A Brief Solution to the Riemann Hypothesis over the Lagarias Transformation

Mesut KAVAK*

In accordance with the transformation of Lagarias [1] which is the equivalent of the Riemann Hypothesis, for a positive integer n, let $\sigma(n)$ denote the sum of the positive integers that divide n. Let H_n denote the nth harmonic number by

$$H_n = \sum_{n=1}^{n} \frac{1}{n}$$

Does the following inequality hold for all $n \geq 1$ where $\sigma(n)$ is the sum of divisors function?

$$H_n + \ln(H_n)e^{H_n} \geq \sigma(n)$$

1 Definition for the solutions

Theorem: *First of all, let's define an imaginary function as $\rho(n)$, and know that this function is the sum of the elements which are not dividable being the result is an integer for a function as nH_n for each n; so according to this definition, it becomes as the following.*

$$H_n = \frac{\sigma(n) + \rho(n)}{n}$$

Here actually $\rho(n)$ is only by definition. There is no function like this and thus the rule of the function is not known. It is imaginary as a catalyzer. It does its work and leaves the actual functions alone without becoming inclusive when it shows us the result. This equation is only for relating n, H_n and $\sigma(n)$ together somehow. $\rho(n)$ can be a negative number that is negative for the values of $\ln H_n > 1$ here as we are going to see it over the below stated operations. If the result is suitable by the assumptions, then we can use it.

Warning

By using the equation, $H_n + \ln(H_n)e^{H_n} \geq \sigma(n)$ inequality turns into (1).

$$H_n + \ln(H_n)e^{H_n} \geq nH_n - \rho(n) \tag{1}$$

If it is edited, it becomes (2) over (2a).

$$\frac{\ln(H_n)e^{H_n} + \rho(n)}{n-1} \geq H_n \tag{2}$$

$$\ln(H_n)e^{H_n} \geq nH_n - H_n - \rho(n) \tag{2a}$$

Condition: *Right this point assume, that the actual inequality is not (2) but is (3).*

$$\frac{e^{H_n}}{n} \geq H_n \tag{3}$$

On (2), assume that actually the numerator is always bigger than e^{H_n}, and thus also if the divisor was $n-1$, this would increase the possibility of to be greater than H_n of the division; so for the worst possibility, let's use this as (3). If the following operations are not verified over these above stated definitional assumptions, then we must redetermine the conditions and definitions.

Here if the numerator is bigger than e^{H_n}, then the equation becomes $\frac{\rho(n)}{1-\ln H_n} > e^{H_n}$ over $\ln(H_n)e^{H_n} + \rho(n) > e^{H_n}$; so $\rho(n)$ is negative for $\ln H_n > 1$.

Warning

Now, let (3) be (4).

$$\sqrt[n]{e} \geq \sqrt[nH_n]{nH_n} \tag{4}$$

For $e = \lim_{n\longrightarrow\infty}\left(1 + \frac{1}{n}\right)^n$, (4) becomes (5).

$$\lim_{n\longrightarrow\infty}\left(1 + \frac{1}{n} \geq \sqrt[nH_n]{nH_n}\right) \tag{5}$$

For this, it can be written as (6)

$$\lim_{n\longrightarrow\infty}\left(n + 1 \geq nk\right) \tag{6}$$

where $k = \sqrt[nH_n]{nH_n}$. For $n \geq nk - 1$ it becomes $\frac{1}{n} \geq k - 1$; so what ever the direction of the inequality is, even if both sides were equal to each other, k would not become a number smaller than 1 since n is always positive. It is always $k > 1$. Here assume, that is (7)

$$n = nk - 1 + b \tag{7}$$

since it is $n \geq nk-1$ over (6), where b is a number being $b \in \mathbb{R}^+$ and thus being $b > 0$; thus it becomes (8) over (7).

