrive from ever more distant regions and ultimately max-out our patch of space's information storage capacity. By comparison to Olber's paradox it is if the sky were filled with wall-to-wall gravity instead of wall-to-wall light. In mathematical terms the Hubble radius (R_H) must always be greater than the Schwarzschild radius of the matter within the cosmic horizon:

$R_H > R_S = 2GM/c^2$, *therefore*

$R_H > 2GM/c^2$ *Equation 3*

In essence this is another way of saying that the kinetic energy of expansion must be slightly greater than the gravitational potential pulling everything together. Now if we accept that $R_H = cT$, as argued in my earlier paper, then we get:

$cT > 2GM/c^2$

where T again represents the age of the universe and M represents the total mass within it. Re-arranging we get:

$G < c^3T/2M$ *Equation 4*

What this tells us is that, if the speed of light (c) and total mass (M) are to remain constant, the value of G would depend upon the age of the universe (T). Substituting the measured values of c, T and M into equation 4 indicates that G must be less than 3.25×10^{-11} m^3/kg-s^2 (for M = 1.8×10^{53} kg) and 4.01×10^{-11} (for M = 1.46×10^{53} kg). The measured value of G is 6.673×10^{-11} m^3/kg-s^2, which is too high in both cases. What this suggests is that, if Equation 4 has any validity, the amount of matter present is significantly less than current estimates. In other words, much of what we see out there in deep space is, in fact, antimatter.

We must also address the situation when the universe was much younger, and the value of T in Equation 4 was much smaller. Obviously this implies that with fixed values of c and M, G must have had a much lower value in the distant past; and as T goes to zero then G must have also gone to zero at the Big Bang. One way that this could occur is suggested by the Higgs mechanism, which is to give mass to certain fundamental particles, but is to go to zero at an extremely high symmetry-breaking temperature. In this regard, the mass of a particle may be understood to arise from its tendency to couple with space, with Big G as the coupling 'constant'. Therefore, a lower value of G in the past equates to a lower rest mass and relativistic mass for all particles.

If we accept that G had a lesser value in the past, then we must also accept that it has been increasing over time. And if it has increased up until the present there is no reason to think it will not continue to do so in the future. Ultimately at some distant time, when G becomes great enough, all of the stars, planets and even un-bound subatomic particles would reach the Planck mass (2.176×10^{-8}kg) and collapse into blackholes. Free streaming protons would collapse when the universe is 10^{19} times its current age, and electrons when it is 10^{22} times older. Assuming neutrinos are about one millionth the mass of electrons they would collapse when the universe is 10^{28} times as old. And given what we know of the Beckenstein-Hawking entropy formula: this highly expanded space, full of large and small blackholes, represents a very high entropy condition.

In regard to the maximization of entropy, the idea of an increasing value of G that is just a smidgen below what would halt the expansion of space also makes a lot of sense. Having a value of G that was any weaker than allowed, by Equation 4, would represent a lower entropy condition in terms of the number of blackholes created: And having it stronger would stop the parallel increase in entropy arising from expansion —

ultimately stopping the flow of time and putting us on or inside a blackhole. This maximization of the two entropic processes combines to provide for an 'entropic sweet-spot', which would, among other things, help address the fine-tuning problem. In other words, if the physical 'constants' we observe arise from the co-maximization of a variety of entropic processes, this may explain why they have their current values. For example, the fact that the universe appears to be at the critical density, may be evidence that an equilibrium between its kinetic energy of expansion and its gravitational potential energy is a higher entropic state than if these phenomena were out of balance. And in order to maintain this balance in the future, the value of G must increase accordingly.

Finally we must also recognize the case where:

$G = c^3T/2M$

Considering the arguments made above, this would seem to represent a situation where we find ourselves on an event horizon. However, we could still avoid the event horizon if our patch of the universe is rotating with respect to the surrounding space, which could cancel just enough gravity to do the job. Indeed this 'skimming-the-edge' scenario would represent the highest possible entropy in that, we can have the entropy of expansion occurring very near the highest entropic density of mass. Therefore equation 4 should more properly be written as:

$G \leq c^3T/2M$

to allow for the case of rotation.

3. Evidence For An Increasing G

3.1 Red Shift

Do we see evidence of an increasing G? One might expect that if G was much less in the past we would see differences in the orbital rates of distant stars and galaxies etc. However, this would not be the case, as a lesser value of G would decrease inertial mass to exactly the same degree that it decreased gravitational mass. However, if we assume that electromagnetic constants were the same in the past as they are today, a weaker G in the past does imply less gravitational red shifting of light in earlier times. For very distant objects it may be difficult to discern the gravitational red shifting from the expansional red shift. However, for very massive stars that are close enough to us to not be affected by expansion we may look for a net change in red shift over time. Alternatively, if we could find a number of stars of roughly the same mass but at different distances within our galaxy and plot their gravitational red shifts, we may find that it is, on average, weaker in the more distant stars — although the changes would be very small. For example, within the last million years G would have only increased by about (10^6years)/[Age of universe (10^{10} years)] = one part in ten thousand.

On the same note it may become possible, at some point, to measure an increase in red shift over time of the of light moving upwards in Earth's gravity, via a Pound-Rebka type experiment taken at different times.

3.2 Type 1a Supernova

Type 1a supernova explode when a white dwarf accretes enough matter to exceed 1.4 solar masses and the gravitational attraction overcomes electron degeneracy pressure. Generally this happens because the white dwarf has a partner from which it can steal the needed matter to reach critical mass. However, if we assume that the strong nuclear force parameters as well as the electromagnetic parameters have remained constant, while G has increased, we may observe white dwarfs exploding without accreting matter: which would be evidence that they could have been pushed over the limit by an increasing value of G.

