

A Proof of Goldbach's Conjecture

Chongxi Yu

Techfields Inc.

1679 S Dupont HYW, Dover, Delaware, USA

Abstract

Prime numbers are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years. Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. A kaleidoscope can produce an endless variety of colorful patterns and it looks like a magic, but when you open it, it contains only very simple, loose, colored objects such as beads or pebbles and bits of glass. Goldbach's conjecture is about all numbers, the pattern of prime numbers likes a "kaleidoscope" of numbers, here we divided any even numbers into 10 groups and primes into 4 groups, Goldbach's conjecture will be much simpler. Here we give a clear proof for Goldbach's conjecture based on the fundamental theorem of arithmetic and Euclid's proof that the set of prime numbers is endless.

Key words: Goldbach's conjecture, fundamental theorem of arithmetic, Euclid's proof of infinite primes

Introduction

Prime numbers¹ are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years and many "advanced mathematics tools" are used to solve them, but they are still unsolved.

I believe that prime numbers are "basic building blocks" of the natural numbers and they must follow some very simple basic rules and do not need "advanced mathematics tools" to solve them. One of the basic rules is the "fundamental theorem of arithmetic" and the "simplest tools" are Euclid's proof of endless prime numbers and fundamental theorem of arithmetic.

Fundamental theorem of arithmetic:

The crucial importance of prime numbers to number theory and mathematics in general stems from the fundamental theorem of arithmetic², which states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.^[1] Primes can thus be considered the "basic building blocks" of the natural numbers.

Euclid's proof³ that the set of prime numbers is endless

The proof works by showing that if we assume that there is a biggest prime number, then there is a contradiction.

We can number all the primes in ascending order, so that $P_1 = 2$, $P_2 = 3$, $P_3 = 5$ and so on. If we assume that there are just n primes, then the biggest prime will be labeled P_n . Now we can form the number Q by multiplying together all these primes and adding 1, so

$$Q = (P_1 \times P_2 \times P_3 \times P_4 \dots \times P_n) + 1$$

Now we can see that if we divide Q by any of our n primes there is always a remainder of 1, so Q is not divisible by any of the primes, but we know that all positive integers are either primes or can be decomposed into a product of primes. This means that either Q must be a prime or Q must be divisible by primes that are larger than P_n .

Our assumption that P_n is the biggest prime has led us to a contradiction, so this assumption must be false, so there is no biggest prime and the set of prime numbers is endless.

Discussions

Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. It states:

Every even integer greater than 2 can be expressed as the sum of two primes.

If N is an even integer:

$N = N/2 + N/2 = (N/2+m) + (N/2-m)$; $m = 0, 1, 2, 3, \dots, M$. We need to prove $[(N/2+m)]$ and $[(N/2-m)]$ can be primes at same time.

A kaleidoscope can produce an endless variety of colorful patterns and it looks like a magic, but when you open it, it contains only very simple, loose, colored objects such as beads or pebbles and bits of glass. Goldbach's conjecture is about all numbers, the pattern of prime numbers likes a "kaleidoscope" of numbers, if we divide any even numbers into 10 groups and primes into 4 groups, Goldbach's conjecture will be much simpler.

If a number ($N > 3$) is not divisible by 3 or any prime which is smaller or equal to $N/3$, it must be a prime. Any number is divisible by 7, it have 1/3 chance is divisible by 3, any number is divisible by 11, it have 1/3 chance is divisible by 3 and 1/7 chance is divisible by 7, any number is divisible by 13, it has 1/3 chance to be divisible 3 and 1/7 chance to be divisible by 7, and 1/11 chance to be divisible by 11, so on, so we have terms: $1/3, 1/7 \times 2/3, 1/11 \times 2/3 \times 6/7, 1/13 \times 2/3 \times 6/7 \times 10/11 \dots$,

Let N_o represent any odd number, the chance of N_o to be a non-prime is: $[(1/3) + (1/7 \times 2/3) + (1/11 \times 2/3 \times 6/7) + (1/13 \times 2/3 \times 6/7 \times 10/11) + (1/17 \times 2/3 \times 6/7 \times 10/11 \times 12/13) + (1/19 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17) + (1/23 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19) + (1/29 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23) + (1/31 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29) + (1/37 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31) + (1/41 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37) + (1/43 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41) + (1/47 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43) + \dots]$ -----Formula 1

Any odd number cannot be divisible by 2 and any odd number with 5 as its last digit is not a prime except 5.

Let \sum represent the sum of the infinite terms and $\Delta = 1 - \sum$, according to Euclid's proof³ that the set of prime numbers is endless. Δ is the chance of any odd number to be a prime. \sum may be very close to 1 when N is growing to ∞ , but always less than 1. Let $\Delta = 1 - \sum$, when N is growing to ∞ , Δ may be very close to 0, but always more than 0 according to Euclid's proof³ that the set of prime numbers is endless. If Δ is 0, then there is no prime, that is not true.

The sum of first 20 terms = $[(1/3) + (1/7 \times 2/3) + (1/11 \times 2/3 \times 6/7) + (1/13 \times 2/3 \times 6/7 \times 10/11) + (1/17 \times 2/3 \times 6/7 \times 10/11 \times 12/13) + (1/19 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17) + (1/23 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19) + (1/29 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23) + (1/31 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29) + (1/37 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31) + (1/41 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37) + (1/43 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41) + (1/47 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43) + (1/53 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47) +$

$$\begin{aligned}
& (1/59 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53) + \\
& (1/61 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 58/59) + \\
& (1/67 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 58/59 \times 60/61) + \\
& (1/71 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 58/59 \times 60/61 \times 66/67) + \\
& (1/73 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 58/59 \times 60/61 \times 66/67 \times 70/71) + \\
& (1/79 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 58/59 \times 60/61 \times 66/67 \times 70/71 \times 72/73) \\
& = [0.333333 + 0.095238 + 0.051948 + 0.039960 + 0.028207 + 0.023753 + 0.018590 + 0.014102 + 0.012738 + 0.010328 + 0.009370 + \\
& 0.008436 + 0.007538 + 0.006543 + 0.005766 + 0.005483 + 0.004910 + 0.004564 + 0.004377 + 0.003831] = 0.689015
\end{aligned}$$

For the first 20 term: $\Sigma = 0.689015$, $\Delta = 1 - \Sigma = 0.310985$

The chance of N_0 to be a prime is: $\Delta = 1 - [(1/3) + (1/7 \times 2/3) + (1/11 \times 2/3 \times 6/7) + (1/13 \times 2/3 \times 6/7 \times 10/11) + (1/17 \times 2/3 \times 6/7 \times 10/11 \times 12/13) + (1/19 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17) + (1/23 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19) + (1/29 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23) + (1/31 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29) + (1/37 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31) + (1/41 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37) + (1/43 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41) + (1/47 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 + \dots)]$ -----Formula 2

Let us consider the following cases:

1. When any even integer (N) has 0 as its last digit, such as 10, 20, 30, 40, 110, 120, 1120, 1130, ..., then $N/2$ has only 0 or 5 as its last digit:

1a. Except 5 and 2, any prime must have 1, 3, 7, or 9 as its last digit. When both N and $N/2$ have 0 as their last digit, then N must be 20, 30, 40, 60, 80, 100, 120, ..., N . For enough large number N , Let's consider $N = O_1 + O_2 = (N/2 + L + 3) + (N/2 - L - 3)$, O_1 and O_2 is an odd number. $O_1 > O_2$, $O_1 - O_2 = 2L + 6$, $L = 0, 5, 10, 15, 20, 25, 30, \dots, L$, $O_1 - O_2 = 2L + 6 = 6, 26, 46, 66, 86, 106, 126, \dots, (2L + 6)$, then O_1 is an odd number with 3 as its last digit, O_2 is an odd number with 7 as its last digit.

Also we can have $N=O_1+O_2=(N/2+L+7)+(N/2-L-7)$, O_1 and O_2 is an odd number. $O_1>O_2$, $O_1-O_2=2L+14$, $L=0, 10, 20, 30\dots L$, $O_1-O_2=2L+14=14, 34, 54, 74, 94, 114, 134,\dots(2L+14)$, then O_1 is an odd number with 7 as its last digit, O_2 is an odd number with 3 as its last digit.

Then, we have odd number pairs as listed in table 1:

Table 1. The odd number pairs in $N=O_1+O_2=(N/2+L+3)+(N/2-L-3)$ and $N=O_1+O_2=(N/2+L+7)+(N/2-L-7)$

N-7	N-17	N-37	N-47	N-67	N-97	...	$N/2+L+3$	$N/2-L-7$...	83	73	53	43	23	13	3
7	17	37	47	67	97	...	$N/2-L-3$	$N/2+L+7$...	N-83	N-73	N-53	N-43	N-23	N-13	N-3

Let \$1 represents a prime with 1 as its last digit, such as 11, 31, 41, 61, 71, 101, 131, 151, 181, 191,...; \$3 represents a prime with 3 as its last digit, such as 3, 13, 23, 43, 53, 73, 83, 103, 113, 163, 193....; \$7 represents a prime with 7 as its last digit, such as, 7, 17, 37, 47, 67, 97, 107, 127, 137, 157, 167, 197....; and \$9 represents a prime with 9 as its last digit, such as 19, 29, 59, 79, 89, 109, 139, 149, 179, 199,....

