

FORTY INEDITED SEQUENCES OF INTEGERS INVOLVING PRIMES AND FERMAT PSEUDOPRIMES

(1)

Definition:

$a(n)$ is the least prime obtained concatenating, from the left to the right, q with p_2 , then with p_1 , then again with q , where p_1 and p_2 are consecutive primes having the same number of digits, $p_2 > p_1 > 43$ and q a prime, $q > 5$.

Example:

$a(1) = 23534723$ because $q = 23$ is the least prime $q > 5$ that satisfies the condition that concatenated with $p_2 = 53$ then with $p_1 = 47$ (where p_1 and p_2 are consecutive primes) then again with $q = 23$ produces a prime.

The first twenty terms of the sequence:

23534723, 41595341, 761597, 767617, 13716713, 23737123, 61797361, 11837911, 17898317, 797897, 71031017, 1110710311, 4310910743, 1711310917, 1112711311, 1113112711, 8913713189, 71391377, 1114913911, 1715114917.

Note:

I conjecture that for any pair of consecutive primes $[p_1, p_2]$, $p_2 > p_1 > 43$, p_1 and p_2 having the same number of digits, there exist a prime q , $5 < q < p_1$, such that the number n obtained concatenating (from the left to the right) q with p_2 , then with p_1 , then again with q is prime.

Reference:

Conjecture on the pairs of consecutive primes having the same number of digits involving concatenation, M. Coman.

(2)

Definition:

The terms of this sequence are the squares of primes that can be written as $(p - q - 1) * p - q - 1$ where q and p are successive primes.

Example:

The number $169 = 13^2$ is a term of this sequence because it can be written as $169 = (83 - 79 - 1) * 83 - 79 - 1$, where 79 and 83 are successive primes.

The first sixteen terms of the sequence (ordered by the size of p and q):

25, 49, 121, 169, 361, 961, 2209, 6241, 9409, 7921, 24649, 4489, 10201, 5329, 11881, 32041.

The corresponding pairs $[p, q]$ for the terms above:

[11, 7], [23, 19], [29, 23], [83, 79], [89, 83], [239, 233], [367, 359], [1559, 1553], [1567, 1559], [1979, 1973], [2053, 2039], [2243, 2239], [2549, 2543], [2663, 2659], [2969, 2963], [3203, 3191].

Note:

I conjecture that there are an infinity of primes which can be written as $\text{sqr}((p - q - 1) * p - q - 1)$, where p and q are successive primes, $p > q$.

Reference:

Squares of primes that can be written as $(p - q - 1) * p - q - 1$ where p and q are successive primes, M. Coman.

(3)

Definition:

$a(n)$ is the least prime which can be deconcatenated in three numbers, i.e., from left to right, a , b and $a + b + n$.

Example:

$a(5) = 139$ because 139 is the least prime that satisfies this condition for $n = 5$ ($1 + 3 + 5 = 9$).

The first eight terms of the sequence:

101, 113, 103, 137, 127, 139, 107, 3313.

Note:

I conjecture that for any n positive integer there exist an infinity of primes which can be deconcatenated in three numbers, i.e., from left to right, a , b and $a + b + n$.

Reference:

Conjecture on the primes obtained concatenating three numbers, id est a , b and $a + b + n$, M. Coman.

(4)

Definition:

$a(n)$, defined only for n even, is the least prime which can be deconcatenated in three numbers, i.e., from left to right, p , n and $p + n$, where p and $p + n$ are primes.

Example:

$a(12) = 51217$ because 51217 is the least prime that satisfies this condition for $n = 12$ ($5 + 12 = 17$).

The first seven terms of the sequence:

11213, 347, 11617, 5813, 31013, 51217, 51419.

Note:

I conjecture that for any n even there exist an infinity of primes which can be deconcatenated in three numbers, i.e., from left to right, p , n and $p + n$, where p and $p + n$ are primes.

Reference:

Conjecture on primes obtained concatenating p , n and $p + n$, where p and $p + n$ primes, M. Coman.

(5)

Definition:

$a(n) = (4^n - 1)/3$ where n is odd greater than 3.

The first twenty terms of the sequence:

341, 5461, 87381, 1398101, 1398101, 1398101, 5726623061, 91625968981,
1466015503701, 23456248059221, 375299968947541, 6004799503160661,
96076792050570581, 1537228672809129301, 24595658764946068821,
393530540239137101141, 6296488643826193618261, 100743818301219097892181,
1611901092819505566274901, 25790417485112089060398421.

Notes:

For n prime greater than 3 it is known that $a(n)$ it is a Poulet number, called Cipolla pseudoprime to base 2 (see the sequence A210454 in OEIS).

I conjecture that, for n odd greater than 3, $a(n)$ is divisible by a Poulet number.

Reference:

Conjecture that states that numbers $(4^n - 1)/3$ where n is odd are divisible by Poulet numbers, M. Coman.

(6)

Definition:

$a(n) = 16^n - 4^n + 1$ where n is positive integer.

The first eighteen terms of the sequence :

13, 241, 4033, 65281, 1047553, 16773121, 268419073, 4294901761, 68719214593,
1099510579201, 17592181850113, 281474959933441, 4503599560261633,
72057593769492481, 1152921503533105153, 18446744069414584321,
295147905162172956673, 4722366482800925736961.

