AN INTRODUCTION TO THE n-IRREDUCIBLE SEQUENTS AND THE n-IRREDUCIBLE NUMBER

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Abstract. In this work, we introduce the n-irreducible sequents and the n-irreducible numbers defined with the help of the second order logic. We give many concrete examples of n-irreducible numbers and n-irreducible sequents with the Peano's axioms and the axioms of the real numbers. Shortly, a sequent is n-irreducible iff the sequent is composed by some closed hypotheses and a n-irreducible formula (a close formula with one internal variable such that the formula is only true when we set that variable to the unique natural number n), and it does not exist some strict sub-sequent which are composed by some closed sub-hypotheses and some sub-m-irreducible formula with $m > 1$. The definition is motivated by the intuition that the "Nature's hypotheses"do not carry natural numbers or "hidden natural numbers" except for the numbers 0 and 1, i.e., they can be used in a n-irreducible sequent. Moreover, we postulate at second order of logic that the "Nature's hypotheses"are not chosen randomly: the "Nature's hypotheses"has the propriety to give the largest n-irreducible number $N_Z \sim 2^{2.205\times10^{61}}$ among a finite number of n-irreducible sequents. The Collatz conjecture, the Goldbach's conjecture, the Polignac's conjecture, the Firoozbakht's conjecture, the Oppermann's conjecture, the Agoh-Giuga conjecture, the generalized Fermat's conjecture and the Schinzel's hypothesis H are reviewed with this new (second order logic) n-irreducible axiom. Finally, two open questions remain: Can we prove that a natural number is not n-irreducible? If a n-irreducible number n is found with a function symbol f where its outputs values are only 0 and 1, can we always replace the function symbol f by a another function symbol $\tilde{f}$ such that $\tilde{f} = 1 - f$ and the new sequent is still n-irreducible?

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Contents

1. Introduction

The present article is motivated by the consequences (in number theory and in fundamental physics) of the definition of a n-irreducible number, the definition of a n-irreducible sequent, the (second order logic) n-irreducible axiom which states the existence of the largest n-irreducible number N_Z among a finite number of n-irreducible sequent and the (second order logic) n-irreducible hypothesis on the “Nature's hypotheses”(the required hypotheses to explain the physical measurements) which states that the “Nature's hypotheses”are the hypotheses of a N_Z-irreducible sequent. The first set of consequences are: the Collatz conjecture, the Goldbach's conjecture, the Polignac's conjecture, the Firoozbakht's conjecture, the Oppermann's conjecture, the Agoh-Giuga conjecture, the generalized Fermat's conjecture (which requires computational resources which are not reached today even for checking the simple case: $a = 2$ and $b = 1$) and the Schinzel's hypothesis H (which requires computational resources impossible to reach: $\left(\pi\left(N_Z\right)2^{N_Z}\right)^{N_Z}$ cases) are solved by the (second order logic) n-irreducible axiom. To prove the previous conjectures, it requires almost unaccessible computational resources for checking about $N_Z \sim! \left(3 \times 4 \times (2N)^{N^2}\right) \sim 2^{2.205\times10^{61}}$ cases with $N = 7$ since the number of atoms in the visible universe is 2×10^{79}. A universe-sized quantum computer would perform ”only” $2^{2\times10^{79}}$ operations. The second set of consequences are the “Nature's hypotheses”generated by a N_Z-irreducible hypothesis on the “Nature's hypotheses”.

From researches in fundamental physics, the n-irreducible numbers and the n-irreducible sequents definitions arise from the intuition that the “Nature's hypotheses”do not carry natural numbers or ”hidden natural numbers” except for 0 and 1, i.e. the “Nature's hypotheses”can be used in a n-irreducible sequent. Shortly, a sequent is n-irreducible iff the sequent is composed by some closed hypotheses and a n-irreducible formula (a close formula with one internal variable such that the formula is only true when we set that variable to the unique natural number n), and it does not exist some strict sub-sequent which are composed by some closed sub-hypotheses and some sub-m-irreducible formula with $m > 1$. Moreover, we postulate (at second order of logic) that the “Nature's hypotheses”are not chosen randomly: the “Nature's hypotheses”are the hypotheses which give the largest n-irreducible number $N_Z \sim 2^{2.205\times10^{61}}$ among a finite number of N_Z-irreducible sequents.

The article is organized as follow: firstly, we present the notations used throughout this article. Secondly, we define what is an explicit sub-formula in order to define what is a n-irreducible number and a n-irreducible sequent. Thirdly, we present some n-irreducible number examples. Fourthly, we present the Collatz conjecture, the Goldbach's conjecture, the Polignac's conjecture, the Firoozbakht's conjecture, the Oppermann's conjecture, the

Agoh-Giuga conjecture, the generalized Fermat's conjecture and the Schinzel's hypothesis H as n-irreducible sequents. Fifthly, we present the (second order logic) n-irreducible axiom, the (second order logic) hypothesis on the "Nature's hypotheses"and their consequences. Sixthly, we ask ourselves two open questions about the n-irreducible numbers. Seventhly, we present some larger n-irreducible number examples. Eighthly, we present some n-irreducible number examples with the axioms of the real numbers and ninethly, the conclusion and the acknowledgment.

2. Notations

In the present article:

1- In general, the variable $x_n \equiv \underbrace{||...|}_{n \text{ times}} [\,] \underbrace{||...|}_{n \text{ times}}$ with the unique variable label number $0 \leq n$, can be replaced in explicit sub-formulas by $x_{n'} \equiv \underbrace{||...|}_{n' \text{ times}} [\,] \underbrace{||...|}_{n' \text{ times}}$ with the unique variable label number $0 \leq n' \leq n$.

In general, the function notation $f_n(x_{n_0}, ..., x_{n_m})$ should be read:

$$f_n(t_1, t_2, ..., t_n) \equiv \underbrace{||...|}_{n \text{ times}} (t_1, t_2, ..., t_n) \underbrace{||...|}_{n \text{ times}} \tag{2.1}$$

where $0 \leq n$ is a unique function label number. The previous function notion which can be replaced in explicit sub-formulas by the following one:

$$f_{n'}(t'_1, t'_2, ..., t'_n) \equiv \underbrace{||...|}_{n' \text{ times}} (t'_1, t'_2, ..., t'_n) \underbrace{||...|}_{n' \text{ times}} \tag{2.2}$$

In general, the relation notation $\mathcal{R}_n(t_1, t_2, ..., t_n)$ should be read:

$$\mathcal{R}_n(t_1, t_2, ..., t_n) \equiv \underbrace{||...|}_{n \text{ times}} (t_1, t_2, ..., t_n) \underbrace{||...|}_{n \text{ times}} \tag{2.3}$$

where $0 \leq n$ is a unique relation label number. The previous relation notion which can be replaced in explicit sub-formulas by the following one:

$$\mathcal{R}_{n'}(t'_1, t'_2, ..., t'_n) \equiv \underbrace{||...|}_{n' \text{ times}} (t'_1, t'_2, ..., t'_n) \underbrace{||...|}_{n' \text{ times}} \tag{2.4}$$

The notation do not need to distinguish between function and relation since the functions can not be some arguments of the boolean operator and the relations are always some arguments of the boolean operators.

In general, the logic operators of the truth table should be read:

$$\begin{aligned} & B_{00} \wedge \neg A \wedge \neg B \vee B_{01} \wedge \neg A \wedge B \vee \\ & B_{10} \wedge A \wedge \neg B \vee B_{11} \wedge A \wedge B \\ & \equiv (A, B)^{B_{00} B_{01} B_{10} B_{11}} \end{aligned} \tag{2.5}$$

The logic operator $(A,B)^{B_{00}B_{01}B_{10}B_{11}}$ can be replaced by A or B in explicit sub-formulas. The 8 logic operators:

$$\begin{aligned}
(A,B)^{FFFF} &\equiv F \equiv_F (A,B)_F \\
(A,B)^{TTTT} &\equiv T \equiv_T (A,B)_F \\
(A,B)^{FFTT} &\equiv A \equiv_A (A,B)_A \\
(A,B)^{FTFT} &\equiv B \equiv_B (A,B)_B \\
(A,B)^{TTFF} &\equiv \neg A \equiv_{\bar{A}} (A,B)_{\bar{A}} \\
(A,B)^{TFTF} &\equiv \neg B \equiv_{\bar{B}} (A,B)_{\bar{B}} \\
(A,B)^{FTFF} &\equiv \neg A \wedge B \equiv A \nleftarrow B \equiv_{\nleftarrow} (A,B)_{\nleftarrow} \\
(A,B)^{TFTT} &\equiv A \vee \neg B \equiv A \leftarrow B \equiv_{\leftarrow} (A,B)_{\leftarrow}
\end{aligned} \tag{2.6}$$

are not necessary since we can write every formulas with the other 8 logic operators:

$$\begin{aligned}
(A,B)^{TFFF} &\equiv \neg(A \vee B) \equiv_{\bar{\vee}} (A,B)_{\bar{\vee}} \\
(A,B)^{FFTF} &\equiv A \rightarrow B \equiv_{\rightarrow} (A,B)_{\rightarrow} \\
(A,B)^{FFFT} &\equiv \neg(A \wedge B) \equiv_{\bar{\wedge}} (A,B)_{\bar{\wedge}} \\
(A,B)^{FTTF} &\equiv A \veebar B \equiv_{\veebar} (A,B)_{\veebar} \\
(A,B)^{TFFT} &\equiv \neg(A \veebar B) \equiv_{\bar{\veebar}} (A,B)_{\bar{\veebar}} \\
(A,B)^{FFFT} &\equiv A \wedge B \equiv_{\wedge} (A,B)_{\wedge} \\
(A,B)^{TTFT} &\equiv A \nrightarrow B \equiv_{\nrightarrow} (A,B)_{\nrightarrow} \\
(A,B)^{TTTF} &\equiv A \vee B \equiv_{\vee} (A,B)_{\vee}
\end{aligned} \tag{2.7}$$

In general, the logic quantifier notation should be read:

$$\begin{aligned}
\exists x A &\equiv_{\exists} (x,A)_{\exists} \\
\nexists x A &\equiv_{\nexists} (x,A)_{\nexists} \\
\forall x A &\equiv_{\forall} (x,A)_{\forall} \\
\not\forall x A &\equiv_{\not\forall} (x,A)_{\not\forall}
\end{aligned} \tag{2.8}$$

We count the number of symbols with minimal symmetric notation:

- 2 symbols for the variables
- 2 symbols for the functions
- 32 symbols for the boolean operators
- 8 symbols for the quantifications
- 1 symbol $|$ for enumeration
- 1 symbol $\vdash$ for sequent

We have a total of 46 symbols with this minimal symmetric notation.