Let O1 represents an odd number with 1 as its last digit, such as 11, 21, 31, 41, 51, 61, 71,...; O3 represents an odd number with 3 as its last digit, such as 3, 13, 23, 33, 43, 53, 63, 73,...; O7 represents an odd number with 7 as its last digit, such as, 7, 17, 27, 37, 47, 57, 67, 77....; and O9 represents an odd number with 9 as its last digit, such as 9, 19, 29, 39, 49, 59, 69, 79,....

Fundamental theorem of arithmetic states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.

Every odd number with 3 as its last digit is a product of \$3x\$1 or \$7x\$9; \$1 is decided by \$3 or \$3 is decided by \$1 and \$9 is decided by \$7 or \$7 is decided by 9, so we need to consider only \$3 and \$7, \$1 and \$9, \$3 and \$9, or \$1 and \$7.

For number N, there are 5 x N/10 odd numbers, N/10 odd numbers with 1 as its last digit, N/10 odd numbers with 3 as its last digit, N/10 odd numbers with 5 as its last digit, N/10 odd numbers with 7 as its last digit, and N/10 odd numbers with 9 as its last digit. Odd numbers with 5 as its last digit is not primes except 5. According to Euclid's proof, primes are endless and it is easy to prove that prime with 1, 3, 7, or 9 as its last digit is also endless.

Let's select \$3 to be a product of \$3 and \$1 or \$7 and \$9. If a number (N>3) is not divisible by 3 or any prime which is smaller or equal to N/3, it must be a prime; any number is divisible by 7, it have 1/3 chance is divisible by 3; any number is divisible by 13, it has 1/3 chance to be divisible 3 and 1/7 chance to be divisible by 7, so on, so we have terms: 1/3, 1/7x2/3, 1/13x2/3x6/7.... For number N, there are N/10 odd number with 1 as its last digit, N/10 odd number with 3 as its last digit, N/10 odd number with 7 as its last digit, and N/10 odd number with 9 as its last digit.

The chance of any odd number with 3 as its last digit to be a non-prime is: $[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + \dots]$ -----(Formula 3)

The number (n) of primes in N/10 odd number with 3 as its last digit: $n_3 = N/10 - \{N/10[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + \dots]\}$ -----(Formula 4)

For infinite terms, the number will grow slowly and will be close to 1, but never equal to 1 (if it equal to 1, we will have 0 prime) according to Euclid's proof of endless prime numbers. Let \sum_3 represents the sum of the above infinite terms and Δ_3 represents the chance of any odd number to be a prime. When N is growing to ∞ , and $\Delta_3 = 1 - \sum_3$ may be close to 0, but never be 0.

The sum of first 20 terms = $[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + (1/53 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47) + (1/67 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53) + (1/73 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67) + (1/83 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73) +$

$$\begin{aligned}
& (1/97 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83) + \\
& (1/103 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97) + \\
& (1/107 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103) + \\
& (1/113 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107) + \\
& (1/127 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113) + \\
& (1/137 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127) + \\
& (1/157 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137) + \\
& (1/163 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137 \times 156/157) = \\
& [1/3 + 1/10.5 + 1/22.75 + 1/32.23 + 1/46.33 + 1/77.92 + 1/93.07 + 1/104.15 + 1/120 + 1/164.61 + 1/171.01 + 1/197.14 + 1/233.2 + 1/250.2 + 1/262.47 \\
& + 1/279.80 + 1/317.27 + 1/344.97 + 1/398.24 + 1/416.11 = [0.333333 + 0.095238 + 0.043956 + 0.031028 + 0.021585 + 0.012834 + 0.010745 + 0.009602 \\
& + 0.008333 + 0.006075 + 0.005848 + 0.0050773 + 0.004288 + 0.003997 + 0.003810 + 0.003574 + 0.003152 + 0.002899 + 0.002511 + 0.002403 \\
& + 0.002331] = 0.6102883
\end{aligned}$$

For the first 20 term: $\sum_3 = 0.6103$, $\Delta_3 = 1 - \sum_3 = 0.3897$

For $N=600$, the smallest prime $\$1$ is 11, it decided the possible largest prime $\$3$ is 53, the smallest prime $\$9$ is 19, but 3×3 is 9, so the possible largest prime $\$7$ is 47, $47 \times 3 \times 3 = 423$ (the next will $67 \times 3 \times 3 = 603 > 600$), so we have: Prime number with 3 as its last digit = $600/10 - \{600/10[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + (1/53 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47)]\} = 600/10 - 600/10[0.333333 + 0.095238 + 0.043956 + 0.031028 + 0.021585 + 0.012834 + 0.010745 + 0.009602 + 0.008333 = 60 - 60 \times 0.566654 = 60 - 34 = 26$, it is 3 less than 29 primes (in total 60 odd number) with 3 as their last digit from 1 to 600 because 600 is not a big enough number, when N is big enough, the calculated number will be very close to the real number of primes. For $N=600$, we have $\Delta_3 = 1 - \sum_3 = 1 - 0.566654 = 0.433346$, every odd number with 3 as its last digit has almost 43% chance to be a prime number smaller than 600, every odd number with 3 as its last digit has more than 43% chance to be a prime; for a number bigger than 600, every odd number with 3 as its last digit has less than 43% chance to be a prime.

Every odd number with 7 as its last digit is a product of \$3x\$9 or \$7x\$1; \$1 is decided by \$7 and \$9 is decided by \$3, so we need to consider only \$3 and \$7 (we have other selections too, \$9 and \$7, \$9 and \$1, or \$3 and \$1).

The chance of any odd number with 7 as its last digit to be a non-prime is: $[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + \dots]$ -----(Formula 3)

The number (n) of primes in N/10 odd number with 7 as its last digit is: $n = N/10 - \{N/10[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + \dots]\}$ -----Formula 4.

That mean we have almost same number of primes with 7 as their last digit as the number of primes with 3 as their last digit. From above formula, we can know smaller number N has high percentage to be primes than bigger number N.

Let \sum_7 represent the sum of the above infinitely terms. $\sum_7 = \sum_3$, when N is growing to ∞ , and $\Delta_7 = 1 - \sum_7$ may be close to 0, but never be 0.

For simple, to find out at least one pair primes of $(N/2+m) + (N/2-m)$, we need to fix $(N/2+m)$ or $(N/2-m)$ to be a prime as the list (half in left side and half in right side) in table 1. Let see $\$7=7$ first (left side of table 1), we can know there is a bigger chance for $(N-7)$ to be a prime with 3 as its last digit than $(N-07)$. If $N-7$ can be divisible by 7, then $(N-7)+7 (=N)$ [$7a + 7 = 7(a+1)$, $7a$ and $7(a+1)$ must be divisible by 7] will be divisible by 7, but N with 0 as its last digit and only 70, 140, 210, 280, 350, 420, 490, 560, 630,... are divisible by 7, but we worked on only N and N/2 with 0 as their last digit, only 140, 280, 420, 560,...,(1 in 14) can be divisible by 7, so the term $(1/7 \times 13/14)$ should be taken off from Formula 3.

For the next prime $\$7=17$, $(N-17)$ cannot be divisible by 17 except 340, 680,..., so $(1/17 \times 33/34)$ should be taken off from Formula 3, so on.

Let n_7 represents the total number of primes ($\$7$) with 7 as their last digit in any number N, the chance of every N- $\$7$ with 3 as its last digit to be a prime is: $\Delta_3 = 1 - \sum_3 = 1 - \{[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + \dots] - [(1/7 \times 13/14) + (1/17 \times 33/34) + (1/37 \times 73/74) \dots]\}$ -----Formula 5

When the number of \$7 is 5 or more, $5 \times [(1/7 \times 13/14) + (1/17 \times 33/34) + (1/37 \times 73/74) \dots] > 1$, so every 5 \$7 will have at least 1 prime of N-\$7 to form 1 pair of primes in which one has 7 as its last digit and another has 3 as its last digit and their sum is any number N in which N and N/2 have 0 as its last digit.

For enough large number N, Let's consider $N = O_1 + O_2 = (N/2 + L + 1) + (N/2 - L - 1)$, O_1 and O_2 are odd numbers. $O_1 > O_2$, $O_1 - O_2 = 2L + 2$, $L = 0, 10, 20, 30 \dots L$, $O_1 - O_2 = 2L + 2 = 2, 22, 42, 62, 82, 102, 122, \dots (2L + 2)$, then O_1 is an odd number with 1 as its last digit, O_2 is an odd number with 9 as its last digit.