Note:

I conjecture that $a(n)$ is either prime either divisible by a Poulet number.

Reference:

Conjecture that states that numbers $16^n - 4^n + 1$ are either primes either divisible by Poulet numbers, M. Coman.

(7)

Definition:

$a(n) = 2^n * 2^{(2^n)} - 1$ where n is larger than or equal to 3. This is a subset of Woodall numbers of the form $k * 2^k - 1$ (A003261 in OEIS).

The first six terms of the sequence :

2047, 1048575, 137438953471, 1180591620717411303423,
43556142965880123323311949751266331066367,
29642774844752946028434172162224104410437116074403984394101141506025761
187823615.

Note:

I conjecture that $a(n)$ is either prime either divisible by a Poulet number.

Reference:

Conjecture on a subset of Woodall numbers divisible by Poulet numbers, M. Coman.

(8)

Definition:

$a(n) = 2^{(n-1)} - 1$ where n is prime, $n \geq 11$, $n \neq 13$. This is a subset of Mersenne numbers of the form $2^k - 1$ (A000225 in OEIS).

The first eighteen terms of the sequence :

1023, 65535, 262143, 4194303, 268435455, 1073741823, 68719476735,
1099511627775, 4398046511103, 70368744177663, 4503599627370495,
288230376151711743, 1152921504606846975, 73786976294838206463,
1180591620717411303423, 472236648286964523695, 302231454903657293676543,
4835703278458516698824703.

Note:

I conjectured that $a(n)$ is either prime either divisible by a Poulet number.

Reference:

Conjecture on a subset of Mersenne numbers divisible by Poulet numbers, M. Coman.

(9)

Definition:

$a(n) = (4^n + 1)/5$ where n is prime greater than 5.

The first thirteen terms of the sequence :

3277, 838861, 13421773, 3435973837, 54975581389, 14073748835533,
57646075230342349, 922337203685477581, 3777893186295716170957,
967140655691703339764941, 15474250491067253436239053,
3961408125713216879677197517, 16225927682921336339157801028813.

Note:

I conjecture that any Poulet number of the form $(4^n + 1)/5$ where n is prime is either 2-Poulet number either a product of primes $p(1)*p(2)*...*p(k)$ such that all the semiprimes $p(i)*p(j)$, where $1 \leq i < j \leq k$, are 2-Poulet numbers.

Reference:

Conjecture on the Poulet numbers of the form $(4^n + 1)/5$ where n is prime, M. Coman.

(10)

Definition:

$a(n) = 4*n^2 + 8*n + 3$ where n is positive integer.

The first thirty terms of the sequence :

15, 35, 63, 99, 143, 195, 255, 323, 399, 483, 575, 675, 783, 899, 1023, 1155, 1295, 1443, 1599, 1763, 1935, 2115, 2303, 2499, 2703, 2915, 3135, 3363, 3599, 3843.

Note:

I conjecture that any term $a(n)$ is Fermat pseudoprime to base $2*n + 2$.

Reference:

Conjecture that states that numbers $4n^2 + 8^n + 3$ are Fermat pseudoprimes to base $2n + 2$, M. Coman.

(11)

Definition:

The terms of this sequence are the Poulet numbers which can be written as $P*2 - d$, where P is another Poulet number and d one of the prime factors of P .

Example:

The number 1105 is a term of this sequence because it can be written as $1105 = 561*2 - 17$ and 561, 1105 are Poulet numbers and 17 a prime factor of 561.

The first thirteen terms of the sequence :

1105, 2701, 7957, 8481, 15841, 16705, 31609, 33153, 46657, 62745, 129889, 181901, 323713.

The corresponding Poulet numbers for the terms above:

561, 1387, 4033, 4369, 7957, 8481, 15841, 16705, 23377, 31417, 65281, 91001, 162193.

Note:

I conjecture that this sequence is infinite.

Reference:

On the recurrence $((((P*2-d)*2-d)*2-d)...) on Poulet numbers P having a prime factor d , M. Coman.$

(12)

Definition:

The terms of this sequence are the Poulet numbers which can be written as $(P*2 - d)*2 - d$, where P is another Poulet number and d one of the prime factors of P .

Example:

The number 4369 is a term of this sequence because it can be written as $4369 = (1105*2 - 17)*2 - 17$ and 1105, 4369 are Poulet numbers and 17 a prime factor of 1105.

The first seventeen terms of the sequence :

4369, 10585, 16705, 31609, 33153, 60787, 126217, 164737, 196093, 241001, 256999, 271951, 318361, 452051, 481573, 486737, 745889.

The corresponding Poulet numbers for the terms above:

1105, 2701, 4369, 7957, 8481, 15709, 31609, 41665, 49141, 60701, 65077, 68101, 80581, 113201, 121465, 123251, 188057.

Note:

I conjecture that this sequence is infinite.

Reference:

On the recurrence $((((P*2-d)*2-d)*2-d)...)$ on Poulet numbers P having a prime factor d, M. Coman.

(13)

Definition:

The terms of this sequence are the Poulet numbers which can be written as a sum of two successive primes p and q plus one.