2- We omit some parentheses to improve the readability but they are necessary for writing the related explicit formulas and explicit sub-formulas.

3- If the formula ϕ is previously defined, the formula $\phi_{[y/x]}$ is a shortcut for the formula ϕ written with the variable y instead of the variable x with respect to the explicit sub-formulas of $\phi_{[y/x]}$.

3. Definitions

Let consider a language $\mathcal{L}$ of first order logic which contains the language needed for the Peano hypotheses (except the recursion hypothesis).

Let introduce the necessary preliminary definitions and lemmas:

1- Preliminary definitions and lemmas about the explicit sub-formulas of a formula ϕ:

a- A formula ϕ containing l pair of parentheses is an explicit formula iff the i^{th} opening parenthesis and the corresponding i^{th} closing parenthesis are labeled unambiguously with respect to the other parentheses with an injection f_{inj} : $\{1, ..., l\} \subset \mathbb{N} \longrightarrow \mathbb{N}$ such that: $... \underset{f_{inj}(i)}{\Big(} \; .... \; \underset{f_{inj}(i)}{\Big)} \; ... ..$ Moreover the function

b- Preliminary lemma:

Every formula ϕ can be written as an explicit formula.

c- An explicit formula ψ is an explicit sub-formula of a formula ϕ iff the formula ψ is an explicit formula and ψ is a sub-sequence of the symbol sequence of the formula ϕ written as an explicit formula.

Remark: an explicit sub-formula ψ of a formula ϕ may contain a function symbol f of arity strictly smaller than the function symbol f in the formula ϕ. Roughly speaking, an explicit sub-formula can be written by removing the same number of argument for each function symbol f of the original formula.

d- Preliminary lemma about the explicit sub-formulas of a formula ϕ:

An explicit sub-formula of an explicit sub-formula of a formula ϕ is an explicit sub-formula of the formula ϕ.

2- Preliminary definition about the n-irreducible formulas:

A formula $\phi_{n-irreducible}$ is a n-irreducible formula iff $\phi_{n-irreducible}$ is a closed formula and a formula ϕ exists such that:

$$\phi_{n-irreducible} \equiv \phi_{[\underbrace{f_s(...f_s(}_{n \text{ times}} c_0 \underbrace{)...)}_{n \text{ times}} /x]} \wedge \exists ! y \left(\phi_{[y/x]}\right) \ . \tag{3.1}$$

We rewrite the previous equation without the shortcut symbol $\exists!$:

$$\phi_{n-irreducible} \equiv \phi_{[\underbrace{f_s(...f_s(}_{n \text{ times}} c_0 \underbrace{)...)}_{n \text{ times}} /x]} \wedge \neg \exists y \exists y' \left(\neg y = y' \wedge \phi_{[y/x]} \wedge \phi_{[y'/x]}\right) \ . \tag{3.2}$$

The main definition of the present article, the following sequent:

$$\Gamma \vdash \phi_{n-irreducible} \ , \tag{3.3}$$

where $\vdash$ means "it exists a model such that the n-irreducible formula $\phi_{n-irreducible}$ is verified under the hypotheses Γ", is a n-irreducible sequent and n is a n-irreducible number iff:

1- the hypotheses Γ are closed formulas and the formula $\phi_{n-irreducible}$ is a n-irreducible formula (see the equation 3.1),

2- and for every closed and explicit sub-formula Δ of the hypotheses Γ and for every m-irreducible formula $\psi_{m-irreducible}$ where ψ is an explicit sub-formula of the formula ϕ, we have the following relation:

$$\Delta \vdash \psi_{m-irreducible} \text{ and } m = c_0 \tag{3.4}$$

or

$$\Delta \vdash \psi_{m-irreducible} \text{ and } m = f_s\left(c_0\right) \tag{3.5}$$

or

$$\Delta \equiv \Gamma \text{ and } \psi_{m-irreducible} \equiv \phi_{n-irreducible} \tag{3.6}$$

or it exists a model such that,

$$\Delta \nvDash \psi_{m-irreducible} \tag{3.7}$$

Important remark: for all n-irreducible numbers in the present article, we include the Peano hypotheses inside the n-irreducible hypotheses Γ or either the following Peano sub-hypotheses:

$$\begin{aligned} &\forall x \exists y \left(x = f_s\left(y\right)\right) \\ &\forall x_1 ... \forall x_n \left(\left(\phi_{[c_0/x_0, x_1, ..., x_n]} \wedge \forall x_0 \left(\phi_{[x_0, x_1, ..., x_n]} \rightarrow \phi_{[f_s(x_0), x_1, ..., x_n]}\right)\right) \rightarrow \left(\forall x_0 \phi_{[x, x_1, ..., x_n]}\right)\right) \end{aligned} \tag{3.8}$$

with $\phi = \tilde{\phi} \vee \neg x_0 < 0$, from the following Peano hypotheses:

$$\begin{aligned} &\forall x \left(\neg f_s\left(x\right) = c_0\right) \\ &\forall x \exists y \left(\neg x = c_0 \rightarrow x = f_s\left(y\right)\right) \\ &\forall x_1 ... \forall x_n \left(\left(\phi_{[c_0/x_0, x_1, ... x_n]} \wedge \forall x_0 \left(\phi_{[x_0, x, ..., x_n]} \rightarrow \phi_{[f_s(x_0), x_1, ..., x_n]}\right)\right) \rightarrow \left(\forall x_0 \phi_{[x_0, x_1, ..., x_n]}\right)\right) . \end{aligned} \tag{3.9}$$

If we use the Peano sub-hypotheses above, we use the integers rather than the natural numbers. The use of Peano hypotheses in a n-irreducible sequent with respect to the Peano sub-hypotheses should be motivated by other definitions like the prime number function definition.

4. Some n-irreducible number examples

We give in this section some examples of n-irreducible numbers. Firstly, we write the preliminary formulas satisfied by the following function symbols:

0- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the ordering function symbol $\mathcal{R}_<$:

$$\begin{aligned} &\forall x \forall y \left(\exists z \left(x = f_+\left(y, z\right) \wedge \neg z = c_0\right) \rightarrow \mathcal{R}_<\left(x, y\right) = True\right) \\ &\forall x \forall y \left(\neg \exists z \left(x = f_+\left(y, z\right) \wedge \neg z = c_0\right) \rightarrow \mathcal{R}_<\left(x, y\right) = False\right) \end{aligned} \tag{4.1}$$

1- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the prime function symbol f_{Prime}:

$$\begin{aligned}&\forall x\left(\exists y\exists z\left(y<x\wedge z<x\wedge x=f_{\times}\left(y,z\right)\right)\rightarrow f_{Prime}\left(x\right)=c_{0}\right)\\&\forall x\left(\neg\exists y\exists z\left(y<x\wedge z<x\wedge x=f_{\times}\left(y,z\right)\right)\rightarrow f_{Prime}\left(x\right)=f_{s}\left(c_{0}\right)\right)\ .\end{aligned}\tag{4.2}$$

Important remark: If we use the prime function f_{prime} in a n-irreducible sequent, we should use the Peano hypotheses (natural numbers) rather than the Peano sub-hypotheses (integers). Using the above prime function f_{prime} with the Peano sub-hypotheses implies that all strictly positive integers are prime numbers and all negative integers are not prime numbers.