Also we can have $N = O_1 + O_2 = (N/2 + L + 9) + (N/2 - L - 9)$, O_1 and O_2 are odd numbers. $O_1 > O_2$, $O_1 - O_2 = 2L + 18$, $L = 0, 10, 20, 30 \dots L$, $O_1 - O_2 = 2L + 18 = 18, 38, 58, 78, 98, 118, 138, \dots (2L + 18)$, then O_1 is an odd number with 9 as its last digit, O_2 is an odd number with 1 as its last digit.

These odd number pairs are listed in table 2:

Table 2. The odd number pairs in $N = O_1 + O_2 = (N/2 + L + 1) + (N/2 - L - 1)$ and $N = O_1 + O_2 = (N/2 + L + 9) + (N/2 - L - 9)$

N-9	N-19	N-29	N-39	N-59	N-79	N-89	...	N/2+L+1	N/2-L-9	...	101	71	61	41	31	11
9(3x3)	19	29	39	59	79	89	...	N/2-L-1	N/2+L+9	...	N-101	N-71	N-61	N-41	N-31	N-11

Every odd number (O_1) with 1 as its last digit is a product of \$1x\$1, \$3x\$7, and \$9x\$9. The first \$9 is 19, but the odd number $9 = 3 \times 3$, so 3 is the smallest prime for \$9.

The chance of any odd number with 1 as its last digit to be a non-prime is: $[(1/3) + (1/11 \times 2/3) + (1/13 \times 2/3 \times 10/11) + (1/19 \times 2/3 \times 10/11 \times 12/13) + (1/23 \times 2/3 \times 10/11 \times 12/13 \times 18/19) + (1/29 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23) + (1/31 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29) + (1/41 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31) + (1/43 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41) + \dots]$ -----Formula 6

The number (n) of primes in N/10 odd number with 1 as its last digit is: $n = N/10 - \{N/10[(1/3) + (1/11 \times 2/3) + (1/13 \times 2/3 \times 10/11) + (1/19 \times 2/3 \times 10/11 \times 12/13) + (1/23 \times 2/3 \times 10/11 \times 12/13 \times 18/19) + (1/29 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23) + (1/31 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29) + (1/41 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31) + (1/43 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41) + \dots]\}$ -----Formula 7

The sum of first 20 terms = $[(1/3) + (1/11 \times 2/3) + (1/13 \times 2/3 \times 10/11) + (1/19 \times 2/3 \times 10/11 \times 12/13) + (1/23 \times 2/3 \times 10/11 \times 12/13 \times 18/19) + (1/29 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23) + (1/31 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29) + (1/41 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31) + (1/43 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41) + (1/53 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43) + (1/59 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53) + (1/61 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59) + (1/71 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59 \times 60/61) + (1/73 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59 \times 60/61 \times 70/71) + (1/79 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59 \times 60/61 \times 70/71 \times 72/73) + (1/83 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59 \times 60/61 \times 70/71 \times 72/73 \times 78/79) + (1/89 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59 \times 60/61 \times 70/71 \times 72/73 \times 78/79 \times 82/83) + (1/101 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59 \times 60/61 \times 70/71 \times 72/73 \times 78/79 \times 82/83 \times 88/89) + (1/103 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59 \times 60/61 \times 70/71 \times 72/73 \times 78/79 \times 82/83 \times 88/89 \times 100/101) + (1/109 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59 \times 60/61 \times 70/71 \times 72/73 \times 78/79 \times 82/83 \times 88/89 \times 100/101 \times 102/103)] = 0.333333 + 0.060606 + 0.046620 + 0.029444 + 0.046086 + 0.01748 + 0.01568 + 0.011966 + 0.010747 + 0.008517 + 0.007506 + 0.006484 + 0.006032 + 0.005783 + 0.00529 + 0.004953 + 0.004564 + 0.003976 + 0.003861 + 0.003612 = 0.63254$

Every odd number (O9) with 9 as its last digit is a product terms of \$1 x\$9, \$3x\$3 or \$7x\$7. We need to consider only \$1, \$3, and \$7.

The chance of any odd number with 9 as its last digit to be a non-prime is: $[(1/3) + (1/7 \times 2/3) + (1/11 \times 2/3 \times 6/7) + (1/13 \times 2/3 \times 6/7 \times 10/11) + (1/17 \times 2/3 \times 6/7 \times 10/11 \times 12/13) + (1/23 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17) + (1/31 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23) + (1/37 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31) + (1/41 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37) + (1/43 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41) + (1/47 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43) \dots]$ -----Formula 8

the number (n) of primes in N/10 odd number with 9 as its last digit: $n = N/10 - \{N/10[(1/3) + (1/7 \times 2/3) + (1/11 \times 2/3 \times 6/7) + (1/13 \times 2/3 \times 6/7 \times 10/11) + (1/17 \times 2/3 \times 6/7 \times 10/11 \times 12/13) + (1/23 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17) + (1/31 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23) + (1/37 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31) + (1/41 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37) + (1/43 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41) + (1/47 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43) \dots]\}$ -----Formula 9

The sum of first 20 terms = $[(1/3) + (1/7 \times 2/3) + (1/11 \times 2/3 \times 6/7) + (1/13 \times 2/3 \times 6/7 \times 10/11) + (1/17 \times 2/3 \times 6/7 \times 10/11 \times 12/13) + (1/23 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17) + (1/31 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23) + (1/37 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31) + (1/41 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37) + (1/43 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41) + (1/47 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43) + (1/53 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47) + (1/61 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53) + (1/67 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 60/61) + (1/71 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 60/61 \times 66/67) + (1/73 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 60/61 \times 66/67 \times 70/71) + (1/83 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 60/61 \times 66/67 \times 70/71 \times 72/73) + (1/97 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 60/61 \times 66/67 \times 70/71 \times 72/73 \times 82/83) + (1/101 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 60/61 \times 66/67 \times 70/71 \times 72/73 \times 82/83 \times 96/97) + 1/103 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 60/61 \times 66/67 \times 70/71 \times 72/73 \times 82/83 \times 96/97 \times 100/101] = 0.333333 + 0.095238 + 0.051948 + 0.039960 + 0.030558 + 0.021258 + 0.013926 + 0.011291 + 0.010740 + 0.009222 + 0.008928 + 0.007323 + 0.006097 + 0.005460 + 0.005076 + 0.004867 + 0.004323 + 0.003569 + 0.003393 + 0.003294 = 0.669804$

Let see \$9=9 (3 x 3) first (left side of table 2), we can know there is a bigger chance for (N-\$9) to be a prime with 1 as its last digit than (N-09). If N-9 can be divisible by 9, then (N-9)+9 (=N) [9a + 9 = 9(a+1), 9a and 9(a+1) must be divisible by 3 or 9] will be divisible by 3 or 9, but N with 0 as its last digit and only 30, 60, 90, 120, 150, 180, 210, 240, 260,... are divisible by 3 or 9,..., and we worked on only N and N/2 with 0 as their last digit, only 60, 120, 180, 240,..., (1 in 6) can be divisible by 3 or 9, so the term (1/3x5/6) should be taken off from Formula 6.

For the next prime \$9=19, (N-19) cannot be divisible by 19 except 380, 760,..., so (1/19x37/38) should be taken off from Formula 6, so on.

Let n_9 represents the total number of primes (\$9) with 9 as their last digit in any number N, the chance of every N-\$9 with 1 as its last digit to be a prime is: $\Delta_1=1-\sum_1=1- \{[(1/3) + (1/11x2/3) + (1/13x2/3x10/11) + (1/19x2/3x10/11x12/13) + (1/23x2/3x10/11x12/13x18/19) + (1/29x2/3x10/11x12/13x18/19x22/23) + (1/31x2/3x10/11x12/13x18/19x22/23x28/29) + (1/41x2/3x10/11x12/13x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41) + \dots] - [(1/3x5/6) + (1/19x37/38) + (1/29x57/58) \dots]\}$ -----Formula 10

When the number of \$9 is 4 or more, $4 \times [(1/3x5/6) + (1/19x37/38) + (1/29x57/58) \dots] > 1$, so every 4 \$9 will have at least 1 prime of N-\$9 to form 1 pair of primes in which one has 9 as its last digit and another has 1 as its last digit and their sum is any number N in which N and N/2 have 0 as its last digit.

For N = 600 (see table 3), 600 can be expressed as the sum of 15 pairs of primes in which one prime with 3 as its last digit and another prime with 7 as its last digit and 600 can be expressed as the sum of 16 pairs of primes in which one prime with 1 as its last digit and another prime with 9 as its last digit.

