

The physical nature of the basic concepts of physics

5. Pressure, Temperature and Thermal Energy

Guido. F. Nelissen

guido.nelissen@gmail.com

Abstract

The kinetic theory defines the temperature of an ideal monatomic gas as a measure for the average translational kinetic energy of its particles. This definition ignores the fact that temperature is inevitably characterized by an isotropic distribution of the velocities of the particles over all possible directions and the continuous collisions and thermal radiation that this brings about.

In this paper I first demonstrate that ‘the thermal energy’ of an ideal gas is in fact a mathematical expression of the total amount of isotropic momentum flow.

This allows me to conclude that the pressure in an ideal gas is a measure for the average two-sided momentum flow across any unit area of the particle system and that the temperature of an ideal gas is a measure for the average two-sided momentum flow across any unit area of that gas, for a unit number density of its molecules, so that the temperature is in fact the pressure for a unit number density.

In that way I am able to demonstrate that the Maxwell-Boltzmann speed distribution function is not the fundamental characteristic of thermal motion, but that it is a result of the all-sided collisions that are typical for systems consisting of elastic particles with isotropic motion.

1. The incompleteness of the present thermal concepts

1.1 The present concept of ‘heat’

Temperature is classically defined as an indicator of the direction in which “heat” flows when two bodies at different temperatures are brought into contact with each other ^[1], or as a measure for the amount of “heat” that is absorbed by a reversible engine working between the temperature of an object and the unit temperature ^[2], etc.

All the classic definitions of “temperature” have in common that:

- they define the “temperature” of a body in function of the “temperature” of another body. This means that the definition of temperature is intrinsically based on a circular reasoning: one must already know what “temperature” is, in order to understand its definition.
- they introduce the biased notion of the flow of “heat”, in which “heat” is defined as the kinetic ‘energy’ of the motion of the molecules.

The inseparable relationship between “temperature” and “energy” finds its historical grounds in the beginning of the nineteenth century, when it was postulated that a material substance named “caloric” existed in every body ^[3]: *“It was believed that a body at high temperature contained more “caloric” than one at a low temperature. When the two bodies were put together, the body rich in “caloric” lost some to the other, until both bodies reached the same temperature.”*

The concept of “heat” as a physical entity called “caloric” could however not stand the test of experiment and it has been replaced by the concept of ‘energy’. So today we still describe temperature changes as the transfer of ‘something’ but now we don’t call it ‘caloric’ anymore but we call it ‘energy’ and we conclude that a body at high temperature contains more of this ‘energy’ than one at low temperature and when the two bodies are put together, ‘energy’ is transferred from the hotter to the colder body.

It must be clear that in this way the problem is not really solved because in fact the present textbooks don’t really tell us what ‘energy’ is. The progress that has been made by the use of ‘kinetic energy’ instead of caloric is that temperature is now defined mathematically, as a property of the molecules that is related to their speed (i). In his paper “The long arm of the second law”^[4] Miguel Rubi of the university of Barcelona writes that thermodynamics is one of the most widely misunderstood branches of physics: *“laypeople and scientist alike regularly use concepts such as ‘temperature’, ‘pressure’ and ‘energy’ without knowing their rigorous meaning and subtleties”*.

All these elements oblige us to formulate a new, more accurate definition of ‘pressure’ and ‘temperature’ for ideal gases. To do this, we first have to take a closer look at the physical meaning of these fundamental characteristics of physics.

1.2 The present definition of pressure

In section 5 “Repetitive impulsive forces” of my paper “Part 2: The true nature of force”, I have demonstrated that the ‘force’ exerted by a gas on an external wall of the box in which it is enclosed, is equal to momentum flow to that wall, which is expressed as the momentum transfer per impulse (Δp_1) times the impulse frequency (v):

$$\mathbf{F} = \Delta Q_p = \Delta p_1 \cdot v$$

In the textbooks of physics, the classic case is given of an ideal monatomic gas that is enclosed in a cubic box with perfectly elastic walls with size “L” (and area $A = L^2$) and of which the average RMS-speed of the molecules (their speed relative to the center of mass of the gas) is “ v_{RMS} ”, so that the average internal linear momentum of each molecule is equal to “ $m v_{RMS}$ ” (ii).

When a particle hits one of the elastic walls, its velocity and consequently its momentum is reversed, so that the momentum transfer to the wall at each collision is equal to $2mv$ (iii).

When there is no preferential direction, the velocities of the molecules are isotropically distributed over all possible directions so that averaged over time, one sixth of the total number of molecules ($N/6$) will move in the direction of each wall, and the frequency of the collisions with each wall is equal to: $v = Nv/6L$

In section 5.2.1. of my paper “Part 2 The true nature of force”, I have demonstrated that in the case of collisions with an elastic wall, the momentum change for a unit mass particle ‘ m_1 ’ is equal to $\Delta p_1 = 2m_1v$, so that the force on each wall of the box is:

$$F = \Delta p_1 \cdot v = (2m_1v) \cdot v = (2m_1v)(Nv/6L) = Nm_1v^2/3L = mv^2/3L = mA v^2/3V = \rho A v^2/3$$

in which “ $\rho = m/V$ ” is the mass density of the gas.