$$n = \frac{b-1}{1-k} \tag{8}$$

Since the inequality is $k > 1$, then b must always be smaller number than 1 to be positive of the division; thus it becomes $1 > b > 0$; so k cannot take random values since n is positive integer. If it is $k > 1$, for the greatest value of k, it becomes $\lim_{b\to 0} k = 2$. For this value, equality of (7) becomes $n = 2n-1$ and thus becomes $n = 1$. It means, actually k decreases as long

*kavakmesut@outlook.com.tr

as n increased; thus it means it is always (9),

$$1 = \lim_{m\longrightarrow\infty} \sqrt[m]{m} \tag{9}$$

where $m \in \mathbb{Z}^+$; thus also means it is (10),

$$1 = \lim_{n\longrightarrow\infty} \sqrt[nH_n]{nH_n} \tag{10}$$

since the inequality is $H_n \geq 1$ and thus is $nH_n \geq 1$.

2 The Result: RH$^+$

By the above stated defined elements, in accordance with (6), if it is $\lim_{n\to\infty} n+1 \geq nk$, then it becomes $\lim_{n\to\infty} \frac{1}{k-1} \geq n$. Assume that it is actually $k = n-p$ where p is real number. For this, the previous inequality becomes $\lim_{n\to\infty} \frac{1}{n-p-1} \geq n$ and thus becomes $\lim_{n\to\infty} p \geq \frac{n^2-n-1}{n}$. Here assume that actually for each n, the inequality always turns into $p = \frac{n^2-n-1}{n}$ equality. For this, $k = n-p$ becomes $k = \frac{n+1}{n}$ and thus (6) becomes (11).

$$\lim_{n\longrightarrow\infty} (n+1 \geq n+1) \tag{11}$$

(11) shows us that for each n, $\lim_{n\longrightarrow\infty} n+1 \geq nk$ inequality is defined; thus the above stated assumptions and imaginary functions are also suitable. Since (11) is also equivalent of (3), also it is equivalent of (12).

$$H_n + \ln(H_n)e^{H_n} \geq \sigma(n) \tag{12}$$

> I again noticed that the solution is not completely wrong, as also is not completely false. namely if the numerator is bigger than e^{H_n}, then the equation becomes $\frac{\rho(n)}{1-\ln H_n} > e^{H_n}$ over $\ln(H_n)e^{H_n} + \rho(n) > e^{H_n}$; so $\rho(n)$ is negative for $\ln H_n > 1$. It means we must take that imaginary function as negative for the definition at the beginning as
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> $$H_n = \frac{\sigma(n) - \rho(n)}{n}$$
>
> After that the simplified inequality again becomes as the following,
>
> $$\frac{e^{H_n}}{n} \geq H_n$$
>
> where $n \geq 9$. Hence the rest is the same, and the following inequality holds for all $n \geq 9$ where $\sigma(n)$ is the sum of divisors function.
>
> $$H_n + \ln(H_n)e^{H_n} \geq \sigma(n)$$
>
> As a result, the result of RH is still RH^+. If also this is wrong, I promise I shall not work again forever about RH the calamitous problem.
>
> **Note on 10.11.2018**

Acknowledgment

I have been working about some unknown problems for a time [2] that Riemann Hypothesis is included as well, and a short time ago I supposed that I found a solution out to the Riemann Hypothesis; but I noticed that there is a stupid mistake; after that I published a brief approach; for a long time I did not work about it; but today I remembered it and just wanted to work because of boredom, and finally I could bring a simple solution out indirectly in a few hours again after midnight even if it is not so sexy and enlightening about the functions to determine relation with prime separation. The solution includes indirect and tricky definitions and operations. Even so, solution is solution always.

Good bye!

References

1. Jeffrey C. Lagarias. 2002 *An Elementary Problem Equivalent to the Riemann Hypothesis*, The American Mathematical Monthly. Vol. 109, No. 6, pp. 534-543
2. Kavak M. 2018, *Complement Inferences on Theoretical Physics and Mathematics*, OSF Preprints, Available online: https://osf.io/tw52w/