7	17	27	37	47	57	67	77	87	97	107	117	127	137	147	157	167	177	187
Prime	Prime	3x9	Prime	prime	3x19	Prime	7x11	3x29	Prime	prime	3x3x13	Prime	prime	3x7x7	prime	prime	3x59	11x17
593	583	573	563	553	543	533	523	513	503	493	483	473	463	453	443	433	423	413
Prime	11x53	3x191	Prime	7x79	3x181	13x41	prime	3x3x3x19	Prime	17x29	3x7x23	11x43	prime	3x151	Prime	prime	3x3x47	7x59

223	233	243	253	263	273	283	293	303	313	323	333	343	353	363	373	383	393	403
prime	prime	3x3x3 x3x3	11x23	prime	3x7x13	prime	prime	3x101	Prime	17x19	3x111	7x7x7	Prime	3x11x 11	Prime	Prime	3x131	13x31
377	367	357	347	337	327	317	307	297	287	277	267	257	247	237	227	217	207	197
13x29	prime	3x7x1 7	prime	prime	3x109	Prime	prime	3x3x33	7x41	Prime	3x89	Prime	13x19	3x79	Prime	7x31	3x3x2 3	Prime
387	397	407	417	427	437	447	457	467	477	487	497	507	517	527	537	547	557	567
3x3x43	prime	11x37	3x139	7x61	23x19	3x149	prime	prime	3x3x5 3	prime	7x71	3x13x 13	11x47	17x31	3x179	prime	prime	3x3x3 x3x7
213	203	193	183	173	163	153	143	133	123	113	103	93	83	73	63	53	43	33
3x71	7x29	prime	3x61	Prime	prime	3x3x1 7	11x13	7x19	3x41	3x3x1 3	prime	3 x31	prime	7x11	3x3x7	prime	prime	3x11
																3	13	23
																Prime	Prime	prime
																597	587	577
																3x199	prime	prime
589	579	569	559	549	539	529	519	509	499	489	479	469	459	449	439	429	419	409
19x31	3x193	Prime	13x43	3x3x6 1	7x7x11	23x23	3x178	Prime	Prime	3x163	Prime	7x67	3x3x3 x17	Prime	Prime	3x11x 13	Prime	Prime
11	21	31	41	51	61	71	81	91	101	111	121	131	141	151	161	171	181	191
Prime	3x7	Prime	Prime	3x17	Prime	Prime	3x3x3	7x13	Prime	3x37	11x11	Prime	3x47	Prime	7x23	3x3x1 9	Prime	Prime

381	371	361	351	341	331	321	311	301	291	281	271	261	251	241	231	221	211	201
3x127	7x53	19x19	3x3x3x13	11x31	Prime	3x107	Prime	7x43	3x97	Prime	Prime	3x3x29	Prime	Prime	3x7x11	13x17	Prime	3x67
219	229	239	249	259	269	279	289	299	309	319	329	339	349	359	369	379	389	399
3x73	Prime	Prime	3x83	7x37	Prime	3x3x31	17x17	13x23	3x103	11x29	7x47	3x113	Prime	Prime	3x3x41	prime	Prime	3x7x19
209	199	189	179	169	159	149	139	129	119	109	99	89	79	69	59	49	39	29
11x19	Prime	3x3x7	Prime	13x13	3x53	Prime	Prime	3x43	7x17	Prime	3x3x11	Prime	Prime	3x23	Prime	7x7	3x13	Prime
391	401	411	421	431	441	451	461	471	481	491	501	511	521	531	541	551	561	571
17x23	Prime	3x137	Prime	Prime	3x3x7x7	11x41	Prime	3x157	13x37	Prime	3x167	7x73	Prime	3x3x59	Prime	19x29	3x11x17	Prime
																	591	581
																	3x197	7x81
																	9	19
																	3x3	Prime

1b. When both N has 0 as its last digit, and N/2 has 5 as its last digit.

Table 3. The odd number pairs in $N=O_1+O_2=(N/2+L-2)+(N/2-L+2)$ and $N=O_1+O_2=(N/2+L+2)+(N/2-L-2)$

N-7	N-17	N-37	N-47	N-67	N-97	...	N/2+L-2	N/2-L-2	...	83	73	53	43	23	13	3
7	17	37	47	67	97	...	N/2-L+2	N/2+L+2	...	N-83	N-73	N-53	N-43	N-23	N-13	N-3

Let see $\$7=7$ first (left side of table 3), we can know there is a bigger chance for (N- $\$7$) to be a prime with 3 as its last digit than (N-07). If N-7 can be divisible by 7, then (N-7)+7 (=N) [$7a + 7 = 7(a+1)$, 7a and 7(a+1) must be divisible by 7] will be divisible by 7, but N with 0 as its last digit and only 70, 140, 210, 280, 350, 420, 490, 560, 630,... are divisible by 7, but we worked on only N with 0 as their last digit and N/2 with 5 as their last digit, so only 70, 210, 350, 490,...,(1 in14) can be divisible by 7, so the term $(1/7 \times 13/14)$ should be taken off from Formula 3.

For the next prime $\$7=17$, (N-17) cannot be divisible by 17 except 170, 510,..., so $(1/17 \times 33/34)$ should be taken off from Formula 3, so on.

Let n_7 represents the total number of primes ($\$7$) with 7 as their last digit in any number N, the chance of every N- $\$7$ with 3 as its last digit to be a prime is: $\Delta_3 = 1 - \sum_3 = 1 - \{[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + \dots] - [(1/7 \times 13/14) + (1/17 \times 33/34) + (1/37 \times 73/74) \dots]\}$

When the number of $\$7$ is 5 or more, $5 \times [(1/7 \times 13/14) + (1/17 \times 33/34) + (1/37 \times 73/74) \dots] > 1$, so every 5 $\$7$ will have at least 1 prime of N- $\$7$ to form 1 pair of primes in which one has 7 as its last digit and another has 3 as its last digit and their sum is any number N in which N have 0 as its last digit and N/2 have 5 as its last digit.

Table 4. The odd number pairs in $N = O_1 + O_2 = (N/2 + L - 4) + (N/2 - L + 4)$ and $N = O_1 + O_2 = (N/2 + L - 6) + (N/2 - L + 6)$

N-9	N-19	N-29	N-59	N-79	N-89	...	N/2+1-4	N/2-L+6	...	101	71	61	41	31	11
9(3x3)	19	29	59	79	89	...	N/2-L+4	N/2+L-6	...	N-101	N-71	N-61	N-41	N-31	N-11

Let see \$9=9 (3 x 3) first (left side of table 4), we can know there is a bigger chance for (N-\$9) to be a prime with 1 as its last digit than (N-O9). If N-9 can be divisible by 9, then (N-9)+9 (=N) [9a + 9 = 9(a+1), 9a and 9(a+1) must be divisible by 3 or 9] will be divisible by 3 or 9, but N with 0 as its last digit and only 30, 60, 90, 120, 150, 180, 210, 240, 260,... are divisible by 3 or 9,..., but we worked on only N with 0 as their last digit and N/2 with 5 as its last digit, only 30, 90, 150, 210,..., (1 in 6) can be divisible by 3 or 9, so the term (1/3x5/6) should be taken off from Formula 6.

For the next prime \$9=19, (N-19) cannot be divisible by 19 except 190, 570,..., so (1/19x37/38) should be taken off from Formula 6, so on.

Let n_9 represents the total number of primes (\$9) with 9 as their last digit in any number N, the chance of every N-\$9 with 1 as its last digit to be a prime is: $\Delta_1 = 1 - \sum_1 = 1 - \{[(1/3) + (1/11 \times 2/3) + (1/13 \times 2/3 \times 10/11) + (1/19 \times 2/3 \times 10/11 \times 12/13) + (1/23 \times 2/3 \times 10/11 \times 12/13 \times 18/19) + (1/29 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23) + (1/31 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29) + (1/41 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31) + (1/43 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41) + \dots] - [(1/3 \times 5/6) + (1/19 \times 37/38) + (1/29 \times 57/58) \dots]\}$

When the number of \$9 is 4 or more, $4 \times [(1/3 \times 5/6) + (1/19 \times 37/38) + (1/29 \times 57/58) \dots] > 1$, so every 4 \$9 will have at least 1 prime of N-\$9 to form 1 pair of primes in which one has 9 as its last digit and another has 1 as its last digit and the sum is any number N in which N has 0 as its last digit and N/2 has 5 as its last digit.

2. When any even integer (N) has 2 as its last digit, such as 12, 22, 32, 42, 112, 122, 1122, 1132,..., then N/2 has only 6 or 1 as its last digit.

2a. When any even integer (N) has 2 as its last digit, such as 12, 32, 52, 112, 1132,..., then N/2 has 6 as its last digit:

Table 5. The odd number pairs in $N=O_1+O_2=(N/2+L-3)+(N/2-L+3)$ and $N=O_1+O_2=(N/2+L+3)+(N/2-L-3)$

N-9	N-19	N-29	N-59	N-79	N-89	...	N/2+L-3	N/2-L-3	...	83	73	53	43	23	13	3
9(3x3)	19	29	59	79	89	...	N/2-L+3	N/2+L+3	...	N-83	N-73	N-53	N-43	N-23	N-13	N-3

Let see \$9 = 9 first (left side of table 5), if N-9 can be divisible by 9, then (N-9)+9 (=N) [9a + 9 = 9(a+1), 9a and 9(a+1) must be divisible by 3 or 9] will be divisible by 3 or 9, but N with 2 as its last digit and only 12, 42, 72, 102, 132, 162, 192, 222, 252, 282, 312, 342, 372, 402, 432, 462, 492, 522, 552,... are divisible by 3 or 9,..., but we worked on only N with 2 as their last digit and N/2 with 6 as their last digit, only 12, 72, 132, 192, 252,..., (1 in 6) can be divisible by 3 or 9, so the term (1/3x5/6) should be taken off from Formula 3.