Trivially, 0 and 1 are n-irreducible numbers with the following formulas ϕ:

$$\phi\equiv x=c_{0}\text{ and }\phi\equiv x=f_{s}\left(c_{0}\right)\ .\tag{4.3}$$

2 is a n-irreducible number with the following formula ϕ:

$$\phi\equiv x=f_{+}\left(f_{s}\left(c_{0}\right),f_{s}\left(c_{0}\right)\right)\tag{4.4}$$

or for instance, the following formula ϕ:

$$\phi\equiv\forall y\left(y<x\rightarrow\left(y=c_{0}\vee y=f_{s}\left(c_{0}\right)\right)\right)\wedge\neg\exists x'\left(x<x'\wedge\forall y\left(y<x'\rightarrow\left(y=c_{0}\vee y=f_{s}\left(c_{0}\right)\right)\right)\right)\ .\tag{4.5}$$

If we would like to include the Peano hypotheses (except the recursion hypothesis) for the n-irreducible number 2, we should look at the following formula ϕ which requires the multiplication hypothesis:

$$\begin{aligned}\phi=&\exists y\exists z\left(x=f_{\times}\left(y,z\right)\wedge\left(f_{s}(c_{0})<y\vee f_{s}(c_{0})<z\right)\right)\wedge\\&\neg\exists x'\left(x<x'\wedge\exists y\exists z\left(x'=f_{\times}\left(y,z\right)\wedge\left(f_{s}(c_{0})<y\vee f_{s}(c_{0})<z\right)\right)\right)\ .\end{aligned}\tag{4.6}$$

In order to prove that some other natural numbers are n-irreducible, we use the prime function f_{Prime} (see 4.2).

3 is a n-irreducible number with the following formula ϕ (see 4.2):

$$\phi\equiv\forall y\left(f_{Prime}\left(y\right)=c_{0}\rightarrow x<y\right)\wedge\neg\exists x'\left(x<x'\wedge\forall y\left(f_{Prime}\left(y\right)=c_{0}\rightarrow x'<y\right)\right)\ .\tag{4.7}$$

4 is a n-irreducible number with the following formula ϕ (see 4.2):

$$\phi\equiv f_{Prime}\left(x\right)=c_{0}\wedge\neg\exists x'\left(x'<x\wedge f_{Prime}\left(x'\right)=c_{0}\right)\ .\tag{4.8}$$

6 is a n-irreducible number with the following formula ϕ:

$$\phi=\exists y\exists z\left(x=f_{\times}\left(y,z\right)\wedge y<z\wedge z<x\right)\wedge\neg\exists x'\exists y\exists z\left(x'<x\wedge x'=f_{\times}\left(y,z\right)\wedge y<z\wedge z<x'\right)\tag{4.9}$$

5. Conjectures which induce monster n-irreducible numbers if counterexamples exist

In the previous section, we introduced some n-irreducible numbers that are small and easy to find. In this section, we examine how some monster n-irreducible numbers can be produced if some conjectures are false. Firstly, we write the preliminary formulas satisfied by the following function symbols:

1- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the unary inverse function symbol f_s^{-1}:

$$\begin{aligned}&\forall x\left(\neg x=c_0\rightarrow f_s\left(f_s^{-1}\left(x\right)\right)=x\right)\\&\forall x\left(x=c_0\rightarrow f_s^{-1}\left(x\right)=c_0\right)\ .\end{aligned}\tag{5.1}$$

2- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the subtraction function symbol f_-:

$$\begin{aligned}&\forall x\forall y\left(y<x\rightarrow f_+\left(f_-\left(x,y\right),y\right)=x\right)\\&\forall x\forall y\left(\neg y<x\rightarrow f_-\left(x,y\right)=c_0\right)\ .\end{aligned}\tag{5.2}$$

3- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the twin prime function symbol f_{Twin} (see 4.2 and 5.1):

$$\begin{aligned}&\forall x\left(\left(f_{Prime}\left(f_s^{-1}\left(x\right)\right)=f_s\left(c_0\right)\wedge f_{Prime}\left(f_s\left(x\right)\right)=f_s\left(c_0\right)\right)\rightarrow f_{Twin}\left(x\right)=f_s\left(c_0\right)\right)\\&\forall x\left(\neg\left(f_{Prime}\left(f_s^{-1}\left(x\right)\right)=f_s\left(c_0\right)\wedge f_{Prime}\left(f_s\left(x\right)\right)=f_s\left(c_0\right)\right)\rightarrow f_{Twin}\left(x\right)=c_0\right)\ .\end{aligned}\tag{5.3}$$

4- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the ceiling prime function symbol f_{CP} (see 4.2):

$$\forall x\exists y\left(f_{CP}\left(x\right)=y\wedge f_{Prime}\left(y\right)=f_s\left(c_0\right)\wedge\neg\exists z\left(x<z\wedge z<y\wedge f_{Prime}\left(z\right)=f_s\left(c_0\right)\right)\right)\ .\tag{5.4}$$

5- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the function symbol $f_{p-Prime}$ which give the n^{th} prime number (see 4.2)

$$\begin{aligned}&f_{p-Prime}\left(f_s\left(c_0\right)\right)=c_0\\&\forall x\left(f_{p-Prime}\left(f_s\left(x\right)\right)=f_{CP}\left(f_{p-Prime}\left(x\right)\right)\right)\ .\end{aligned}\tag{5.5}$$

6- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the coprime function symbol $f_{Coprime}$:

$$\begin{aligned}&\forall x\forall x'\left(\exists y\exists z\exists z'\left(\neg y=f_s\left(c_0\right)\wedge x=f_\times\left(y,z\right)\wedge x'=f_\times\left(y,z'\right)\right)\rightarrow f_{Coprime}\left(x,x'\right)=c_0\right)\\&\forall x\forall x'\left(\neg\exists y\exists z\exists z'\left(\neg y=f_s\left(c_0\right)\wedge x=f_\times\left(y,z\right)\wedge x'=f_\times\left(y,z'\right)\right)\rightarrow f_{Coprime}\left(x,x'\right)=f_s\left(c_0\right)\right)\ .\end{aligned}\tag{5.6}$$

7- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the power function symbol $f_\wedge$:

$$\begin{aligned}&\forall x\forall y\left(y=c_0\rightarrow f_\wedge\left(x,y\right)=f_s\left(c_0\right)\right)\\&\forall x\forall y\left(\neg y=c_0\rightarrow f_\wedge\left(x,f_s\left(y\right)\right)=f_\times\left(f_\wedge\left(x,y\right),x\right)\right)\ .\end{aligned}\tag{5.7}$$

5.1. *Collatz conjecture.* The only conjecture where we use the Zermelo-Fraenkel axioms:

$$\forall n \left(\exists k \left(n = f_{\times}\left(2, k\right)\right) \rightarrow f_{\times}\left(2, f\left(n\right)\right)\right) = n$$
$$\forall n \left(\exists k \left(n = f_{+}\left(f_{\times}\left(2, k\right) + 1\right)\right) \rightarrow f\left(n\right) = f_{+}\left(f_{\times}(3, n), 1\right)\right)$$

$$\forall x \left(x \in g\left(1\right) \leftrightarrow x = 1\right)$$
$$\forall x \left(x \in g\left(f_{+}\left(n, 1\right)\right) \leftrightarrow \left(x \in g\left(n\right) \vee f\left(x\right) = n\right)\right)$$

$$\forall n \left(\neg f_{+}\left(h\left(n\right), 1\right) \in g\left(n\right)\right)$$
$$\forall n \forall n' \left(\neg h\left(n\right) < n' \rightarrow n' \in g\left(n\right)\right)$$

$$\exists n \left(h\left(x\right) = n\right) \wedge \nexists n \left(h\left(x\right) < h\left(n\right)\right) \tag{5.8}$$

5.2. *Goldbach's conjecture.* If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the sub-function symbol $g_{Goldbach-1}$ and the function symbol $f_{Goldbach-1}$ which gives the minimal number of prime numbers necessary to express a natural number as a sum of prime number minus one (see 4.2 and 5.2):

$$\begin{aligned}
&\forall x \forall y \left(f_{Prime}\left(y\right) = f_s\left(c_0\right) \rightarrow g_{Goldbach-1}(x, y) = c_0\right) \\
&\forall x \forall y \Bigg(f_{Prime}\left(y\right) = c_0 \rightarrow \exists z \\
&\left(z < y \wedge f_{Prime}\left(z\right) = f_s\left(c_0\right) \wedge g_{Goldbach-1}(x, y) = f_s\left(g_{Goldbach-1}(x, f_{-}\left(y, z\right))\right) \wedge \neg \exists z' \right.\\
&\left.\left(z' < y \wedge f_{Prime}\left(z'\right) = f_s\left(c_0\right) \wedge g_{Goldbach-1}(x, f_{-}\left(y, z'\right)) < g_{Goldbach-1}(x, f_{-}\left(y, z\right))\right)\right)\Bigg) \\
&\forall x \left(f_{Goldbach-1}(x) = g_{Goldbach-1}(x, x)\right) \ .
\end{aligned} \tag{5.9}$$

If a first counterexample m_Z exists for the Goldbach's conjecture [Hel13], we can show that m_Z is a n-irreducible number with the following formula ϕ (see the previous equation):

$$\begin{aligned}
\phi \equiv &\neg x = c_0 \wedge \neg x = f_s\left(c_0\right) \wedge \forall y \left(y < x \rightarrow f_{Goldbach-1}(y) < f_{Goldbach-1}(x)\right) \wedge \\
&x < y \rightarrow f_{Goldbach-1}(x) = f_{Goldbach-1}(y) \ .
\end{aligned} \tag{5.10}$$

5.3. *Polignac's conjecture.* If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the Polignac function symbol $f_{Polignac}$ which gives the difference between the two next prime numbers of a natural number (see 5.2 and 5.4):

$$\forall x \left(f_{Polignac}\left(x\right) = f_{-}\left(f_{CP}\left(f_{CP}\left(x\right)\right), f_{CP}\left(x\right)\right)\right) \ . \tag{5.11}$$