The first twenty terms of the sequence :

341, 1105, 4681, 5461, 6601, 7957, 11305, 13741, 14491, 18721, 23001, 39865, 42799, 63973, 65281, 72885, 88561, 91001, 101101, 107185.

The corresponding pairs [p, q] for the terms above:

[167, 173], [547, 557], [2339, 2341], [2729, 2731], [3299, 3301], [3967, 3989], [5651, 5653], [6869, 6871], [7243, 7247], [9349, 9371], [11497, 11503], [19927, 19937], [21397, 21403], [31981, 31991], [32633, 32647], [36433, 36451], [44279, 44281], [45497, 45503], [50549, 50551], [53591, 53593].

Note:

I conjecture that this sequence is infinite.

Twenty from the first 81 Poulet numbers can be written this way.

See the sequence A001043 in OEIS for numbers that are the sum of two successive primes.

Reference:

Poulet numbers which can be written as a sum of two successive primes plus one, M. Coman.

(14)

Definition:

The terms of this sequence are the Poulet numbers P for which the number $q = (P - 1)/2^n - 2^n$ is prime, where n is the integer for which the number $(P - 1)/2^n$ is odd.

The first eighteen terms of the sequence :

561, 645, 1105, 1387, 1905, 2047, 2821, 4369, 4681, 5461, 8481, 13747, 14491, 15709, 15841, 16705, 18705, 19951.

The corresponding pairs $[q, n]$ for the terms above:

[19, 4], [157, 2], [53, 4], [691, 1], [103, 4], [1021, 1], [701, 2], [257, 4], [577, 3], [1361, 2], [233, 5]. [6871, 1], [7243, 1], [3923, 2], [463, 5], [197, 6], [153, 4], [9973, 1].

Note:

I conjecture that this sequence has an infinite number of terms.

Reference:

Conjecture on Poulet numbers of the form $(q + 2^n) \cdot 2^n + 1$ where q prime, M. Coman.

(15)

Definition:

The terms of this sequence are the Poulet numbers which admit a deconcatenation in two prime numbers p and q where $q = p + 30 \cdot k$, where k integer.

Examples:

The number 4917331 is a term of this sequence because can be deconcatenated in two primes, 491 and 7331, and $7331 = 491 + 228 \cdot 30$; also 6617929 because can be deconcatenated in two primes, 66179 and 29, and $29 = 66179 - 2205 \cdot 30$.

The first twenty terms of the sequence :

13747, 49141, 101101, 294409, 401401, 711361, 1052929, 1141141, 1373653, 1472353, 1730977, 3581761, 4917331, 6617929, 6779137, 9371251, 11157721, 15139199, 16349477, 16435747.

The corresponding $[p, q, k]$ for the terms above:

[137, 47, -3], [491, 41, -15], [101, 101, 0], [29, 4409, 146], [401, 401, 0], [71, 1361, 43], [10529, 29, -350], [11, 41141, 457], [1373, 653, -24], [14723, 53, -489], [17, 30977, 1032], [3581, 761, -94], [491, 7331, 228], [66179, 29, -2205], [677, 9137, 282], [9371, 251, -304], [11, 157721, 5257], [15139, 199, -498], [1634947, 7, -54498], [164357, 47, -5477].

Note:

I conjecture that this sequence is infinite.

Reference:

Poulet numbers obtained concatenating two primes p and $p \pm 30k$, M. Coman.

(16)

Definition:

The terms of this sequence are the primes obtained concatenating four consecutive numbers, the largest one from them being a Poulet number.

Example:

The number 1726172717281729 is such a prime, obtained concatenating the numbers 1726, 1727, 1728 and 1729, where 1729 is a Poulet number.

The first ten terms of the sequence :

1726172717281729, 2044204520462047, 2818281928292821, 4678467946804681,
8318831983208321, 13978139791398013981, 15706157071570815709,
15838158391584015841, 19948199491995019951, 30118301193012030121.

Note:

I conjecture that this sequence is infinite.

Reference:

Primes obtained concatenating four consecutive numbers, the largest one being a Poulet number, M. Coman.

(17)

Definition:

The terms of this sequence are the Poulet numbers which can be written as $x^3 + y^3$.

The first fifteen terms of the sequence :

341, 1729, 10261, 15841, 46657, 126217, 188461, 228241, 617093, 688213, 1082809,
1157689, 1773289, 2628073.

The corresponding [x, y] for the terms above:

[5, 6], [1, 12] and [9, 10], [10, 21], [6, 25], [1, 36], [25, 48], [45, 46], [48, 49], [29, 84],
[42, 85], [81, 82], [4, 105], [12, 121], [1, 138], [40, 141].

Notes:

I conjecture that this sequence is infinite.

The numbers 341, 1729, 188461, 228241, 1082809 are centered cube numbers (equal to $2*n^3 + 3*n^2 + 3*n + 1$, see the sequence A005898 in OEIS). I conjecture that there are infinite Poulet numbers which are also centered cube numbers.

I also conjecture that there are infinite Poulet numbers of the form $n^3 + 1$.

Reference:

Poulet numbers which can be written as $x^3 \pm y^3$, M. Coman.

(18)

Definition:

The terms of this sequence are the Poulet numbers which can be written as $x^3 - y^3$.