For the next prime \$9=19, (N-19) cannot be divisible by 19 except 552, 932,..., so (1/19x37/38) should be taken off from Formula 3, so on.

Let n_9 represents the total number of primes (\$9) with 9 as their last digit in any number N, the chance of every N-\$9 with 3 as its last digit to be a prime is: $\Delta_3 = 1 - \sum_3 = 1 - \{[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + \dots] - [(1/3 \times 5/6) + (1/19 \times 37/38) + (1/29 \times 57/58) \dots]\}$

When the number of \$9 is 4 or more, $4 \times [(1/3 \times 5/6) + (1/19 \times 37/38) + (1/29 \times 57/58) \dots] > 1$, so every 4 \$9 will have at least 1 prime of N-\$9 to form 1 pair of primes in which one has 9 as its last digit and another has 3 as its last digit and their sum is any number N in which N has 2 as its last digit and N/2 has 6 as its last digit.

Table 6. The odd number pairs in $N=O_1+O_2=(N/2+L-5)+(N/2-L+5)$ and $N=O_1+O_2=(N/2+L+5)+(N/2-L+5)$

N-11	N-31	N-41	N-61	N-71	N-101	...	N/2+L-5	N/2-L+5	...	101	71	61	41	31	11
11	31	41	61	71	101	...	N/2-L+5	N/2+L-5	...	N-101	N-71	N-61	N-41	N-31	N-11

Let see \$1=11 first (left side of table 6), we can know there is a bigger chance for (N-\$1) to be a prime with 1 as its last digit than (N-O1). If N-11 can be divisible by 11, then (N-11)+11, [11a + 11 = 11(a+1), 11a and 11(a+1) must be divisible by 11] will be divisible by 11, but N with 2 as its last digit and only 132, 242, 352, 462, 572, 682, 792, 902,... are divisible by 11,..., but we worked on only N with 2 as their last digit and N/2 with 6 as its last digit, only 132, 352, 572, 792,...(1 in 22) can be divisible by 11, so the term (1/11x21/22) should be taken off from Formula 6.

For the next prime \$1=31, (N-31) cannot be divisible by 31 except 372, 992,..., so (1/31x61/62) should be taken off from Formula 6, so on.

Let n_1 represents the total number of primes (\$1) with 1 as their last digit in any number N, the chance of every N-\$1 with 1 as its last digit to be a prime is: $\Delta_1 = 1 - \sum_1 = 1 - \{[(1/3) + (1/11 \times 2/3) + (1/13 \times 2/3 \times 10/11) + (1/19 \times 2/3 \times 10/11 \times 12/13) + (1/23 \times 2/3 \times 10/11 \times 12/13 \times 18/19) + (1/29 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23) + (1/31 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29) + (1/41 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31) + (1/43 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41) + \dots] - [(1/11 \times 21/22) + (1/31 \times 61/62) + (1/41 \times 81/82) \dots]\}$

When the number of \$1 is 11 or more, $[(1/11 \times 21/22) + (1/31 \times 61/62) + (1/41 \times 81/82) \dots] > 1$, so every 11 \$1 will have at least 1 prime of N-\$1 to form 1 pair of primes in which both have 1 as its last digit and their sum is any number N in which N has 2 as its last digit and N/2 has 6 as its last digit.

2b. When any even integer (N) has 2 as its last digit, such as 22, 42, 62, 82, 102, 1122,..., then N/2 has 1 as its last digit:

Table 7. The odd number pairs in $N = O_1 + O_2 = (N/2 + L + 2) + (N/2 - L + 8)$ and $N = O_1 + O_2 = (N/2 - L + 2) + (N/2 + L + 8)$

N-9	N-19	N-29	N-39	N-49	N-59	...	N/2+L+2	N/2-L+2	...	73	63	53	43	33	23	13	3
9	19	29	59	79	89	...	N/2-L+8	N/2+L+8	...	N-73	N-63	N-53	N-43	N-33	N-23	N-13	N-3

Let see \$9=9 first (left side of table 7), if N-9 can be divisible by 9, then (N-9)+9 (=N) [9a + 9 = 9(a+1), 9a and 9(a+1) must be divisible by 3 or 9] will be divisible by 3 or 9, N with 2 as its last digit and only 12, 42, 72, 102, 132, 162, 192, 222, 252, 282, 312, 342, 372, 402, 432, 462, 492, 522, 552,... are divisible by 3 or 9,..., but we worked on only N with 2 as their last digit and N/2 with 1 as their last digit, only 42, 102, 162, 222, 282, 342, 402,...(1 in 6) can be divisible by 3 or 9, so the term (1/3x5/6) should be taken off from Formula 3.

For the next prime \$9=19, (N-19) cannot be divisible by 19 except 552, 932,..., so (1/19x37/38) should be taken off from Formula 3, so on.

Let n₉ represents the total number of primes (\$9) with 9 as their last digit in any number N, the chance of every N-\$9 with 3 as its last digit to be a prime is: $\Delta_3 = 1 - \sum_3 = 1 - \{[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + \dots] - [(1/3 \times 5/6) + (1/19 \times 37/38) + (1/29 \times 57/58) \dots]\}$

When the number of \$9 is 4 or more, $4 \times [(1/3 \times 5/6) + (1/19 \times 37/38) + (1/29 \times 57/58) \dots] > 1$, so every 4 \$9 will have at least 1 prime of N-\$9 to form 1 pair of primes in which one has 9 as its last digit and another has 3 as its last digit and their sum is any number N in which N has 2 as its last digit and N/2 has 1 as its last digit.

Table 8. The odd number pairs in $N = O_1 + O_2 = (N/2 + L + 0) + (N/2 - L - 0)$ and $N = O_1 + O_2 = (N/2 + L - 0) + (N/2 - L + 0)$

N-11	N-21	N-31	N-41	N-51	N-61	N-71	...	N/2+L+0	N/2-L+0	...	81	71	61	51	41	31	21	11
11	21	31	41	51	61	71	...	N/2-L-0	N/2+L-0	...	N-81	N-71	N-61	N-51	N-41	N-31	N-21	N-11

Let see \$1=11 first (left side of table 8), we can know there is a bigger chance for (N-\$1) to be a prime with 1 as its last digit than (N-O1). If N-11 can be divisible by 11, then (N-11)+11 (=N) [11a + 11 = 11(a+1), 11a and 11(a+1) must be divisible by 11] will be divisible by 11, N with 2 as its last digit and only 132, 242, 352, 462, 572, 682, 792, 902,... are divisible by 11,..., but we worked on only N with 2 as their last digit and N/2 with 1 as its last digit, only 22, 242, 462, 682,..., (1 in 22) can be divisible by 11, so the term (1/11x21/22) should be taken off from Formula 6.

For the next prime \$1=31, (N-31) cannot be divisible by 31 except 62, 682,..., so (1/31x61/62) should be taken off from Formula 6, so on.

Let n_1 represents the total number of primes (\$1) with 1 as their last digit in any number N, the chance of every N-\$1 with 1 as its last digit to be a prime is: $\Delta_1=1-\sum_1=1- \{[(1/3) + (1/11x2/3) + (1/13x2/3x10/11)+ (1/19x2/3x10/11x12/13) + (1/23x2/3x10/11x12/13x18/19) + (1/29x2/3x10/11x12/13x18/19x22/23) + (1/31x2/3x10/11x12/13x18/19x22/23x28/29) + (1/41x2/3x10/11x12/13x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41)+...]- [(1/11x21/22)] + (1/31x61/62) + (1/41x81/82)...]\}$

When the number of \$1 is 11 or more, $[(1/11x21/22)] + (1/31x61/62) + (1/41x81/82)...] > 1$, so every 11 \$1 will have at least 1 prime of N-\$1 to form 1 pair of primes in which both have 1 as its last digit and their sum is any number N in which N has 2 as its last digit and N/2 has 1 as its last digit.