If a first counterexample m_Z exists for the Polignac's conjecture [dP51], we can show that m_Z is a n-irreducible number with following formula ϕ (see the previous equation):

$$\begin{aligned}
\phi \equiv &\neg x = c_0 \wedge \neg x = f_s\left(c_0\right) \wedge \\
&\exists y \left(f_{Polignac}\left(x\right) = y \wedge \neg \exists z \left(x < z \wedge f_{Polignac}\left(x\right) = f_{Polignac}\left(z\right)\right)\right) \wedge \\
&\neg \exists x' \left(x' < x \wedge f_{Polignac}\left(x'\right) = y \wedge \neg \exists z \left(x' < z \wedge f_{Polignac}\left(x'\right) = f_{Polignac}\left(z\right)\right)\right) \ .
\end{aligned} \tag{5.12}$$

Since the set of prime numbers is infinite, the following explicit sub-formula will not work (see 5.2 and 5.4):

$$\forall x \left(f_{Polignac}(x) = f_{-}\left(f_{CP}(x), x\right)\right) . \tag{5.13}$$

5.4. *Firoozbakht's conjecture.* If a first counterexample m_Z exists for the Firoozbakht's conjecture [20004], we can show that m_Z is a n-irreducible number with the following formula ϕ (see 5.5):

$$\begin{aligned} \phi \equiv & \neg x = c_0 \wedge \neg f_{\wedge}\left(f_{p-Prime}\left(f_s(x)\right), x\right) < f_{\wedge}\left(f_{p-Prime}(x), f_s(x)\right) \wedge \\ & \forall x' \left(x' < x \rightarrow f_{\wedge}\left(f_{p-Prime}\left(f_s\left(x'\right)\right), x'\right) < f_{\wedge}\left(f_{p-Prime}\left(x'\right), f_s\left(x'\right)\right)\right) . \end{aligned} \tag{5.14}$$

5.5. *Oppermann's conjecture.* We define the first variant of the Oppermann's conjecture [vsOFS83]:

For all natural numbers x such that $x > 1$, there is at least one prime number between $x(x-1)$ and x^2.

If a first counterexample m_Z exists for the Oppermann's conjecture [vsOFS83], we can show that m_Z is a n-irreducible number with the following formula ϕ (see 4.2 and 5.1):

$$\begin{aligned} \phi \equiv & \neg x = c_0 \wedge \neg x = f_s(c_0) \wedge \neg \exists y \left(f_{\times}\left(x, f_s^{-1}(x)\right) < y \wedge y < f_{\times}(x, x) \wedge f_{Prime}(y) = f_s(c_0)\right) \\ & \wedge \neg \exists x' \neg \exists y \left(x' < x \wedge f_{\times}\left(x', f_s^{-1}\left(x'\right)\right) < y \wedge y < f_{\times}\left(x', x'\right) \wedge f_{Prime}(y) = f_s(c_0)\right) . \end{aligned} \tag{5.15}$$

If a first counterexample m_Z exists for the Oppermann's conjecture [vsOFS83] and the first variant of the Oppermann's conjecture [vsOFS83] is true, we can show that m_Z is a n-irreducible number with the following formula ϕ (see 4.2 and 5.1):

$$\begin{aligned} & \phi \equiv \neg x = c_0 \wedge \neg x = f_s(c_0) \wedge \\ & \neg \exists y \exists y' \left(f_{\times}\left(x, f_s^{-1}(x)\right) < y \wedge y < f_{\times}(x, x) \wedge f_{\times}(x, x) < y' \wedge y' < f_{\times}\left(x, f_s(x)\right) \wedge \right. \\ & \left. f_{Prime}(y) = f_s(c_0)\right) \wedge \neg \exists x' \neg \exists y \exists y' \\ & \left(x' < x \wedge f_{\times}\left(x', f_s^{-1}\left(x'\right)\right) < y \wedge y < f_{\times}\left(x', x'\right) \wedge f_{\times}\left(x', x'\right) < y' \wedge y' < f_{\times}\left(x', f_s\left(x'\right)\right) \wedge \right. \\ & \left. f_{Prime}(y) = f_s(c_0)\right)) . \end{aligned} \tag{5.16}$$

5.6. *Agoh-Giuga conjecture.* If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the Giuga sub-function symbol g_{Giuga} and the Giuga function symbol f_{Giuga}(see 4.2, 5.1 and 5.7):

$$\begin{aligned} & \forall x \left(g_{Giuga}(x, c_0) = f_s(c_0)\right) \\ & \forall x \forall y \left(g_{Giuga}\left(x, f_s(y)\right) = f_{+}\left(g_{Giuga}(x, y), f_{\wedge}\left(y, f_s^{-1}(x)\right)\right)\right) \\ & \forall x \left(\left(f_{Prime}(x) = f_s(c_0) \rightarrow \exists y \left(f_s\left(g_{Giuga}(x, x)\right) = f_{\times}(x, y)\right)\right) \rightarrow f_{Giuga} = f_s(c_0) \right) \\ & \forall x \left(\neg \left(f_{Prime}(x) = f_s(c_0) \rightarrow \exists y \left(f_s\left(g_{Giuga}(x, x)\right) = f_{\times}(x, y)\right)\right) \rightarrow f_{Giuga} = c_0 \right) . \end{aligned} \tag{5.17}$$

If m_Z is the last natural number where the Agoh-Giuga conjecture [Giu51] is true, we can show that m_Z is a n-irreducible number with the following formula ϕ (see the previous equation):

$$\phi \equiv f_{Giuga}(x) = c_0 \wedge \neg\exists x' \left(x' < x \wedge f_{Giuga}\left(x'\right) = c_0\right) . \tag{5.18}$$

5.7. *Generalized Fermat's conjecture.* We define the generalized Fermat's conjecture [Rie11]:

Let be some natural number a and c, there is an infinite number of natural numbers b such that $a^b + c^b$ is a prime number.

If m_Z is the last number where the generalized Fermat's conjecture [Rie11] for some fixed natural number a and c is true and every explicit sub-formulas which are equivalent to the Generalized Fermat's conjecture [Rie11] with the fixed natural number a' and c' are true , we can show that m_Z is a monster n-irreducible number with the following formula ϕ (see 4.2 and 5.7):

$$\begin{aligned}\phi \equiv & f_{Prime}\left(f_+\left(f_\wedge\left(n_a, x\right), f_\wedge\left(n_c, x\right)\right)\right) = f_s\left(c_0\right) \wedge \\ & \neg\exists x'\left(x < x' \wedge f_{Prime}\left(f_+\left(f_\wedge\left(n_a, x'\right), f_\wedge\left(n_c, x'\right)\right)\right) = f_s\left(c_0\right)\right) \wedge \\ & \neg\exists x''\Bigg(x'' < x \wedge f_{Prime}\left(f_+\left(f_\wedge\left(n_a, x''\right), f_\wedge\left(n_c, x''\right)\right)\right) = f_s\left(c_0\right) \wedge \\ & \neg\exists x'\left(x'' < x' \wedge f_{Prime}\left(f_+\left(f_\wedge\left(n_a, x''\right), f_\wedge\left(n_c, x''\right)\right)\right) = f_s\left(c_0\right)\right)\Bigg) ,\end{aligned} \tag{5.19}$$

where $n_a = \underbrace{f_s(...f_s(}_{a \text{ times}} c_0 \underbrace{)...)}_{a \text{ times}}$ and $n_c = \underbrace{f_s(...f_s(}_{c \text{ times}} c_0 \underbrace{)...)}_{c \text{ times}}$.

If we can show that the generalized Fermat's conjecture is true for many fixed natural numbers a and c, we can show that m_Z is a n-irreducible number with the following formula ϕ (see 4.2 and 5.7):

$$\begin{aligned}\phi \equiv & \exists y\Bigg(f_{Prime}\left(f_+\left(f_\wedge\left(n_a, y\right), f_\wedge\left(x, y\right)\right)\right) = f_s\left(c_0\right) \wedge \\ & \neg\exists x'\left(x < x' \wedge f_{Prime}\left(f_+\left(f_\wedge\left(n_a, y\right), f_\wedge\left(x', y\right)\right)\right) = f_s\left(c_0\right)\right)\Bigg) \wedge \\ & \neg\exists x''\exists y\Bigg(x'' < x \wedge f_{Prime}\left(f_+\left(f_\wedge\left(n_a, y\right), f_\wedge\left(x'', y\right)\right)\right) = f_s\left(c_0\right) \wedge \\ & \neg\exists x'\left(x'' < x' \wedge f_{Prime}\left(f_+\left(f_\wedge\left(n_a, y\right), f_\wedge\left(x', y\right)\right)\right) = f_s\left(c_0\right)\right)\Bigg) .\end{aligned} \tag{5.20}$$

If we can show that the generalized Fermat's conjecture is true for many fixed natural numbers a, we can show that m_Z is a n-irreducible number with the following formula ϕ (see 4.2 and