The first twenty terms of the sequence :

1387, 4681, 7957, 8911, 13741, 14491, 63973, 93961, 115921, 126217, 172081, 341497, 488881, 748657, 873181, 1397419, 2113921, 2455921, 2628073, 2867221.

The corresponding [x, y] for the terms above:

[22, 21], [40, 39], [52, 51], [24, 17] and [55, 54], [29, 22], [70, 69], [40, 3], [46, 15], [49, 12], [81, 74], [94, 87] and [240, 239], [100, 87], [84, 47], [145, 132], [540, 539], [683, 682], [129, 32] and [166, 135] and [202, 183], [145, 84] and [217, 198], [144, 71] and [172, 135], [373, 366].

Notes:

I conjecture that this sequence is infinite.

The numbers 1387, 4681, 7957, 8911, 14491, 172081, 873181, 1397419 are centered hexagonal numbers (equal to $3*n^2 + 3*n + 1$, see the sequence A003215 in OEIS). I conjecture that there are infinite Poulet numbers which are also centered hexagonal numbers.

Reference:

Poulet numbers which can be written as $x^3 \pm y^3$, M. Coman.

(19)

Definition:

The terms of this sequence are the Poulet numbers of the form $1860*n + 961$.

The first twelve terms of the sequence :

2821, 4681, 10261, 13981, 15841, 75361, 93961, 172081, 285541, 399001, 512461, 625921.

The corresponding n for the terms above:

1, 2, 5, 7, 8, 40, 50, 92, 153, 214, 275, 336.

Note:

I came to this formula in the following way: let d be a factor (not necessarily prime) of the Poulet number P such that $d < \sqrt{P}$ and m the least number such that $m*d*(d - 1) > (P - 1)/2$; let n be equal to $P - m*d*(d - 1)$; then often exist a set of Poulet numbers Q such that $Q \bmod(m*d*(d - 1)) = n$. For example, for $P = 2047 = 23*89$ and $d = 23$, where $d < \sqrt{2047}$, the least m such that $m*23*22 > (P - 1)/2$ is equal to 3 ($1518 > 1023$, while, for 2, $1012 < 1023$); so, $n = 2047 - 3*23*22 = 2047 - 1518 = 529$ and indeed there exist a set of Poulet numbers Q such that $Q \bmod 1518 = 529$; the formula $1518*x + 529$ gives the Poulet numbers 2047, 6601, 15709, 30889 (...) for $x = 1, 4, 10, 20$ (...).

Reference:

Formula to generate a set of Poulet numbers from a Poulet number P and its factor d lesser than \sqrt{P} , M. Coman.

(20)

Definition:

The set of primes which are the sum of three consecutive Poulet numbers.

The first fifteen terms of the sequence :

2311, 3137, 5021, 7213, 13421, 27653, 37847, 40289, 61673, 72139, 78479, 85223, 99719, 116239, 178909 (...)

The corresponding triplets of Poulet numbers for the terms above:

[561, 645, 1105], [645, 1105, 1387], [1387, 1729, 1905], [2047, 2465, 2701, 4369, 4371, 4681], [8481, 8911, 10261], [11305, 12801, 13741], [12801, 13741, 13747], [18721, 19951, 23001], [23001, 23377, 25761, 23377, 25761, 29341, 25761, 29341, 30121, 31621, 33153, 34945, 35333, 39865, 41041], [57421, 60701, 60787].

Note:

I conjecture that this sequence is infinite.

Reference:

Three sequences of primes obtained from Poulet numbers, M. Coman.

(21)

Definition:

The set of the primes which are partial sums of the sequence of Poulet numbers.

The first fifteen terms of the sequence :

7673, 17707, 33757, 270763, 484621, 615949, 691147, 863309, 962431, 1070309, 2576293, 4260049, 5542423, 5900473.

The Poulet numbers up to which the terms above are partial sums:

1905, 2821, 4371, 18721, 31417, 34945, 39865, 46657, 49981, 55245, 91001, 149281, 172081, 181901.

Note:

I conjecture that this sequence is infinite.

Reference:

Three sequences of primes obtained from Poulet numbers, M. Coman.

(22)

Definition:

The set of the primes which are obtained concatenating four consecutive 2-Poulet numbers.

The first ten terms of the sequence :

341138720472701, 795783211026113747, 14491157091872119951,
31417316093162135333, 104653123251129889130561, 220729226801233017241001,
458989481573486737489997, 657901665281665333672487,
665281665333672487679729, 688213710533721801722201.

Notes:

I conjecture that this sequence is infinite.

Ten such primes are obtained using just the first hundred of 2-Poulet numbers (from 341 to 722201).

Reference:

Three sequences of primes obtained from Poulet numbers, M. Coman.

(23)

Definition:

The set of the primes which are obtained concatenating to the left a prime with its digital sum.

The first seventeen terms of the sequence :

211, 1019, 523, 1129, 431, 541, 743, 853, 1459, 761, 1367, 1789, 1697, 5113, 14149,
7151, 10163.

Note:

I conjecture that this sequence is infinite.

Reference:

Three sequences of primes obtained using the digital root and the digital sum of a prime, M. Coman.

(24)

Definition:

The set of the primes which are obtained concatenating to the left a prime with its digital root.