3a. When any even integer (N) has 4 as its last digit, such as 24, 44, 64, 84, 104, 1124,..., then N/2 has 2 as its last digit:

Table 9. The odd number pairs in $N=O_1+O_2=(N/2+L+1)+(N/2-L+7)$ and $N=O_1+O_2=(N/2-L+1)+(N/2+L+7)$

N-7	N-17	N-37	N-47	N-67	N-97	N-107	...	N/2+L-5	N/2-L-5	...	107	97	67	47	37	17	7
7	17	37	47	67	97	107	...	N/2-L+5	N/2+L+5	...	N-107	N-97	N-67	N-47	N-37	N-17	N-7

Let see $\$7=7$ first (left side of table 9), we can know there is a bigger chance for (N- $\$7$) to be a prime with 7 as its last digit than (N-07). If N-7 can be divisible by 7, then (N-7)+7 (=N) [$7a + 7 = 7(a+1)$, 7a and 7(a+1) must be divisible by 7] will be divisible by 7, but N with 4 as its last digit and only 14, 84, 154, 224, 294, 364, 434, 504, 574,... are divisible by 7, but we worked on only N with 4 as their last digit and N/2 with 2 as their last digit, only 84, 224, 364, 504,..., (1 in 14) can be divisible by 7, so the term $(1/7 \times 13/14)$ should be taken off from Formula 3.

For the next prime $\$7=17$, (N-17) cannot be divisible by 17 except 204, 544, 884,..., so $(1/17 \times 33/34)$ should be taken off from Formula 3, so on.

Let n_7 represents the total number of primes ($\$7$) with 7 as their last digit in any number N, the chance of every N- $\$7$ with 7 as its last digit to be a prime is: $\Delta_7 = 1 - \sum_7 = 1 - \{[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + \dots] - [(1/7 \times 13/14) + (1/17 \times 33/34) + (1/37 \times 73/74) \dots]\}$

When the number of $\$7$ is 5 or more, $5 \times [(1/7 \times 13/14) + (1/17 \times 33/34) + (1/37 \times 73/74) \dots] > 1$, so every 5 $\$7$ will have at least 1 prime of N- $\$7$ to form 1 pair of primes in which both have 7 as its last digit and the sum is any number N in which N have 4 as its last digit and N/2 have 2 as its last digit.

Table 10. The odd number pairs in $N = O_1 + O_2 = (N/2 + L + 1) + (N/2 - L - 1)$ and $N = O_1 + O_2 = (N/2 + L - 1) + (N/2 - L + 1)$

N-11	N-21	N-31	N-41	N-51	N-61	N-71	N-81	...	N/2+L+1	N/2-L+1	...	103	83	53	43	23	13	3
11	21	31	41	51	61	71	81	...	N/2-L-1	N/2+L-1	...	N-71	N-61	N-51	N-41	N-31	N-21	N-11

Let see \$1=11 first (left side of table 10), we can know there is a bigger chance for (N-\$1) to be a prime with 1 as its last digit than (N-O1). If N-11 can be divisible by 11, then (N-11)+11 (=N) [11a + 11 = 11(a+1), 11a and 11(a+1) must be divisible by 11] will be divisible by 11, N with 4 as its last digit and only 44, 154, 264, 374, 484, 594, 704, 814,... are divisible by 11,..., but we worked on only N with 4 as their last digit and N/2 with 2 as its last digit, only 44, 264, 484, 704,..., (1 in 22) can be divisible by 11, so the term (1/11x21/22) should be taken off from Formula 3.

For the next prime \$1=31, (N-31) cannot be divisible by 31 except 124, 744,..., so (1/31x61/62) should be taken off from Formula 6, so on.

Let n_1 represents the total number of primes (\$1) with 1 as their last digit in any number N, the chance of every N-\$1 with 3 as its last digit to be a prime is: $\Delta_3=1-\sum_3=1- \{[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23) + (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + \dots] - [(1/11x21/22)] + (1/31x61/62) + (1/41x81/82) \dots\}$

When the number of \$1 is 11 or more, $[(1/11x21/22)] + (1/31x61/62) + (1/41x81/82) \dots > 1$, so every 11 \$1 will have at least 1 prime of N-\$1 to form 1 pair of primes in which one prime has 1 as its last digit and another has 3 as its last digit and their sum is any number N in which N has 4 as its last digit and N/2 has 2 as its last digit.

3b. When any even integer (N) has 4 as its last digit, such as 14, 34, 54, 74, 94, 1114,..., then N/2 has 7 as its last digit:

Table 11. The odd number pairs in $N=O_1+O_2=(N/2+L+0)+(N/2-L+0)$ and $N=O_1+O_2=(N/2-L+0)+(N/2+L+0)$

N-7	N-17	N-37	N-47	N-67	N-97	N-107	...	N/2+L+0	N/2-L+0	...	107	97	67	47	37	17	7
7	17	37	47	67	97	107	...	N/2-L+0	N/2+L+0	...	N-107	N-97	N-67	N-47	N-37	N-17	N-7

Let see $\$7=7$ first (left side of table 11), we can know there is a bigger chance for $(N-\$7)$ to be a prime with 7 as its last digit than $(N-O7)$. If $N-7$ can be divisible by 7, then $(N-7)+7 (=N)$ [$7a + 7 = 7(a+1)$, $7a$ and $7(a+1)$ must be divisible by 7] will be divisible by 7, but N with 4 as its last digit and only 14, 84, 154, 224, 294, 364, 434, 504, 574,... are divisible by 7, but we worked on only N with 4 as their last digit and $N/2$ with 7 as their last digit, only 14, 154, 294, 434, 574,..., (1 in 14) can be divisible by 7, so the term $(1/7 \times 13/14)$ should be taken off from Formula 3.

For the next prime $\$7=17$, $(N-17)$ cannot be divisible by 17 except 34, 374, 714,..., so $(1/17 \times 33/34)$ should be taken off from Formula 3, so on.

Let n_7 represents the total number of primes ($\$7$) with 7 as their last digit in any number N , the chance of every $N-\$7$ with 7 as its last digit to be a prime is: $\Delta_7 = 1 - \sum_7 = 1 - \{[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + \dots] - [(1/7 \times 13/14) + (1/17 \times 33/34) + (1/37 \times 73/74) \dots]\}$

When the number of $\$7$ is 5 or more, $5 \times [(1/7 \times 13/14) + (1/17 \times 33/34) + (1/37 \times 73/74) \dots] > 1$, so every 5 $\$7$ will have at least 1 prime of $N-\$7$ to form 1 pair of primes in which both have 7 as its last digit and the sum is any number N in which N have 4 as its last digit and $N/2$ have 7 as its last digit.

Table 12. The odd number pairs in $N=O_1+O_2=(N/2+L-4)+(N/2-L+4)$ and $N=O_1+O_2=(N/2+L+4)+(N/2-L-4)$

N-11	N-31	N-41	N-51	N-61	N-71	...	N/2+L-4	N/2-L-4	...	73	53	43	23	13	3
11	31	41	51	61	71	...	N/2-L+4	N/2+L+4	...	N-73	N-53	N-43	N-23	N-13	N-3

Let see \$1=11 first (left side of table 12), we can know there is a bigger chance for (N-\$1) to be a prime with 1 as its last digit than (N-O1). If N-11 can be divisible by 11, then (N-11)+11 (=N) [11a + 11 = 11(a+1), 11a and 11(a+1) must be divisible by 11] will be divisible by 11, N with 4 as its last digit and only 44, 154, 264, 374, 484, 594, 704, 814,... are divisible by 11,..., but we worked on only N with 4 as their last digit and N/2 with 7 as its last digit, only 154, 374, 594, 814,... (1 in 22) can be divisible by 11, so the term (1/11x21/22) should be taken off from Formula 3.

For the next prime \$1=31, (N-31) cannot be divisible by 31 except 434, 1054,..., so (1/31x61/62) should be taken off from Formula 6, so on.

Let n_1 represents the total number of primes (\$1) with 1 as their last digit in any number N, the chance of every N-\$1 with 3 as its last digit to be a prime is: $\Delta_3=1-\sum_3=1- \{[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + \dots] - [(1/11 \times 21/22) + (1/31 \times 61/62) + (1/41 \times 81/82) \dots]\}$

When the number of \$1 is 11 or more, $[(1/11 \times 21/22) + (1/31 \times 61/62) + (1/41 \times 81/82) \dots] > 1$, so every 11 \$1 will have at least 1 prime of N-\$1 to form 1 pair of primes in which one prime has 1 as its last digit and another have 3 as its last digit and their sum is any number N in which N has 4 as its last digit and N/2 has 7 as its last digit.

4a. When any even integer (N) has 6 as its last digit, such as 26, 46, 66, 86, 106, 1126,..., then N/2 has 3 as its last digit:

Table 13. The odd number pairs in $N=O_1+O_2=(N/2+L+0)+(N/2-L+0)$ and $N=O_1+O_2=(N/2-L+0)+(N/2+L+0)$

N-3	N-13	N-23	N-43	N-53	N-73	N-83	...	N/2+L+0	N/2-L+0	...	83	73	53	43	23	13	3
3	13	23	43	53	73	83	...	N/2-L+0	N/2+L+0	...	N-83	N-73	N-53	N-43	N-23	N-13	N-3

Let see $\$3=3$ first (left side of table 13), we can know there is a bigger chance for (N- $\$3$) to be a prime with 3 as its last digit than (N-03). If N-3 can be divisible by 3, then (N-3)+3 (=N) [$3a + 3 = 3(a+1)$, 3a and 3(a+1) must be divisible by 3] will be divisible by 3, but N with 6 as its last digit and only 6, 36, 66, 96, 126, 156, 186, 216, 246,... are divisible by 3, but we worked on only N with 6 as their last digit and N/2 with 3 as their last digit, only 6, 66, 126, 186, 246,...,(1 in 6) can be divisible by 3, so the term $(1/3 \times 5/6)$ should be taken off from Formula 3.