5.7):

$$\begin{aligned}
\phi \equiv &\exists y \exists z \Bigg(f_{Prime}\left(f_{+}\left(f_{\wedge}\left(x,y\right),f_{\wedge}\left(z,y\right)\right)\right)=f_{s}\left(c_{0}\right)\wedge \\
&\neg\exists x'\left(x<x'\wedge f_{Prime}\left(f_{+}\left(f_{\wedge}\left(x',y\right),f_{\wedge}\left(z,y\right)\right)\right)=f_{s}\left(c_{0}\right)\right)\Bigg)\wedge \\
&\neg\exists x''\exists y\exists z\Bigg(x''<x\wedge f_{Prime}\left(f_{+}\left(f_{\wedge}\left(x'',y\right),f_{\wedge}\left(z,y\right)\right)\right)=f_{s}\left(c_{0}\right)\wedge \\
&\neg\exists x'\left(x''<x'\wedge f_{Prime}\left(f_{+}\left(f_{\wedge}\left(x',y\right),f_{\wedge}\left(z,y\right)\right)\right)=f_{s}\left(c_{0}\right)\right)\Bigg)\,.
\end{aligned} \tag{5.21}$$

5.8. *Schinzel's hypothesis H.* If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the r polynomials function symbol $f_{i,Schinzel}$ of maximal degree d (see 5.7):

$$\begin{aligned}
&\forall x\, (\\
&f_{1,Schinzel}\left(x\right)=f_{+}\left(f_{+}\left(...f_{+}\left(f_{\times}\left(a_{10},f_{\wedge}\left(x,b_{0}\right)\right),f_{\times}\left(a_{11},f_{\wedge}\left(x,b_{1}\right)\right)\right)...\right),f_{\times}\left(a_{1d},f_{\wedge}\left(x,b_{d}\right)\right)\right) \\
&\qquad\vdots \\
&f_{r,Schinzel}\left(x\right)=f_{+}\left(f_{+}\left(...f_{+}\left(f_{\times}\left(a_{r0},f_{\wedge}\left(x,b_{0}\right)\right),f_{\times}\left(a_{r1},f_{\wedge}\left(x,b_{1}\right)\right)\right)...\right),f_{\times}\left(a_{rd},f_{\wedge}\left(x,b_{d}\right)\right)\right) \\
&\qquad)
\end{aligned} \tag{5.22}$$

where $a_{ij}=\underbrace{f_s(...f_s(}_{a(i,j)\text{ times}}\ c_0\ \underbrace{)...)}_{a(i,j)\text{ times}}$ and $b_i=\underbrace{f_s(...f_s(}_{i\text{ times}}c_0\underbrace{)...)}_{i\text{ times}}$.

Since the r polynomials $f_{i,Schinzel}$ are irreducible, the polynomial coefficients a_{ij} satisfy the first following constraint (see the previous equation):

$$\left(\nexists x f_{1,Schinzel}\left(x\right)=c_{0}\right)\wedge ...\wedge\left(\nexists x f_{r,Schinzel}\left(x\right)=c_{0}\right)\,. \tag{5.23}$$

Since the product of the r polynomials $f_{i,Schinzel}$ has not a fixed prime divisor, the polynomial coefficients a_{ij} satisfy the second following constraint (see 5.23):

$$\begin{aligned}
&\nexists x\forall y\exists z\Bigg(f_{Prime}\left(x\right)=f_{s}\left(c_{0}\right)\wedge \\
&f_{\times}\left(f_{\times}\left(...f_{\times}\left(f_{1,Schinzel}\left(y\right),f_{2,Schinzel}\left(y\right)\right)...\right),f_{r,Schinzel}\left(y\right)\right)=f_{\times}\left(x,z\right)\Bigg)\,.
\end{aligned} \tag{5.24}$$

If m_Z is the last number where the Schinzel's hypothesis H [Guy04] for some fixed polynomial is true and every explicit sub-formulas which are equivalent to the Schinzel's hypothesis H [Guy04] for some fixed polynomials are true, we can show that m_Z is a monster n-irreducible number with the following formula ϕ (see 4.2 and 5.23):

$$\begin{aligned}
\phi \equiv &f_{Prime}\left(f_{1,Schinzel}\left(x\right)\right)=f_{s}\left(c_{0}\right)\wedge ...\wedge f_{Prime}\left(f_{r,Schinzel}\left(x\right)\right)=f_{s}\left(c_{0}\right)\wedge \\
&\nexists x'\left(x<x'\wedge f_{Prime}\left(f_{1,Schinzel}\left(x'\right)\right)=f_{s}\left(c_{0}\right)\wedge ...\wedge f_{Prime}\left(f_{r,Schinzel}\left(x'\right)\right)=f_{s}\left(c_{0}\right)\right)\,.
\end{aligned} \tag{5.25}$$

We build new formulas for new monster n-irreducible numbers like in the generalized Fermat's conjecture:

If we can show that the Schinzel's hypothesis H [Guy04] is true for a fixed number of polynomial r, a fixed maximal degree d, many fixed polynomial coefficients a_{ij} and some running polynomial coefficients a_{ij}, we can show that m_Z is a monster n-irreducible number with a formula ϕ.

We build new formulas for new monster n-irreducible numbers like in the generalized Fermat's conjecture:

If we can show that the Schinzel's hypothesis H [Guy04] is true for a fixed number of polynomial r, many maximal degrees d and the running polynomial coefficients a_{ij}, we can show that m_Z is a monster n-irreducible number with a formula ϕ.

Finally, we build new formulas for new monster n-irreducible numbers like in the generalized Fermat's conjecture:

If we can show that the Schinzel's hypothesis H [Guy04] is true for many numbers of polynomial r, the running maximal degree d and the running polynomial coefficients a_{ij}, we can show that m_Z is a monster n-irreducible number with a formula ϕ.

6. The second order logic n-irreducible axiom and the second order logic hypothesis on the "Nature's hypotheses"

We introduce one important axiom on n-irreducible numbers and one important hypothesis on the "Nature's hypotheses"at second order logic for both of them:

The (second order logic) n-irreducible axiom:

There is a finite number of n-irreducible sequent and the largest n-irreducible number is:

$$N_Z \sim 2^{2.205 \times 10^{61}} \tag{6.1}$$

. The (second order logic) hypothesis on the "Nature's hypotheses":

the "Nature's hypotheses"which explain the physical measurements are the hypotheses of a N_Z-irreducible sequent.

Some consequences:

1- The physical measurements confirm but do not prove that the "Nature's hypotheses", the mathematical explorations over the n-irreducible numbers confirm but do not prove that N_Z is the largest n-irreducible number.

2- The Collatz conjecture, the Goldbach's conjecture, the Polignac's conjecture, the Firoozbakht conjecture's, the Oppermann's conjecture, the Agoh-Giuga conjecture can be checked with quantum computers with 2.205×10^{61} qubits or less (2×10^{79} atoms in the visible universe) since the computation can be fully parallel [LS14], the generalized Fermat's conjecture (which requires computational resources which are far from what we can imagine technically even for the simplest case: $a = 2$ and $b = 1$) and the Schinzel's hypothesis H (which requires monster computational resources for checking about $\left(\pi\left(N_Z\right) 2^{N_Z}\right)^{N_Z}$ cases). . Moreover, 27 is a n-irreducible number if the Goldbach's conjecture is true.

3- A article is under preparation in order to present the theory of everything where its hypotheses are the hypotheses of a N_Z-irreducible sequent (N_Z would be the number of Lagrangian terms but experimentally, we can only access to a very small fraction of that terms) and to show that any obvious variant of the theory of everything requires some hypotheses which give a n-irreducible number strictly smaller than N_Z.

7. Some open questions about the n-irreducible sequents and the n-irreducible numbers

1- Can we show that a natural number n is not n-irreducible? The difficulty is to prove that there is no n-irreducible sequent among a infinite set of possible sequents which give the n-irreducible number n.

2- If a n-irreducible number n is found with the help of a function symbol f where its output values are only 0 and 1, can we replace the function symbol f by a function symbol $\tilde{f}$ such that $\tilde{f} = 1 - f$ and the new sequent is still n-irreducible?

3- In quantum field theories with gauge fields, the number of space-time dimensions should be larger or equal to 4 in order to have a "renormalizable" theory. Therefore the Nature's hypotheses are n-irreducible only if there are 4 space-time coordinates. The Nature's hypotheses with 5 space-time coordinates are not n-irreducible and one specific spatial coordinate may be skipped at the multiple places where it is written in the theory. From that example, we conclude that every sub-formulas should be considered when we study some n-irreducible sequent. It confirms that the required number of sub-sequents to explore in order check that a sequent is a n-irreducible sequent is roughly exponential to its number of symbols in the general case. However, for a specific n-irreducible sequent, some specific mathematical tools may be developed in order to explore a set of sub-sequents in one time.

4- Does the logic operator $\veebar$ is required in some n-irreducible sequent?

5- In the definition of a n-irreducible sequent:

$$\Delta \vdash \psi_{m-irreducible} \text{ and } m = c_0 \text{ or } m = f_s\left(c_0\right) \tag{7.1}$$

to

$$\Delta \vdash \psi_{m-irreducible} \text{ and } m = \mathcal{S} \in \mathcal{N} \tag{7.2}$$

Does the second order logic n-irreducible axiom holds for any $\mathcal{S}$?