The first twenty terms of the sequence :

211, 523, 229, 431, 137, 541, 743, 853, 761, 467, 173, 283, 797, 1109, 5113, 2137, 4139,
7151, 4157, 1163.

Note:

I conjecture that this sequence is infinite.

Reference:

Three sequences of primes obtained using the digital root and the digital sum of a prime, M. Coman.

(25)

Definition:

The set of the primes which are equal to the sum of a prime p with the number $s(p)/p$ obtained concatenating to the left p with its digital sum and the number $r(p)/p$ obtained concatenating to the left p with its digital root.

The first fifteen terms of the sequence :

433, 839, 1069, 1123, 1759, 1583, 1901, 1319, 1549, 2767, 2591, 17417, 19447, 17471, 11489.

The corresponding triplets $[p, s(p), r(p)]$ for the terms above:

[11, 211, 211], [13, 413, 413], [23, 523, 523], [41, 541, 541], [53, 853, 853], [61, 761, 761], [67, 1367, 467], [73, 1073, 173], [83, 1183, 283], [89, 1789, 889], [97, 1697, 797], [139, 13139, 4139], [149, 14149, 5149], [157, 13157, 4157], [163, 10163, 1163].

Note:

I conjecture that this sequence is infinite.

Reference:

Three sequences of primes obtained using the digital root and the digital sum of a prime, M. Coman.

(26)

Definition:

The sequence of primes obtained concatenating a Poulet number P to the left with $(s(P) - 1)/6$ where $s(P)$ is the digits sum of P :

The first thirteen terms of the sequence :

31387, 31729, 314491, 130121, 331609, 352633, 357421, 465077, 65077, 3115921, 3196021, 3228241, 6275887, 3334153.

The corresponding digits sum s for the terms above:

19, 19, 19, 7, 19, 19, 19, 25, 19, 19, 19, 37, 19.

Note:

I conjecture that this sequence is infinite.

Reference:

Primes obtained concatenating a Poulet number P with $(s - 1)/n$ where s digits sum of P and n is 2, 3 or 6, M. Coman.

(27)

Definition:

The terms of this sequence are the least primes obtained concatenating each prime p , $p \neq 5$, p having an odd prime digit sum s , to the left with a divisor of $s - 1$ (including 1 and $s - 1$), or, in case that such a prime doesn't exist, the corresponding term in the sequence is 0.

The first thirty terms of the sequence :

13, 17, 223, 229, 241, 643, 547, 661, 167, 0, 2113, 2131, 2137, 4139, 1151, 4157, 0, 2179, 0, 1193, 0, 6199, 1223, 5227, 1229, 0, 0, 2269, 2281, 1283.

The corresponding primes p for the terms above:

3, 7, 23, 29, 41, 43, 47, 61, 67, 89, 113, 131, 137, 139, 151, 157, 173, 179, 191, 193, 197, 199, 223, 227, 229, 241, 263, 269, 281, 283.

Note:

I conjecture that this sequence has an infinity of terms not equal to 0 but also an infinity of terms equal to 0.

Reference:

Primes obtained concatenating to the left a prime having an odd prime digit sum s with a divisor of $s - 1$, M. Coman.

(28)

Definition:

The terms of this sequence are the least primes obtained concatenating each prime p , $p \neq 5$, p having an odd prime digit sum s , to the left with numbers $n \cdot (s - 1)$.

The first thirty terms of the sequence :

23, 67, 823, 2029, 2441, 643, 6047, 661, 2467, 2083, 4889, 12113, 36131, 30137, 60139, 6151, 12157, 20173, 48179, 80191, 48179, 80191, 48193, 48197, 18199, 18223, 50227, 24229, 12241, 50263.

The corresponding values of $[p, s, n]$ for the terms above:

[3, 3, 1], [7, 7, 1], [23, 5, 2], [29, 11, 2], [41, 5, 6], [43, 7, 1], [47, 11, 6], [61, 7, 1], [67, 13, 2], [83, 11, 2], [89, 17, 3], [113, 5, 3], [131, 5, 9], [137, 11, 3], [139, 13, 5], [151, 7, 1], [157, 13, 1], [173, 11, 2], [179, 17, 3], [191, 11, 8], [193, 13, 4], [197, 17, 3], [199, 19, 1], [223, 7, 3], [227, 11, 5], [229, 13, 2], [241, 7, 2], [263, 11, 5].

Notes:

I conjecture that for any prime p , $p \neq 5$, having an odd prime digit sum s there exist an infinity of primes obtained concatenating to the left p with multiples of $s - 1$.

I conjecture that for any prime p , $p \neq 5$, having an odd prime digit sum s there exist at least a prime obtained concatenating to the left p with the number $n \cdot (s - 1)$ such that $n < \sqrt{p}$.

Reference:

Primes obtained concatenating to the left a prime having an odd prime digit sum s with a multiple of $s - 1$, M. Coman.

(29)

Definition:

The terms of this sequence are the least primes obtained inserting a number with digit sum equal to 12 after the first digit of a prime p , $p \geq 5$ or, in case that such a prime doesn't exist, the corresponding term for p in the sequence is 0.