(i) See my paper “The true nature of work, kinetic energy and Planck’s constant”.

(ii) For reasons of facility, I will in this paper designate the RMS-speed as the (common) speed ‘ v ’ of particles of an ideal gas.

(iii) See section 5.2.1 of my paper “Part 2 The physical nature of force”.

This leads the kinetic equation of the pressure on each wall:

$$p = \Delta p_{1.v}/A = Nm_1v^2/3V = mv^2/3V = \rho v^2/3 \quad (\text{which is expressed in Pa} = \text{N/m}^2)$$

According to Newton's third law of motion, the pressure exerted by the gas molecules on the wall is equal but opposite to the pressure exerted by the wall on the molecules so that any object inserted at any point of the gas volume, will be will experience an all sided pressure which is equal to the pressure on the surrounding external walls (Pascal's principle).

In this way the former kinetic equation of the pressure on an external wall can be generalized as the equation of the pressure on the surface of any object that is inserted in any point of the gas volume. According to this definition, in all other points of the gas volume, where there are no objects, there is however no pressure, because there are no material surfaces to which the molecules can transfer their momentum. This problem is a bit like "Schrödinger's cat": it is only when you open the box and put an object in it, that the gas will exert a "pressure" on it.

1.3 The present definition of temperature

1.3.1 The kinetic theory of temperature

In the present kinetic theory, the ideal gas law: $pV = NkT = nRT$

is used to conclude that:

$$\text{or } Nm_1.v^2/3 = NkT =$$

$$\text{and so: } kT = m_1v^2/3$$

$$- \quad kT = m_1.v^2/3 = (2/3)(m_1v^2/2) = (2/3)K_1$$

The temperature of an ideal gas is proportional to 2/3rds of the 'average translational kinetic energy' of one molecule of that gas, in which Boltzmann's constant $k = 1.38065 \times 10^{-23}$ J/K is a conversion factor between the kinetic energy of one molecule (in Joule) and the units of temperature (in degrees Kelvin).

$$- \quad RT = N_A m_1.v^2/3 = (2/3)(N_A m_1 v^2/2) = (2/3)N_A K_1$$

The temperature of an ideal gas is proportional 2/3rds of the 'average translational kinetic energy' of one mole of that gas, in which the universal gas constant $R = 8.31447$ J/mol.K is a conversion factor between the kinetic energy of one mole (in Joule) and the units of temperature (in degrees Kelvin).

It follows logically from both equations that $R = k.N_A$ where $N_A = 6,02 \cdot 10^{23}$ is Avogadro's number, which gives the number of molecules in one mol.

These are very accurate mathematical definitions that allow us to calculate the work that can be generated with a given temperature difference, but it doesn't however give us a profound insight in what 'temperature' really is and why it represents only a 2/3rds of the average kinetic energy of the molecules.

1.3.2 Temperature and thermal radiation

In the kinetic equation of temperature, the monatomic atoms are considered as perfectly elastic, point-like spheres. In reality, they consist of a nucleus and electrons that turn around the nucleus at specific orbits. Collisions between atoms can kick electrons into a larger orbit.

This position is however unstable, because the De Broglie wavelength of the electron will no longer fit in the trajectory of this larger orbit (iv), so that the electron will spontaneously return to a smaller orbit, while giving off a photon with a characteristic frequency. This phenomenon, which is generally known as “thermal radiation”, is another fundamental characteristic of “temperature”, which means that a correct definition of “temperature” must in any case take the frequency of collisions into account. This conclusion is strikingly demonstrated by the fact that:

- one single monatomic atom in an equilibrium state, that is moving at a constant speed, has no thermal radiation and therefore no “temperature”, notwithstanding the fact that it undoubtedly has ‘kinetic energy’
- a group of such atoms that are moving in a congruent way, that is with the same speed in the same direction, has no thermal radiation and therefore no “temperature”, notwithstanding the fact that this group undoubtedly has ‘kinetic energy’.

2. The physical nature of (thermal) pressure

In section 1.2 we came to the conclusion that according to the present definition of ‘pressure’, in all points of the gas volume where there are no objects, there is no pressure, because there are no material surfaces to which the molecules can transfer their momentum.

In the points where there are no obstructions, there is indeed no pressure in the proper sense of its definition, but through any imaginable surface in any point of the gas volume, there are “ $Nv/6V$ ” molecules per unit time and per unit area that isotropically flow with an average linear momentum “ m_1v ” in all directions, that are bumping against one another.

Statistically the system is at rest in the way that there is no net mass flow, but there are however continuous momentum flows in all directions.

It is a fundamental property of thermal systems that the motions of their particles are completely isotropic, so that the thermal velocities of the gas molecules are equally spread over the 2 opposite sides of each of the 3 main axis.