If we investigate the case $\mathcal{S} = \{0\}$, can we find an other relevant 1-irreducible sequent than the 1-irreducible sequents with the following formulas:

$$\begin{aligned}
&\phi \equiv x = f_s(c_0)\\
&\phi \equiv c_0 < x \wedge \neg\exists x' \left(x' < x \wedge c_0 < x'\right)\\
&\phi \equiv c_0 < x \wedge x = f_\times(x, x)\\
\\
&\phi \equiv \neg x = c_0 \wedge \neg\exists x' \left(x' < x \wedge \neg c_0 < x'\right)\\
&\phi \equiv \neg c_0 < x \wedge \neg\exists x' \left(x' < x \wedge \neg x' = c_0\right)\\
&\phi \equiv \neg x = c_0 \wedge \neg\exists x' \left(x' < x \wedge \neg x' = c_0\right)\\
&\phi \equiv \neg x = c_0 \wedge x = f_\times(x, x)
\end{aligned} \tag{7.3}$$

where $x \in \mathbb{Z}$ in the three first equations and $x \in \mathbb{N}$ in the four last equations?

If we investigate the case $\mathcal{S} = \{\}$, can we find an other relevant 0-irreducible sequent than the 0-irreducible sequent with the following formula:

$$\phi \equiv x = c_0 \tag{7.4}$$

5- The second order logic n-irreducible axiom may be stronger? We may introduce the following one instead:

The stronger (second order logic) n-irreducible axiom:

For all n-irreducible sequent, the following inequality holds :

$$n \leq 2^{\lambda^a} \tag{7.5}$$

where λ is the number of symbol in the n-irreducible sequent with the minimal symmetric notation.

The inequality is an equality for the N_Z-irreducible sequents where their hypotheses are the "Nature's hypotheses"which explain the physical measurements:

$$N_z = 2^{\lambda_Z^a} \leftrightarrow a = log\left(2, log\left(2, N_Z\right)\right)/log\left(2, \lambda_Z\right) \tag{7.6}$$

The numer of fields in the theory with the "Nature's hypotheses":

$$3 \times 4 \times (2N)^{N^2} \sim 1.735 \times 10^{57} \text{ with N=7 which gives } a = 1.07169 \tag{7.7}$$

The consequences are: the N_Z-irreducible sequents are equivalent and the number of n-irreductible sequents is finite. We can derive a fixed upper bound for the number of n-irreducible sequents and a derive a finite set of sequents where all n-irreductible sequents are included.

A practical consequence for the Goldbach's conjecture: the Goldbach's n-irreducible sequent has $\lambda_{Goldbach} = 797$ which is the sum of the pair parentheses number, the variable labels, the function labels and the relation labels (without the Peano's recursive axiom). It gives a numerical upper bound for its n-irreducible number: $n \leq 2.214 \times 10^{387}$ (the Glodbach' conjecture is verified for: $n \leq 4 \times 10^{18}$).

6- If we prove a sequent with the second order logic n-irreducible axiom, can we prove it without it?

7- The Peano's recursive axiom may not be longer required (which is a axiom over an infinite number of formulas) and the (second order logic) n-irreducible axiom may be sufficient.

8- Can we reduce the (second order logic) n-irreducible axiom to the following one:

The n-irreducible sequent with the "Nature's hypotheses"is the n-irreducible sequent with the largest sum of the pair parenthesis number, the variable label, the function label and the relation labels.

8. Extra: some larger n-irreducible number examples

In this section, with the help of the formulas satisfied by the symbol function f_{Prime} joined to the hypotheses of some n-irreducible sequents, we try to reach the closest n-irreducible number (1024 in the present section) to the largest one $N_Z \sim 2^{2.205\times 10^{61}}$. Firstly, we write the preliminary formulas satisfied by the following function symbols:

1- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the sub-function symbol g_{σ_0-1} and the function symbol f_{σ_0-1} which gives the number of proper divisor of a natural number:

$$\begin{aligned}
&\forall x\,(g_{\sigma_0-1}(x, f_s\,(c_0)) = c_0)\\
&\forall x\forall y\,(\exists z\,(x = f_\times\,(y,z)) \to g_{\sigma_0-1}\,(x, f_s\,(y)) = f_s\,(g_{\sigma_0-1}(x,y)))\\
&\forall x\forall y\,(\neg\exists z\,(x = f_\times\,(y,z)) \to g_{\sigma_0-1}\,(x, f_s\,(y)) = g_{\sigma_0-1}(x,y))\\
&\forall x\,(f_{\sigma_0-1}(x) = g_{\sigma_0-1}(x,x))\ .
\end{aligned}\tag{8.1}$$

2- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the highly composite function symbol f_{HC} (see the previous equation):

$$\begin{aligned}
&\forall x\,(\forall y\,((\neg y = c_0 \land y < x) \to f_{\sigma_0-1}\,(y) < f_{\sigma_0-1}\,(x)) \to f_{HC}(x) = f_s\,(c_0))\\
&\forall x\,(\neg\,(\forall y\,(\neg y = c_0 \land y < x) \to f_{\sigma_0-1}\,(y) < f_{\sigma_0-1}\,(x)) \to f_{HC}(x) = c_0)\ .
\end{aligned}\tag{8.2}$$

3- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the Euler's totient sub-function symbol g_Φ and the Euler's totient function symbol f_Φ which gives the number of coprime numbers below it (see 5.6):

$$\begin{aligned}
&\forall x\,(g_\Phi(x, f_s\,(c_0)) = c_0)\\
&\forall x\forall y\,(f_{Coprime}\,(x,y) = f_s\,(c_0) \to g_\Phi\,(x, f_s\,(y)) = f_s\,(g_\Phi(x,y)))\\
&\forall x\forall y\,(f_{Coprime}\,(x,y) = c_0 \to g_\Phi\,(x, f_s\,(y)) = g_\Phi(x,y))\\
&\forall x\,(f_\Phi(x) = g_\Phi(x,x))
\end{aligned}\tag{8.3}$$

4- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the highly coprime function symbol f_{HCP}(see the previous equation):

$$\begin{aligned}
&\forall x\,(\forall y\,((\neg y = c_0 \land y < x) \to f_\Phi\,(y) < f_\Phi\,(x)) \to f_{HCP}(x) = f_s\,(c_0))\\
&\forall x\,(\neg\forall y\,((\neg y = c_0 \land y < x) \to f_\Phi\,(y) < f_\Phi\,(x)) \to f_{HCP}(x) = c_0)\ .
\end{aligned}\tag{8.4}$$

5- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the sub-function symbol g_{σ_1-1} and the function symbol f_{σ_1-1} which gives the sum of divisors minus one of a natural number (see 5.1):

$$\forall x\left(g_{\sigma_1-1}(x,f_s\left(c_0\right))=c_0\right)$$
$$\forall x\forall y\left(\left(\neg y=f_s\left(c_0\right)\wedge\exists z\left(x=f_\times\left(y,z\right)\right)\right)\rightarrow g_{\sigma_1-1}\left(x,y\right)=f_+\left(g_{\sigma_1-1}(x,f_s^{-1}\left(y\right)),y\right)\right)$$
$$\forall x\forall y\left(\neg\left(\neg y=f_s\left(c_0\right)\wedge\exists z\left(x=f_\times\left(y,z\right)\right)\right)\rightarrow g_{\sigma_1-1}\left(x,y\right)=g_{\sigma_1-1}(x,f_s^{-1}\left(y\right))\right)$$
$$\forall x\left(f_{\sigma_1-1}(x)=g_{\sigma_1-1}(x,x)\right)\ . \tag{8.5}$$

With the concept of complement, we can show that 10 is a n-irreducible number with the following formula ϕ (see 8.2 and 8.4):

$$\phi\equiv f_{HC}\left(x\right)=c_0\wedge f_{HCP}\left(x\right)=f_s\left(c_0\right)\wedge\neg\exists y\left(y<x\wedge f_{HC}\left(x\right)=c_0\wedge f_{HCP}\left(x\right)=f_s\left(c_0\right)\right) \tag{8.6}$$

and we can show that 24 is a n-irreducible number with the following formula ϕ (see 8.2 and 8.4):

$$\phi\equiv f_{HC}\left(x\right)=f_s\left(c_0\right)\wedge f_{HCP}\left(x\right)=c_0\wedge\neg\exists y\left(y<x\wedge f_{HC}\left(x\right)=f_s\left(c_0\right)\wedge f_{HCP}\left(x\right)=c_0\right)\ . \tag{8.7}$$

In order to find much larger n-irreducible number, we use the concept of amicable numbers:

1- 220 is a n-irreducible number with the following formula ϕ (see 8.5):

$$\phi\equiv\exists y\left(x<y\wedge f_{\sigma_1-1}\left(x\right)=f_{\sigma_1-1}\left(y\right)\right)\wedge\forall z\left(z<x\rightarrow\neg\exists y\left(z<y\wedge f_{\sigma_1-1}\left(z\right)=f_{\sigma_1-1}\left(y\right)\right)\right)\ . \tag{8.8}$$

2- 284 is a n-irreducible number with the following formula ϕ (see 8.5):

$$\phi\equiv\exists y\left(y<x\wedge f_{\sigma_1-1}\left(x\right)=f_{\sigma_1-1}\left(y\right)\right)\wedge\forall z\left(z<x\rightarrow\neg\exists y\left(y<z\wedge f_{\sigma_1-1}\left(z\right)=f_{\sigma_1-1}\left(y\right)\right)\right)\ . \tag{8.9}$$

3- 503 is a n-irreducible number with the following formula ϕ (see 8.5):

$$\phi\equiv\exists y\exists z$$
$$\left(x=f_{\sigma_1-1}\left(y\right)\wedge x=f_{\sigma_1-1}\left(z\right)\right)\wedge\forall w\left(w<x\rightarrow\neg\exists y\exists z\left(w=f_{\sigma_1-1}\left(y\right)\wedge w=f_{\sigma_1-1}\left(z\right)\right)\right)\ . \tag{8.10}$$