The first fifty terms of the sequence :

557, 397, 1481, 1483, 1487, 1399, 2393, 2399, 3391, 3847, 4391, 4483, 4397, 5393, 5399, 6481, 6397, 7481, 7393, 7489, 8573, 8669, 9397, 13901, 13903, 13907, 17509, 13913, 14827, 13931, 113837, 15739, 15749, 14851, 16657, 13963, 13967, 15773, 14879, 17581, 14891, 16693, 13997, 13999, 23911, 212923, 26627, 23929, 25733, 27539, 24841, 24851.

The corresponding values of $[p, n]$ for the terms above:

[5, 57], [7, 39], [11, 48], [13, 48], [17, 48], [19, 39], [23, 39], [29, 39], [31, 39], [37, 84], [41, 39], [43, 48], [47, 39], [53, 39], [59, 39], [61, 48], [67, 39], [71, 48], [73, 39], [79, 48], [83, 57], [89, 66], [97, 39], [101, 39], [103, 39], [107, 39], [109, 75], [113, 39], [127, 48], [131, 39], [137, 138], [139, 57], [149, 57], [151, 48], [157, 66], [163, 39], [167, 39], [173, 57], [179, 48], [181, 75], [191, 48], [193, 66], [197, 39], [199, 39], [211, 39], [223, 129], [227, 66], [229, 39], [233, 57], [239, 75], [241, 48], [251, 48].

Note:

I conjecture that for any prime p , $p \geq 5$, there exist a prime q obtained inserting a number n with the sum of digits equal to 12 after the first digit of p (in other words, that the sequence above doesn't have any term equal to zero).

Reference:

A recreative conjecture on primes obtained inserting n with digit sum 12 after the first digit of a prime, M. Coman.

(30)

Definition:

The terms of this sequence are the least primes obtained inserting a number with digit sum equal to 12 before the last digit of a prime p , $p \geq 7$ or, in case that such a prime doesn't exist, the corresponding term for p in the sequence is 0.

The first fifty terms of the sequence :

397, 1481, 1483, 1487, 1399, 2393, 2399, 3391, 3847, 4391, 4483, 4397, 5393, 5399, 6481, 6397, 7481, 7393, 7489, 8573, 8669, 9397, 10391, 10663, 10487, 10399, 11393,

12487, 13751, 13397, 13399, 14489, 15391, 15667, 16573, 16487, 17393, 17489, 18481, 19391, 19483, 19577, 19489, 21391, 22483, 22397, 22669, 23663, 23399, 24391, 25391, 25577.

The corresponding values of $[p, n]$ for the terms above:

[7, 39], [11, 48], [13, 48], [17, 48], [19, 39], [23, 39], [29, 39], [31, 39], [37, 84], [41, 39], [43, 48], [47, 39], [53, 39], [59, 39], [61, 48], [67, 39], [71, 48], [73, 39], [79, 48], [83, 57], [89, 66], [97, 39], [101, 39], [103, 66], [107, 48], [109, 39], [113, 39], [127, 48], [131, 75], [137, 39], [139, 39], [149, 48], [151, 39], [157, 66], [163, 57], [167, 48], [173, 39], [179, 48], [181, 48], [191, 39], [193, 48], [197, 57], [199, 48], [211, 39], [223, 48], [227, 39], [229, 66], [233, 66], [239, 39], [241, 39], [251, 39], [257, 57].

Note:

I conjecture that for any prime p , $p \geq 7$, there exist a prime q obtained inserting a number n with the sum of digits equal to 12 before the last digit of p (in other words, that the sequence above doesn't have any term equal to zero).

Reference:

A recreative conjecture on primes obtained inserting n with digit sum 12 before the last digit of a prime, M. Coman.

(31)

Definition:

The terms of this sequence are the least primes obtained from the lesser term of a pair of twin primes $[p, q]$, $p \geq 11$, inserting a number n having the digit sum 12 after the first digit of p , such that the number obtained inserting n after the first digit of $q = p + 2$ is also prime or, in case that such a prime doesn't exist, the corresponding term for p in the sequence is 0.

The first forty terms of the sequence :

1481, 1487, 2399, 4481, 5669, 71471, 13901, 125507, 15737, 16649, 17579, 16691, 13997, 27527, 27539, 216569, 26681, 334511, 34847, 425519, 49331, 43961, 521821, 517469, 55799, 65717, 614741, 631859, 84809, 815621, 86627, 84857, 822881, 1291019, 1156031, 1157049, 148061, 1174091, 148151, 157229.

The corresponding values of $[p, q, n]$ for the terms above:

[11, 13, 48], [17, 19, 48], [29, 31, 39], [41, 43, 48], [59, 61, 66], [71, 73, 147], [101, 103, 39], [107, 109, 255], [137, 139, 57], [149, 151, 66], [191, 193, 66], [197, 199, 39], [227, 229, 75], [239, 241, 75], [269, 271, 165], [281, 283, 66], [311, 313, 345], [347, 349, 48], [419, 421, 255], [431, 433, 93], [461, 463, 39], [521, 523, 218], [569, 571, 174], [599, 601, 57], [617, 619, 57], [641, 643, 147], [659, 661, 318], [809, 811, 48], [821, 823, 156], [827, 829, 66], [857, 859, 48], [881, 883, 228], [1019, 1021, 291], [1031, 1033, 156], [1049, 1051, 57], [1061, 1063, 48], [1091, 1093, 174], [1151, 1153, 48], [1229, 1231, 57].