3.3 Ignition of Brown Dwarfs

Brown dwarfs are so-called 'sub-stellar' objects: larger than Jupiter, but not quite large enough to ignite a sustained fusion reaction. Brown dwarfs lying just below the fusion threshold could be caused to ignite into dim stars if an increasing G sufficiently increased their gravitational compression against electromagnetic forces to reach fusion temperatures.

3.3 Compression of Old Rocky Planets

According to Equation 4 the value of G is allowed to have increased by about 51% since the formation of the solar system, about 4.6 billion years ago; and by about 40% since the rocky planets solidified. Even allowing for shrinkage due to cooling, such a significant increase in the value of G would introduce compressional forces on planets that could uplift mountain ranges, even if said planets were too small to allow for plate tectonics. In our own Solar System the only rocky planets that have clearly not experienced any plate tectonics are Mercury and Pluto. Yet Mercury shows significant evidence of compression in the form of numerous crustal folds, as well as ellipsoidal craters that have become deformed from circular by un-even squeezing. At present these compressional artifacts are attributed to Mercury's having shrunk as its core cooled.

As for Pluto, recent images from the New Horizons probe shows youthful mountain ranges up to 3500m high (https://www.nasa.gov/press-release/from-mountains-to-moons-multiple-discoveries-from-nasa-s-new-horizons-pluto-mission) that require 'some other process' to explain. Such compressional folds and thrusts would be most prominent in solid bodies in which the compressive strength of constituent material is relatively weak, and more easily deformed by slight increases in gravity. In this regard, Jupiter's moon Europa, whose surface is made entirely of water ice, shows an exceptionally complex pattern of compressional fractures. However, it is clear that a good deal, if not all, of the compressional artifacts on the various moons are primarily the result planetary tidal forces; and so the phenomenon can be more meaningfully studied on relatively isolated planets and asteroids. Our own moon also shows evidence of compression, as expressed in ridges and thrust faults which are generally attributed to thermal effects. Accepting that shrinkage because of cooling after solidification would also introduce compressional forces, the question of an increasing G might be addressed through mathematical modelling that takes both mechanisms into account.

3.4 Growth of Black Holes

According to Equation 2, the Schwarzschild radii of blackholes must increase with an increasing value of G. This implies than any blackholes that have been around from much earlier time should have increased in both size and gravitational potential. The greatest change would occur for blackholes formed very close to the beginning, as it was during these early times that G would have grown much more rapidly. For example, a ten solar mass blackhole that formed when the universe was 100 million years old would now represent 1380 solar masses, even without accreting any additional matter. This being the case, it would provide an explanation for the so-called intermediate-mass blackholes which range from about one hundred to one million solar masses.

If stellar-size blackholes formed much earlier, perhaps as 'primordial' features within the first few seconds after the Big Bang, then they could be millions to billions of times as massive today, and provide an explanation for the super-massive blackholes that are present at the centres of most galaxies. Primordial blackholes have indeed been proposed by Stephen Hawking [12], and are allowed to be both very large and very small. Also, recent work by Garriga et. al. [13], within the inflationary model, suggests that secondary inflationary bubbles may arise within the inflating bubble that became our universe; the remnants of which can form large gravitational features - including the possible seeds for what are to later become super-massive black holes.

3.5 Could Stars and Galaxies Have Formed With a Weaker G?

Accepting that structural gravity gradients, and the repulsion from adjacent antimatter would have been much more intense in the younger less-expanded universe, it is quite likely that this model can still allow for galaxy formation with a weaker value of G. However, when it comes to the formation of stars the question becomes more difficult. If the electromagnetic repulsion of atoms was as strong then as it is now; and the momentum of photons (= h/λ) was as strong as it is today — while G was considerably weaker: then it would have taken a good deal more matter to collapse against electromagnetic forces to form stars. The first stars are believed to have formed when the universe was about 550 million years old, when G would have been only .04 of its current value, according to the model. Therefore a cloud of primordial gas would have had to be a lot larger to collapse into a star. But once such clouds began to collapse, the process would rapidly accelerate as G increased, leading to extremely large and short-lived stars. The collapse of such giant clouds with an increasing G would further contribute to the creating of exceptionally large blackholes. However, if for example the quantum of action (h) was proportionately smaller at that time, then the interplay between gravity and the pressure from photon momentum (h/λ) would be the same as observed today.

3.6 Other Possible Sources of Evidence

Additional approaches that can, at least in theory, be used to test the conjecture include:

1. Slight changes in gravitationally lensed images (such as the Einstein Cross) that may be observed over time if G is increasing;

2. Comparisons of the amount of gravitational lensing for 'standard masses' at different distances;

3. An increase over time, in geodetic deviation and frame dragging around spinning objects, such as Earth;

4. Star patterns behind the sun recorded during solar eclipses should show more curvature than when measured at earlier times, if G is increasing;

5. Tightly orbiting neutron stars (such as the Hulse-Taylor binary) that are gradually converging because of energy loss to gravity waves, should show a more rapid convergence than would be expected for a constant G; and,

6. An increase the rate at which atomic clocks placed at different altitudes fall out of synchronicity.

4. Summary

This paper is a follow-up to 'Astronomical Evidence for an Alternative to Dark-matter', [11] and builds upon the argument that the passage of time is a manifestation of the expansion of the universe. It also recognizes the novel kind of blackhole entropy articulated by Beckenstein and Hawking, and proposes that: maximizing this gravitational entropy at the same time as maximizing the entropy increase from the expansion of space, amounts to a higher entropic situation than if nature does otherwise. Following on this reasoning, a mathematical relationship between 'Big G' and the age of the universe has been proposed, and suggestions have been made as to where to look for evidence.

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