9. Extra bis: some n-irreducible number examples with the axioms of the real numbers

In this section, with the help of the formulas satisfied by the axioms of the real numbers joined to the hypotheses of some n-irreducible sequents, we try to reach the closest n-irreducible number (1024 in the present section) to the largest one $N_Z\sim 2^{2.205\times 10^{61}}$. Firstly, we write the preliminary formulas satisfied by the following function symbols:

1- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the natural number function symbol $f_{\mathbb{N}}$:

$$f_{\mathbb{N}}\left(c_0\right)=f_s\left(c_0\right)$$
$$\forall x\left(f_{\mathbb{N}}\left(x\right)=f_s\left(c_0\right)\rightarrow f_{\mathbb{N}}\left(f_s\left(x\right)\right)=f_s\left(c_0\right)\right)$$
$$\forall x\left(f_{\mathbb{N}}\left(x\right)=c_0\rightarrow f_{\mathbb{N}}\left(f_s\left(x\right)\right)=c_0\right)\ . \tag{9.1}$$

2- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the integer part function symbol f_{IP} (see the previous equation):

$$\forall x \left(\exists n \left(f_{\mathbb{N}}\left(n\right)=f_s\left(c_0\right) \wedge \neg n<x \wedge x<f_s\left(n\right) \wedge f_{IP}\left(x\right)=n\right)\right) . \tag{9.2}$$

3- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the ceiling function symbol $f_{Ceiling}$ (see 9.1):

$$\forall x \left(\exists n \left(f_{\mathbb{N}}\left(n\right)=f_s\left(c_0\right) \wedge x<n \wedge \neg f_s\left(n\right)<x \wedge f_{Ceiling}\left(x\right)=n\right)\right) . \tag{9.3}$$

4- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the negative function symbol f_-

$$\begin{gathered}\forall x \exists y \left(f_+\left(x,y\right)=c_0 \wedge f_-\left(x\right)=y\right)\\ \forall x \neg \exists y \exists y' \left(\neg y=y' \wedge f_+\left(x,y\right)=c_0 \wedge f_+\left(x,y'\right)=c_0\right) .\end{gathered} \tag{9.4}$$

5 If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the factorial function symbol $f_!$ (see 5.1 and 9.1):

$$\begin{gathered}\forall n \left(f_{\mathbb{N}}\left(n\right)=f_s\left(c_0\right) \wedge n=c_0 \rightarrow f_!\left(n\right)=f_s\left(c_0\right)\right)\\ \forall n \left(f_{\mathbb{N}}\left(n\right)=f_s\left(c_0\right) \wedge \neg n=c_0 \rightarrow f_!\left(n\right)=f_\times\left(n, f_!\left(f_s^{-1}\left(n\right)\right)\right)\right) .\end{gathered} \tag{9.5}$$

6 If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the exponential series function symbol g_{Exp} (see 5.1 and 5.7):

$$\begin{gathered}\forall x \left(g_{Exp}\left(x,c_0\right)=c_0\right)\\ \forall x \forall y \left(g_{Exp}\left(x,f_s\left(y\right)\right)=f_+\left(g_{Exp}\left(x,y\right), f_\times\left(f_\wedge\left(x,y\right), f_{-1}\left(f_!\left(y\right)\right)\right)\right)\right) .\end{gathered} \tag{9.6}$$

7 If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the exponential function symbol f_{Exp} (see 9.1 and the previous equation):

$$\begin{aligned}\forall \epsilon \exists N \forall n \Bigg(& \left(0<\epsilon \wedge N<n \wedge f_{\mathbb{N}}\left(N\right)=f_s\left(c_0\right) \wedge f_{\mathbb{N}}\left(n\right)=f_s\left(c_0\right)\right)\\ & \rightarrow \left(g_{Exp}\left(x,n\right)<f_{Exp}\left(x\right) \wedge f_{Exp}\left(x\right)<f_+\left(\epsilon, g_{Exp}\left(x,n\right)\right)\right)\Bigg) .\end{aligned} \tag{9.7}$$

We suppose we can define the Lebesgue integral or the Riemann integral irreducibly in order to define the function $f_{\sqrt{\pi}/2}$ in a n-irreducible form (see 5.7, 9.1, 9.4 and 9.7):

$$\forall n \left(f_{\mathbb{N}}\left(n\right)=f_s\left(c_0\right) \wedge \neg n=c_0 \rightarrow f_{\sqrt{\pi}/2}\left(n\right)=\int_0^\infty f_{Exp}\left(f_-\left(f_\wedge\left(x,n\right)\right)\right)dx\right) . \tag{9.8}$$

We can define the real number $\sqrt{\pi}/2$ irreducibly with the following formula ϕ:

$$\phi \equiv \exists n \left(f_{\mathbb{N}}\left(n\right)=f_s\left(c_0\right) \wedge x=f_{\sqrt{\pi}/2}\left(n\right)\right) \wedge \neg \exists n \left(f_{\mathbb{N}}\left(n\right)=f_s\left(c_0\right) \wedge f_{\sqrt{\pi}/2}\left(n\right)<x\right) . \tag{9.9}$$

Sketch to prove that 5 and 7 are n-irreducible numbers: The $(n-1)$-sphere of radius R and center $\vec{r}$ can be defined irreducibly by imposing a maximum volume for a fixed surface in $\mathbb{R}^n$ or a minimum surface for a fixed volume. By defining a n-irreducible n-cube with vertex coordinates $\underbrace{(\pm 1, ..., \pm 1)}_{n \text{ times}}$ and taking the biggest $(n-1)$-sphere inside it, we can find the n which maximize the volume $V(n)$ or the surface $S(n)$ of the $(n-1)$-sphere: 5 or 7.

Therefore, we can also define irreducibly the real numbers $16\pi^3/15$ and $8\pi^2/15$ with the following formula ϕ:

$$\phi \equiv \exists n \left(f_{\mathbb{N}}(n) = f_s(c_0) \wedge x = S(n)\right) \wedge \neg\exists n' \left(f_{\mathbb{N}}(n') = f_s(c_0) \wedge x < S(n')\right) \;, \tag{9.10}$$

and the following formula ϕ

$$\phi \equiv \exists n \left(f_{\mathbb{N}}(n) = f_s(c_0) \rightarrow x = V(n)\right) \wedge \neg\exists n' \left(f_{\mathbb{N}}(n') = f_s(c_0) \wedge x < V(n')\right) \;. \tag{9.11}$$

We can also define irreducibly the real number e with the following formula ϕ:

$$\phi \equiv x = f_{Exp}\left(f_s(c_0)\right) \;. \tag{9.12}$$

For some n-irreducible real numbers x and x', we can show that n is a n-irreducible number with the following formula ϕ:

$$\begin{gathered}\phi \equiv \exists m \exists p \exists q \left(f_{\mathbb{N}}(n) = f_s(c_0) \wedge f_{\mathbb{N}}(m) = f_s(c_0) \wedge f_{\mathbb{N}}(p) = f_s(c_0) \wedge f_{\mathbb{N}}(q) = f_s(c_0) \wedge\right. \\ \left. f_{\times}\left(p, f_{\wedge}(x, n)\right) = f_{\times}\left(q, f_{\wedge}(x', m)\right) \wedge f_{Coprime}(p, q) = f_s(c_0) \wedge f_{Coprime}(m, n) = f_s(c_0)\right) \;.\end{gathered} \tag{9.13}$$

For some n-irreducible real numbers x and x', we can show that p is a n-irreducible number with the following formula ϕ:

$$\begin{gathered}\phi \equiv \exists n \exists m \exists q \left(f_{\mathbb{N}}(n) = f_s(c_0) \wedge f_{\mathbb{N}}(m) = f_s(c_0) \wedge f_{\mathbb{N}}(p) = f_s(c_0) \wedge f_{\mathbb{N}}(q) = f_s(c_0) \wedge\right. \\ \left. f_{\times}\left(p, f_{\wedge}(x, n)\right) = f_{\times}\left(q, f_{\wedge}(x', m)\right) \wedge f_{Coprime}(p, q) = f_s(c_0) \wedge f_{Coprime}(m, n) = f_s(c_0)\right) \;.\end{gathered} \tag{9.14}$$

Therefore, from the n-irreducible real numbers, $\sqrt{\pi}/2$, $16\pi^3/15$, $8\pi^2/15$ and the help of the two last formulas, we deduce that 2, 3, 4, 6, 15, 128 and 1024 are n-irreducible numbers.

With the help of the integer part function f_{IP} and the ceiling function $f_{Ceiling}$ on the n-irreducible real number $8\pi^2/15$ and $16\pi^3/15$, we deduce that 5, 6, 33 and 34 are n-irreducible numbers.