Note:

I conjecture that for any pair of twin primes $[p, q]$, $p \geq 11$, there exist a number n having the sum of its digits equal to 12 such that inserting n after the first digit of p respectively q are obtained two primes (in other words, that the sequence above doesn't have any term equal to zero).

Reference:

Conjecture on the pairs of primes obtained inserting n with digit sum 12 after the first digit of twin primes, M. Coman.

(32)

Definition:

$a(n)$ is the least prime obtained inserting a power of 3 before a digit of $a(n-1)$ starting from $a(0) = 7$.

Examples:

$a(1) = 37$ because $37 < 97$ even both 3 and 9 are powers of 3 and both 37 and 97 are primes; $a(2) = 337$ because $337 < 397$; $a(3) = 9337$ because $9337 < 27337$ even both 9 and 27 are powers of 3.

The first twenty terms of the sequence:

7, 37, 337, 9337, 93337, 933397, 9333397, 39333397, 393393397, 39339332797, 339339332797, 3393393392797, 33933393392797, 339333933927397, 3393339339327397, 33393339339327397, 333933393393237397, 3339333933933237397, 33393339339332373937, 33393339339332373937.

Notes:

I conjecture that always is obtained a prime $a(n)$ by the method above from a prime $a(n-1)$ so this sequence is infinite.

I conjecture that always is obtained a prime $a(n)$ by the method above from any prime $a(0)$ greater than 5.

Reference:

A recreative method to obtain from a given prime larger primes based on the powers of 3, M. Coman.

(33)

Definition:

The terms of this sequence are the primes obtained concatenating $30 - d(1)$, $30 - d(2)$, ..., $30 - d(n)$, where $d(1)$, ..., $d(n)$ are the digits of a Poulet number.

Example:

For Poulet number 8911 the number obtained concatenating $22 = 30 - 8$ with $21 = 30 - 9$ with $29 = 30 - 1$ with $29 = 30 - 1$, i.e. the number 22212929, is prime, so is a term in this sequence.

The first fifteen terms of the sequence:

27282323, 23212523, 22272829, 22212929, 2729242829, 2629302629, 2628232121,
293026242527, 292621282229, 292530222529, 282925232621, 282629303029,
282126263021, 272124282329, 242330302727.

The Poulet numbers corresponding to the primes above:

3277, 7957, 8321, 8911, 31621, 41041, 42799, 104653, 149281, 150851, 215749,
241001, 294409, 396271, 670033.

Note:

I conjecture that this sequence is infinite.

Reference:

Primes obtained concatenating the numbers $30 - d(n)$ where $d(1), \dots, d(n)$ are the digits of a Poulet number, M. Coman.

(34)

Definition:

The terms of this sequence are the primes obtained concatenating $30 - d(1)$, $30 - d(2)$, ..., $30 - d(k)$, where $d(1), \dots, d(k)$ are the digits of a square of a prime.

Example:

For 1369 ($= 37^2$) the number obtained concatenating $29 = 30 - 1$ with $27 = 30 - 3$ with $24 = 30 - 6$ with $21 = 30 - 9$, i.e. the number 29272421, is prime., so is a term in this sequence.

The first twenty terms of the sequence:

2621, 282221, 29272421, 28223021, 26262221, 2928232421, 2924292821, 2721243029,
2626252829, 2526282221, 2421292421, 2327262629, 2324232821, 292721292821,
292627242629, 292124282621, 282629302229, 282527303021, 282327252821,
272824302629.

The squares of primes corresponding to the primes above:

49 ($= 7^2$), 289 ($= 17^2$), 1369 ($= 37^2$), 2809 ($= 53^2$), 4489 ($= 67^2$), 12769 ($= 113^2$),
16129 ($= 127^2$), 39601 ($= 199^2$), 44521 ($= 211^2$), 54289 ($= 233^2$), 69169 ($= 263^2$),
73441 ($= 271^2$), 76729 ($= 277^2$), 139129 ($= 373^2$), 143641 ($= 379^2$), 196249 ($= 443^2$),
241081 ($= 491^2$), 253009 ($= 503^2$), 273529 ($= 523^2$), 326041 ($= 571^2$).

Note:

I conjecture that this sequence is infinite.

Reference:

Primes obtained concatenating the numbers $30 - d(k)$ where $d(1), \dots, d(k)$ are the digits of a square of a prime, M. Coman.

(35)

Definition:

The terms of this sequence are the primes obtained concatenating the numbers $P - d(1)$; $P - d(2)$; ...; $P - d(k)$; P , where $d(1) < d(2) < \dots < d(k)$ are the prime factors of a Poulet number P , ordered by the size of P .

Example:

Using the sign “//” with the meaning “concatenated to”, for the Poulet number 129921 (= $3 \cdot 11 \cdot 31 \cdot 127$), the number $(129921 - 3)//(129921 - 11)//(129921 - 31)//(129921 - 127)//129921 = 129918129910129890129794129921$ is prime, so is a term in this sequence.

The first ten terms of the sequence:

558550544561, 266426282701, 2814280827902821, 324831643277, 10230993010261, 198801967019951, 805207926080581, 87246871228702087249, 104424104196104653, 129918129910129890129794129921.