For the next prime $\$3=13$, (N-13) cannot be divisible by 13 except 26, 286, 546,..., so $(1/13 \times 25/26)$ should be taken off from Formula 3, so on.

Let n_3 represents the total number of primes ($\$3$) with 3 as their last digit in any number N, the chance of every N- $\$3$ with 3 as its last digit to be a prime is: $\Delta_3=1-\sum_3 = 1- \{[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + \dots] - [(1/3 \times 5/6) + (1/13 \times 25/26) + (1/23 \times 45/46) + \dots]\}$

When the number of $\$3$ is 3 or more, $3 \times [(1/3 \times 5/6) + (1/13 \times 25/26) + (1/23 \times 45/46) \dots] > 1$, so every 3 $\$3$ will have at least 1 prime of N- $\$3$ to form 1 pair of primes in which both have 3 as its last digit and their sum is any number N in which N has 6 as its last digit and N/2 has 3 as its last digit.

Table 14. The odd number pairs in $N=O_1+O_2=(N/2+L-4)+(N/2-L+4)$ and $N=O_1+O_2=(N/2+L+4)+(N/2-L-4)$

N-7	N-17	N-37	N-47	N-67	N-97	...	N/2+L-4	N/2-L-4	...	89	79	59	29	19
7	17	37	47	67	97	...	N/2-L+4	N/2+L+4	...	N-89	N-79	N-59	N-29	N-19

Let see $\$7=7$ first (left side of table 11), we can know there is a bigger chance for (N- $\$7$) to be a prime with 7 as its last digit than (N-07). If N-7 can be divisible by 7, then (N-7)+7 (=N) [$7a + 7 = 7(a+1)$, 7a and 7(a+1) must be divisible by 7] will be divisible by 7, but N with 6 as its last digit and only 56, 126, 196, 266, 336, 406, 476, 546, 616,... are divisible by 7, but we worked on only N with 6 as their last digit and N/2 with 3 as their last digit, only 126, 266, 406, 546,..., (1 in 14) can be divisible by 7, so the term $(1/7 \times 13/14)$ should be taken off from Formula 8.

For the next prime $\$7=17$, (N-17) cannot be divisible by 17 except 226, 566, 906,..., so $(1/17 \times 33/34)$ should be taken off from Formula 8, so on.

Let n_7 represents the total number of primes ($\$7$) with 7 as their last digit in any number N, the chance of every N- $\$7$ with 9 as its last digit to be a prime is: $\Delta_9 = 1 - \sum_9 = 1 - \{[(1/3) + (1/7 \times 2/3) + (1/11 \times 2/3 \times 6/7) + (1/13 \times 2/3 \times 6/7 \times 10/11) + (1/17 \times 2/3 \times 6/7 \times 10/11 \times 12/13) + (1/23 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17) + (1/31 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23) + (1/37 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31) + (1/41 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37) + (1/43 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41) + (1/47 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43) + \dots] - [(1/7 \times 13/14)] + (1/17 \times 33/34) + (1/37 \times 73/74) + \dots\}$

When the number of $\$7$ is 5 or more, $5 \times [(1/7 \times 13/14) + (1/17 \times 33/34) + (1/37 \times 73/74) \dots] > 1$, so every 5 $\$7$ will have at least 1 prime of N- $\$7$ to form 1 pair of primes in which 1 prime has 7 as its last digit and another has 9 as its last digit and their sum is any number N in which N has 6 as its last digit and N/2 have 3 as its last digit.

4b. When any even integer (N) has 6 as its last digit, such as 16, 36, 56, 76, 96, 1116, ..., then N/2 has 8 as its last digit:

Table 15. The odd number pairs in $N=O_1+O_2=(N/2+L-1)+(N/2-L+1)$ and $N=O_1+O_2=(N/2-L-1)+(N/2+L+1)$

N-9	N-19	N-29	N-59	N-79	N-89	N-109	...	N/2+L-1	N/2-L-1	...	107	97	67	47	37	17	7
9(3x3)	19	29	59	79	89	109	...	N/2-L+1	N/2+L+1	...	N-107	N-97	N-67	N-47	N-37	N-17	N-7

Let see $\$9=9$ first (left side of table 15), if N-9 can be divisible by 9, then (N-9)+9 (=N) [$9a + 9 = 9(a+1)$, 9a and 9(a+1) must be divisible by 3 or 9] will be divisible by 3 or 9, N with 6 as its last digit and only 36, 66, 96, 126, 156, 186, 216, 246, 276, 306, 336, 366, 396, 426, 456, 486, 516, 546, ... are divisible by 3 or 9, ..., but we worked on only N with 6 as their last digit and N/2 with 8 as their last digit, only 36, 96, 156, 216, 276, 336, 396, 456, 516, ... (1 in 6) can be divisible by 3 or 9, so the term $(1/3 \times 5/6)$ should be taken off from Formula 3.

For the next prime $\$9=19$, (N-19) cannot be divisible by 19 except 76, 456, 836, ..., so $(1/19 \times 37/38)$ should be taken off from Formula 3, so on.

Let n_9 represents the total number of primes ($\$9$) with 9 as their last digit in any number N, the chance of every N- $\$9$ with 7 as its last digit to be a prime is: $\Delta_7 = 1 - \sum_7 = 1 - \{[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + \dots] - [(1/3 \times 5/6) + (1/19 \times 37/38) + (1/29 \times 57/58) \dots]\}$

When the number of $\$9$ is 4 or more, $4 \times [(1/3 \times 5/6) + (1/19 \times 37/38) + (1/29 \times 57/58) \dots] > 1$, so every 4 $\$9$ will have at least 1 prime of N- $\$9$ to form 1 pair of primes in which one has 9 as its last digit and another has 7 as its last digit and their sum is any number N in which N has 6 as its last digit and N/2 has 8 as its last digit.

Table 16. The odd number pairs in $N=O_1+O_2=(N/2+L-5)+(N/2-L+5)$ and $N=O_1+O_2=(N/2+L-5)+(N/2-L+5)$

N-3	N-13	N-23	N-43	N-53	N-73	N-83	...	N/2+L-5	N/2-L+5	...	83	73	53	43	23	13	3
3	13	23	43	53	73	83	...	N/2-L+5	N/2+L-5	...	N-83	N-73	N-53	N-43	N-23	N-13	N-3

Let see $\$3=3$ first (left side of table 16), we can know there is a bigger chance for $(N-\$3)$ to be a prime with 3 as its last digit than $(N-O3)$. If $N-3$ can be divisible by 3, then $(N-3)+3 (=N)$ [$3a + 3 = 3(a+1)$, $3a$ and $3(a+1)$ must be divisible by 3] will be divisible by 3, but N with 6 as its last digit and only 6, 36, 66, 96, 126, 156, 186, 216, 246,... are divisible by 3, but we worked on only N with 6 as their last digit and $N/2$ with 8 as their last digit, only 36, 96, 156, 216,...,(1 in 6) can be divisible by 3, so the term $(1/3 \times 5/6)$ should be taken off from Formula 3.

For the next prime $\$3=13$, $(N-13)$ cannot be divisible by 13 except 156, 416, 676,..., so $(1/13 \times 25/26)$ should be taken off from Formula 3, so on.

Let n_3 represents the total number of primes ($\$3$) with 3 as their last digit in any number N , the chance of every $N-\$3$ with 3 as its last digit to be a prime is: $\Delta_3=1-\sum_3 = 1- \{[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + \dots] - [(1/3 \times 5/6) + (1/13 \times 25/26) + (1/23 \times 45/46) + \dots]\}$

When the number of $\$3$ is 3 or more, $3 \times [(1/3 \times 5/6) + (1/13 \times 25/26) + (1/23 \times 45/46) \dots] > 1$, so every 3 $\$3$ will have at least 1 prime of $N-\$3$ to form 1 pair of primes in which both have 3 as its last digit and their sum is any number N in which N has 6 as its last digit and $N/2$ has 8 as its last digit.