Finally, we can derive that 8 and 9 are n-irreducible numbers with the help of the following prime exponential number function $f_{PrimeExpPI}$ and $f_{PrimeExpCeiling}$:

$$\forall n \left(f_{\mathbb{N}}(n) = f_s(c_0) \rightarrow f_{PrimeExpPI}(n) = f_{Prime}\left(f_{PI}\left(f_{Exp}(n)\right)\right)\right) \tag{9.15}$$

$$\forall n \left(f_{\mathbb{N}}(n) = f_s(c_0) \rightarrow f_{PrimeExpCeiling}(n) = f_{Prime}\left(f_{Ceiling}\left(f_{Exp}(n)\right)\right)\right) \tag{9.16}$$

7 is a n-irreducible number with the following formula ϕ:

$$\forall m \left(\neg n < m \rightarrow f_{PrimeExpPI}(m) = f_s(c_0)\right) \wedge \nexists n' \forall m \left(n < n' \wedge \neg n' < m \rightarrow f_{PrimeExpPI}(m) = f_s(c_0)\right) \tag{9.17}$$

20 is a n-irreducible number with the following formula ϕ:

$$\forall m \left(m < n \rightarrow f_{PrimeExpPI}(m) = f_s(c_0)\right) \wedge \nexists n' \forall m \left(n < n' \wedge m < n' \rightarrow f_{PrimeExpPI}(m) = f_s(c_0)\right) \tag{9.18}$$

8 is a n-irreducible number with the following formula ϕ:

$$\begin{gathered}\forall m \left(m < n \rightarrow f_{PrimeExpCeiling}(m) = f_s(c_0)\right) \wedge \\ \nexists n' \forall m \left(n < n' \wedge m < n' \rightarrow f_{PrimeExpCeiling}(m) = f_s(c_0)\right)\end{gathered} \tag{9.19}$$

10. A set of n-irreducible numbers

In this section, we insert every n-irreducible number derived in the present article inside an only one set:

(10.1)
$$\mathcal{S}_{n-irreducible} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 10, 14, 15, 20, 24, 27, 31, 33, 34, 37, 128, 220, 284, 503, 1024\}$$

11. The Riemann hypothesis

In this section, with the help of the formulas satisfied by the axioms of the complex numbers and the new complex constant variable c_i ($c_0 = f_+ (f_s (c_0), f_\times (c_i, c_i))$) joined to the hypotheses of some n-irreducible sequent, we propose a method to prove the Riemann hypothesis.

1- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the log function symbol f_{Log}:

$$\forall x \left(0 < x \to f_{Exp} \left(f_{Log} (x)\right) = x\right) \tag{11.1}$$

2- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the real power function symbol $f_{\wedge\mathbb{R}}$ (see 11.1):

$$\forall x \left(0 < x \to f_{\wedge\mathbb{R}} (x, y) = f_{Exp} \left(f_\times \left(y, f_{Log} (x)\right)\right)\right) \tag{11.2}$$

3- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the Riemann zeta series function symbol g_ζ (see 11.2)

$$\begin{aligned} &\forall x \left(x < 0 \to g_\zeta (x, c_0) = c_0\right) \\ &\forall x \forall y \Bigg(x < 0 \to g_\zeta (x, f_s (y)) = f_+ \Bigg(g_\zeta (x, y), f_{\wedge\mathbb{R}} (y, x) \Bigg)\Bigg). \end{aligned} \tag{11.3}$$

4- If required, we add to the hypotheses of the n-irreducible sequents the following formulas satisfied by the Riemann zeta function symbol f_ζ (see 11.3 and the previous equation):

$$\begin{aligned} &\forall x \forall \epsilon \exists N \forall n \Bigg(\left(0 < \epsilon \wedge N < n \wedge f_{\mathbb{N}} (N) = f_s (c_0) \wedge f_{\mathbb{N}} (n) = f_s (c_0)\right) \\ &\to \left(g_\zeta (x, n) < f_\zeta (x) \wedge f_\zeta (x) < f_+ \left(\epsilon, g_\zeta (x, n)\right)\right) \Bigg). \end{aligned} \tag{11.4}$$

We assume we can define irreducibly the homomorphic functions and the analytic continuation. Therefore, we can define the holomorphic Riemann zeta function $f_{\zeta\mathbb{C}}$.

From here, we define the function $f_{\zeta NTZ}$ which enumerates the imaginary part of the none trivial zeros of the holomorphic zeta function f_ζ:

$$f_{\zeta NTZ}\left(c_0\right)=c_0$$
$$\forall n\left(f_{\mathbb{N}}\left(n\right)=f_s\left(c_0\right)\rightarrow f_{\zeta NTZ}\left(n\right)<f_{\zeta NTZ}\left(f_s(n)\right)\right)$$
$$\forall n\left(f_{\mathbb{N}}\left(n\right)=f_s\left(c_0\right)\rightarrow f_{\zeta\mathbb{C}}\left(f_{\zeta NTZ}(n)\right)=c_0\right)$$

(11.5)
$$\neg\exists a\exists b\exists n\left(f_{\mathbb{N}}\left(n\right)=f_s\left(c_0\right)\wedge f_{\zeta\mathbb{C}}\left(f_+\left(a,f_\times\left(c_i,b\right)\right)\right)=c_0\wedge f_{\zeta NTZ}(n)<b\wedge b<f_{\zeta NTZ}(f_s(n))\right)$$

With the help of the function $f_{\zeta NTZ}$, we can find four n-irreducible numbers (14,15,31,37) derived from four n-irreducible sequents:

(11.6)
$$\phi\equiv x=f_{IP}\left(f_{\zeta NTZ}\left(f_s\left(c_0\right)\right)\right)$$

(11.7)
$$\phi\equiv x=f_{Ceiling}\left(f_{\zeta NTZ}\left(f_s\left(c_0\right)\right)\right)$$

$$\phi\equiv\exists n\left(f_{\mathbb{N}}\left(n\right)=f_s\left(c_0\right)\wedge x=f_{Prime}\left(f_{Ceiling}\left(f_{\zeta NTZ}\left(f_s\left(n\right)\right)\right)=f_s(c_0)\right)\wedge\right.$$

(11.8)
$$\left.\neg\exists n'\left(f_{\mathbb{N}}\left(n'\right)=f_s\left(c_0\right)\wedge 0<n'\wedge n'<n\wedge f_{Prime}\left(f_{Ceiling}\left(f_{\zeta NTZ}\left(f_s\left(n\right)\right)\right)=f_s(c_0)\right)\right)\right)$$

$$\phi\equiv\exists n\left(f_{\mathbb{N}}\left(n\right)=f_s\left(c_0\right)\wedge x=f_{Prime}\left(f_{IP}\left(f_{\zeta NTZ}\left(f_s\left(n\right)\right)\right)=f_s(c_0)\right)\wedge\right.$$

(11.9)
$$\left.\neg\exists n'\left(f_{\mathbb{N}}\left(n'\right)=f_s\left(c_0\right)\wedge c_0<n'\wedge n'<n\wedge f_{Prime}\left(f_{IP}\left(f_{\zeta NTZ}\left(f_s\left(n\right)\right)\right)=f_s(c_0)\right)\right)\right)$$

From the holomorphic Riemann zeta function $f_{\zeta\mathbb{C}}$, if the Riemann hypothesis is false, we can define a monster n-irreducible number m_Z with the following n-irreducible sequent:

$$\phi\equiv\exists a\exists b\exists n\left(f_{\mathbb{N}}\left(n\right)=f_s\left(c_0\right)\wedge x=n\wedge\neg f_\times\left(f_s\left(f_s(c_0)\right),a\right)=f_s(c_0)\wedge f_{\zeta NTZ}\left(f_s\left(n\right)\right)=b\wedge\right.$$

(11.10)
$$\left.\neg\exists a'\exists b'\exists n'\left(f_{\mathbb{N}}\left(n'\right)=f_s\left(c_0\right)\wedge n'<n\wedge\neg f_\times\left(f_s\left(f_s(c_0)\right),a'\right)=f_s(c_0)\wedge f_{\zeta NTZ}\left(f_s\left(n'\right)\right)=b'\right)\right)$$

12. Conclusion

This article may open a new area in second order logic with some important consequences in number theory and in fundamental physics if we do not notice contradictions between the (second order logic) n-irreducible axiom and other well known axioms, and we do not observe experimental contradictions between the hypotheses to produce the largest n-irreducible number ever found and the experimental measurements. It is the first article which gives a hint to solve the Collatz conjecture, the Goldbach's conjecture, the Polignac's conjecture, the Firoozbakht's conjecture, the Oppermann's conjecture, the Agoh-Giuga conjecture with a quantum computer of 2.205×10^{61} qubits or less (2×10^{79} atoms in the visible universe) and with only one (second order logic) n-irreducible axiom. The generalized Fermat's conjecture requires computational resources which are far from what we can imagine technically even for the simplest case: $a=2$ and $b=1$ and the Schinzel's hypothesis H requires also monster computational resources for checking about $\left(\pi\left(N_Z\right)2^{N_Z}\right)^{N_Z}$ cases. It is also the first article which gives a hint to generate the "Nature's hypotheses" with only one (second order logic) hypothesis.

Since I am not a mathematician and I am a lonely human, I may have overseen some mistakes (especially, I could miss an explicit sub-formula in the present article since the number of sub-formulas is roughly 2^n for a formula with n symbols or I do not noticed that a sequent is not n-irreducible or my approach to the generalized Fermat's conjecture and the Schinzel's hypothesis H are sensitive to some mistakes since it is one more level of abstraction from the other prime conjectures). Moreover, N_Z may change after the publication of the next article about the theory of everything. Please send me an email (see it below the references) for any mistake noticed in the present article. Every ideas or comments related to the present article are also very welcome.

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