The Poulet numbers corresponding to the primes above:

561, 2701, 2821, 3277, 10261, 19951, 80581, 87249, 104653, 129921.

Note:

I conjecture that this sequence is infinite.

Reference:

Large primes obtained concatenating the numbers $P - d(k)$ where $d(k)$ are the prime factors of the Poulet number P , M. Coman.

(36)

Definition:

The terms of this sequence are the least primes n obtained from the primes p of the form $6k + 1$ through the following operation: $n = (p \cdot q - p)//(p \cdot q - q)//p \cdot q$, where q is a prime of the form $6h + 1$ or, in case that such a prime doesn't exist, the corresponding term for n in the sequence is 0 (the sign “//” is used with the meaning “concatenated to”).

The first ten terms of the sequence :

210186217, 390372403, 114010981159, 558570589, 666684703, 283828142881, 732780793, 201020462077, 131413681387, 142214821501.

The corresponding values of $[p, q]$ for the terms above:

[7, 31], [13, 31], [19, 61], [31, 19], [37, 19], [43, 67], [61, 13], [67, 31], [73, 19], [79, 19].

Note:

I conjecture that for any prime p of the form $6k + 1$ there exist an infinity of primes q of the form $6h + 1$ such that the number n obtained concatenating $p \cdot q - p$ with $p \cdot q - q$ then with $p \cdot q$ is prime (in other words, that the sequence above doesn't have any term equal to zero).

Reference:

Primes obtained concatenating $p^*q - p$ with $p^*q - q$ then with p^*q where p, q primes of the form $6k + 1$, M. Coman.

(37)

Definition:

The terms of this sequence are the least primes n obtained from the primes p of the form $6^*k + 1$, $p > 7$, through the following operation: $n = (7^*p^*q - 7)/(7^*p^*q - p)/(7^*p^*q - q)/7^*p^*q$, where q is a prime of the form $6^*h + 1$ or, in case that such a prime doesn't exist, the corresponding term for n in the sequence is 0 (the sign “//” is used with the meaning “concatenated to”).

The first ten terms of the sequence :

2814280827902821, 13692136801359613699, 13230132301317613237,
35994359643586236001, 3906387039003913, 15792157381576215799,
45486454264539645493, 101682101616101490101689, 71827110711071767189,
102522102432102378102529.

The corresponding values of $[p, q]$ for the terms above:

[13, 31], [19, 103], [31, 61], [37, 139], [43, 13], [61, 37], [67, 97], [73, 199], [79, 13], [97, 151].

Note:

I conjecture that for any prime p of the form $6k + 1$ there exist an infinity of pairs of primes $[q, r]$, both of the form $6h + 1$, such that the number n obtained concatenating $p^*q^*r - p$ with $p^*q^*r - q$ with $p^*q^*r - r$ then with p^*q^*r is prime (in other words, that the sequence above doesn't have any term equal to zero).

Reference:

Primes obtained concatenating $p^*q^*r - p$ with $p^*q^*r - q$ with $p^*q^*r - r$ then with p^*q^*r where p, q, r primes of the form $6k + 1$, M. Coman.

(38)

Definition:

The terms of this sequence are the Poulet numbers P having the property that the number $(P - 1)/(s(P) - 1)$, where $s(P)$ is the sum of digits of P , is an integer.

The first twenty-five terms of the sequence :

645, 1105, 1387, 1729, 1905, 2465, 2701, 2821, 3277, 4033, 4369, 4681, 5461, 6601, 8321, 8481, 8981, 10261, 10585, 11305, 13741, 14491, 15709, 15841.

Note:

This property is shared by 30 from the first 40 Poulet numbers.

Reference:

Conjecture that there exist an infinity of Poulet numbers which are also Harshad-Coman numbers, M. Coman.

(39)

Definition:

The terms of this sequence are the Poulet numbers having the property that are also Harshad numbers.

The first ten terms of the sequence :

645, 1387, 1729, 1905, 2465, 2821, 8911, 30121, 126217, 204001.

Notes:

I conjecture that this sequence is infinite.

I conjecture that if P is both a Poulet number and a Harshad number, then the number $P - 1$ is also a Harshad number.

Reference:

Three conjectures regarding Poulet numbers and Harshad numbers, M. Coman.

(40)

Definition:

The terms of this sequence are the least numbers n for which $s(p \cdot 2^n)$ is divisible by p , where p is prime, $p > 5$, and $s(p \cdot 2^n)$ is the sum of digits of $p \cdot 2^n$ or, in case that such a number n doesn't exist for a certain prime p , the corresponding term in the sequence is 0.

The first twenty-two terms of the sequence :

14, 6, 6, 6, 19, 12, 12, 12, 149, 30, 37, 24, 24, 24, 194, 97, 54, 97, 54, 54, 54, 54.

The corresponding values of p for the terms above:

7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Notes:

I conjecture that for any prime p , $p > 5$, there exist n positive integer such that the sum of the digits of the number $p \cdot 2^n$ is divisible by p (in other words, that the sequence above doesn't have any term equal to zero).

Reference:

Two conjectures involving Harshad numbers, primes and powers of 2, M. Coman.