5a. When any even integer (N) has 8 as its last digit, such as 28, 48, 68, 88, 108, 1128,..., then N/2 has 4 as its last digit:

Table 17. The odd number pairs in $N=O_1+O_2=(N/2+L-1)+(N/2-L+5)$ and $N=O_1+O_2=(N/2-L-1)+(N/2+L+5)$

N-9	N-19	N-29	N-59	N-79	N-89	...	N/2+L-5	N/2-L+5	...	89	79	59	29	19	9(3x3)
9(3x3)	19	29	59	79	89	...	N/2-L+5	N/2+L-5	...	N-89	N-79	N-59	N-29	N-19	N-9

Let see \$9=9 first (left side of table 17), if N-9 can be divisible by 9, then (N-9)+9 (=N) [9a + 9 = 9(a+1), 9a and 9(a+1) must be divisible by 3 or 9] will be divisible by 3 or 9, N with 8 as its last digit and only 18, 48, 78, 108, 138, 168, 198, 228, 258, 288, 318,... are divisible by 3 or 9,..., but we worked on only N with 8 as their last digit and N/2 with 4 as their last digit, only 48, 108, 168, 228, 288,...(1 in 6) can be divisible by 3 or 9, so the term (1/3x5/6) should be taken off from Formula 3.

For the next prime \$9=19, (N-19) cannot be divisible by 19 except 228, 608, 988...., so (1/19x37/38) should be taken off from Formula 3, so on.

Let n_9 represents the total number of primes (\$9) with 9 as their last digit in any number N, the chance of every N-\$9 with 9 as its last digit to be a prime is: $\Delta_9=1-\sum_9=1- \{[(1/3) + (1/7x2/3)+ (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17) + (1/31x2/3x6/7x10/11x12/13x16/17x22/23) + (1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43) + \dots] - [(1/3x5/6)] + (1/19x37/38) + (1/29x57/58) \dots \}$

When the number of \$9 is 4 or more, $4 \times [(1/3x5/6)] + (1/19x37/38) + (1/29x57/58) \dots > 1$, so every 4 \$9 will have at least 1 prime of N-\$9 to form 1 pair of primes in which both have 9 as its last digit and their sum is any number N in which N has 8 as its last digit and N/2 have 4 as its last digit.

Table 18. The odd number pairs in $N=O_1+O_2=(N/2+L+3)+(N/2-L-3)$ and $N=O_1+O_2=(N/2+L-3)+(N/2-L+3)$

N-11	N-31	N-41	N-61	N-71	N-101	...	N/2+L+3	N/2-L+3	...	97	67	47	37	17	7
11	31	41	61	71	101	...	N/2-L-3	N/2+L-3	...	N-97	N-67	N-47	N-37	N-17	N-7

Let see $\$1=11$ first (left side of table 18), we can know there is a bigger chance for $(N-\$1)$ to be a prime with 1 as its last digit than $(N-O1)$. If $N-11$ can be divisible by 11, then $(N-11)+11 (=N)$ [$11a + 11 = 11(a+1)$, $11a$ and $11(a+1)$ must be divisible by 11] will be divisible by 11, N with 8 as its last digit and only 88, 198, 308, 418, 528, 638, 748, 858, 968,... are divisible by 11,..., but we worked on only N with 8 as their last digit and $N/2$ with 4 as its last digit, only 88, 308, 528, 748, 968,... (1 in 22) can be divisible by 11, so the term $(1/11 \times 21/22)$ should be taken off from Formula 3.

For the next prime $\$1=31$, $(N-31)$ cannot be divisible by 31 except 248, 868,..., so $(1/31 \times 61/62)$ should be taken off from Formula 6, so on.

Let n_1 represents the total number of primes ($\$1$) with 1 as their last digit in any number N , the chance of every $N-\$1$ with 7 as its last digit to be a prime is: $\Delta_7=1-\sum_7=1- \{[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + \dots,] - [(1/11 \times 21/22) + (1/31 \times 61/62) + (1/41 \times 81/82) \dots]\}$

When the number of $\$1$ is 11 or more, $[(1/11 \times 21/22) + (1/31 \times 61/62) + (1/41 \times 81/82) \dots] > 1$, so every 11 $\$1$ will have at least 1 prime of $N-\$1$ to form 1 pair of primes in which one prime has 1 as its last digit and another have 7 as its last digit and their sum is any number N in which N has 8 as its last digit and $N/2$ has 4 as its last digit.

5b. When any even integer (N) has 8 as its last digit, such as 18, 38, 58, 78, 98, 1118, ..., then N/2 has 9 as its last digit:

Table 19. The odd number pairs in $N=O_1+O_2=(N/2+L-0)+(N/2-L+0)$ and $N=O_1+O_2=(N/2-L-0)+(N/2+L+0)$

N-9	N-19	N-29	N-59	N-79	N-89	...	N/2+L-0	N/2-L+0	...	89	79	59	29	19	9(3x3)
9(3x3)	19	29	59	79	89	...	N/2-L+0	N/2+L-0	...	N-89	N-79	N-59	N-29	N-19	N-9

Let see \$9=9 first (left side of table 19), if N-9 can be divisible by 9, then (N-9)+9 (=N) [9a + 9 = 9(a+1), 9a and 9(a+1) must be divisible by 3 or 9] will be divisible by 3 or 9, N with 8 as its last digit and only 18, 48, 78, 108, 138, 168, 198, 228, 258, 288, 318, ... are divisible by 3 or 9, ..., but we worked on only N with 8 as their last digit and N/2 with 9 as their last digit, only 18, 78, 138, 198, 258, 318, ..., (1 in 6) can be divisible by 3 or 9, so the term (1/3x5/6) should be taken off from Formula 3.

For the next prime \$9=19, (N-19) cannot be divisible by 19 except 38, 418, 798..., so (1/19x37/38) should be taken off from Formula 3, so on.

Let n_9 represents the total number of primes (\$9) with 9 as their last digit in any number N, the chance of every N-\$9 with 9 as its last digit to be a prime is: $\Delta_9=1-\sum_9=1- \{[(1/3) + (1/7x2/3)+ (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17) + (1/31x2/3x6/7x10/11x12/13x16/17x22/23) + (1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43) + \dots] - [(1/3x5/6)] + (1/19x37/38) + (1/29x57/58) \dots \}$

When the number of \$9 is 4 or more, $4 \times [(1/3x5/6)] + (1/19x37/38) + (1/29x57/58) \dots > 1$, so every 4 \$9 will have at least 1 prime of N-\$9 to form 1 pair of primes in which both have 9 as its last digit and their sum is any number N in which N has 8 as its last digit and N/2 have 9 as its last digit.

Table 20. The odd number pairs in $N=O_1+O_2=(N/2+L-2)+(N/2-L+2)$ and $N=O_1+O_2=(N/2+L+2)+(N/2-L-2)$

N-11	N-31	N-41	N-61	N-71	N-101	...	N/2+L-2	N/2-L-2	...	97	67	47	37	17	7
11	31	41	61	71	101	...	N/2-L+2	N/2+L+2	...	N-97	N-67	N-47	N-37	N-17	N-7

Let see $\$1=11$ first (left side of table 20), we can know there is a bigger chance for $(N-\$1)$ to be a prime with 1 as its last digit than $(N-O1)$. If $N-11$ can be divisible by 11, then $(N-11)+11 (=N)$ [$11a + 11 = 11(a+1)$, $11a$ and $11(a+1)$ must be divisible by 11] will be divisible by 11, N with 8 as its last digit and only 88, 198, 308, 418, 528, 638, 748, 858, 968,... are divisible by 11,..., but we worked on only N with 8 as their last digit and $N/2$ with 9 as its last digit, only 198, 418, 638, 858,... (1 in 22) can be divisible by 11, so the term $(1/11 \times 21/22)$ should be taken off from Formula 3.

For the next prime $\$1=31$, $(N-31)$ cannot be divisible by 31 except 558, 1178...., so $(1/31 \times 61/62)$ should be taken off from Formula 6, so on.

Let n_1 represents the total number of primes ($\$1$) with 1 as their last digit in any number N , the chance of every $N-\$1$ with 7 as its last digit to be a prime is: $\Delta_7=1-\sum_7=1-\{[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + \dots,] - [(1/11 \times 21/22) + (1/31 \times 61/62) + (1/41 \times 81/82) \dots]\}$

When the number of $\$1$ is 11 or more, $[(1/11 \times 21/22) + (1/31 \times 61/62) + (1/41 \times 81/82) \dots] > 1$, so every 11 $\$1$ will have at least 1 prime of $N-\$1$ to form 1 pair of primes in which one prime has 1 as its last digit and another have 7 as its last digit and their sum is any number N in which N has 8 as its last digit and $N/2$ has 9 as its last digit.

For any even number, Goldbach's conjecture is true.

References:

1. Dudley, Underwood (1978), Elementary number theory(2nd ed.), W. H. Freeman and Co., Section 2, Theorem 2.
2. Dudley, Underwood (1978), Elementary number theory(2nd ed.), W. H. Freeman and Co., Section 2, Lemma 5.
3. Dudley, Underwood (1978), Elementary number theory(2nd ed.), W. H. Freeman and Co., p. 10, section 2.
4. Zhang, Yitang (2014). "Bounded gaps between primes".Annals of Mathematics. 179 (3): 1121–1174