This means that the “thermal” momentum flow “ Q_{pT} ” in each direction is equal to $1/6^{\text{th}}$ of the total momentum flow:

$$Q_{pT} = Q_p/6 = Nm_1Av^2/6V = mAv^2/6V = \rho Av^2/6$$

The thermal force F_T caused by this isotropic thermal motion on an immovable elastic wall is then:

$$F_T = 2Q_{pT} = Nm_1Av^2/3V = mAv^2/3V = \rho Av^2/3$$

So that the (thermal) pressure caused by the thermal motion on this wall is:

$$p_T = F_T/A = Nm_1v^2/3V = mv^2/3V = \rho v^2/3$$

This deduction allows us to define the (thermal) pressure in a particle system as the (average) two-sided momentum transfer per unit area in any given direction (which is independent of

(iv) The physical nature of the De Broglie wavelength will be analyzed in depth in my paper on the physical nature of quantum particles”.

the given reference frame). This is equal to the momentum transfer per collision, times the frequency of the collisions per unit area.

It follows from these equations, that the pressure of a monatomic gas:

- is proportional to the linear momentum “ m_1v ” of one molecule of that gas
- is proportional to the amount of molecules that pass through/collide with a unit area per unit time, which on its turn:
 - is proportional to the circulation frequency ‘ v ’ of one molecule, which is proportional to the average speed “ v ” of the molecules of that gas, and
 - is proportional to the density “ Nm_1/V ” of the gas.

This makes it once more clear that from the physical point of view, thermal “pressure” is a measure for the isotropic momentum flow per unit time and per unit area.

- It follows from this definition that the factor “ mv^2 ” in the equation of pressure is a consequence of the fact that the momentum transfer per unit time is the product of the momentum transfer per collision and the number of the collisions per unit time and that both factors are proportional to the (average) speed “ v ” of the molecules.

- It follows also from this definition of ‘pressure’ that if we make the frequency of the collisions independent from their speed, e.g. by firing molecules with a given speed and a given frequency to a wall, the pressure on that wall will still be equal to the momentum of the molecules (mv) times that imposed frequency, but it will not anymore be proportional to “ mv^2 ”, because the frequency is not proportional to ‘ v ’.

The same is true for a pressurized gas, in which the number of molecules (N) is increased, without increasing their speed (see further section 5).

3. The physical nature of thermal kinetic energy (heat)

The (thermal) kinetic energy of one molecule of an ideal monatomic gas is: $K_1 = m_1q^2/2$ and the total amount of momentum flow is then: $Q = 2K_1 = m_1q^2$

The amount of momentum flow of the total number of ‘ N ’ molecules that are isotropically moving in all directions with an average speed ‘ q ’ is consequently:

$$Q_T = Nm_1q^2 = mq^2$$

From this we can conclude that the thermal kinetic energy of an ideal monatomic gas is a mathematical expression of its total amount of reversibly transferrable isotropic translational motion, which is an absolute, hard physical datum because it is independent of the given reference.

Since the thermal motion of the molecules of an ideal monatomic gas is completely isotropic, the thermal velocities are equally spread over all 6 directions (the 2 opposite sides along the 3 main axes), so that the thermal momentum flow ‘ Q_{pT} ’ in each direction is equal to $1/6^{\text{th}}$ of the momentum flow of congruent bulk motion:

$$Q_{pT} = Q_p/6 = Nm_1Av^2/6V = mAv^2/6V = \rho Av^2/6$$

This means that, in each given direction, total amount of (thermal) momentum flow (Q_{pT}) over the total length of the fluid (L_1), is equal to:

$$Q_{pTL_1} = Q_p \cdot L = Nm_1v^2/6 = mv^2/6$$

So that the total amount of thermal momentum flow in the two opposite directions of each axis is equal to 2/3rds of the average kinetic energy of the particles

$$Q_{pTL_2} = 2Q_{pTL_1} = Nm_1v^2/3 = mv^2/3 = (2/3)(mv^2/2) = (2/3)K_T$$

This means that the thermal kinetic energy of the molecules of an ideal monatomic gas is only equal to 2/3rds of their total kinetic energy, because it represents the total amount of momentum flow in the two directions of each of the 3 axis.

4. The physical nature of temperature

4.1 The kinetic definition of temperature

In the present textbooks the kinetic energy of isotropic thermal motion is expressed in function of the temperature as:

$$K_T = (3/2)NkT$$

$$\text{So that in fact: } Nm_1q^2/2 = (3/2)NkT$$

$$\text{And the kinetic energy of one molecule: } K_{T1} = m_1q^2/2 = (3/2)kT$$

So that the temperature expressed in its natural units (Nm per molecule) is equal to:

$$kT = (2/3)m_1q^2/2 = (2/3)K_1$$

As we have demonstrated in de previous section, the reason why the temperature of the molecules of an ideal monatomic gas is equal to (only) 2/3rds of their kinetic energy is due to the fact that the total amount of thermal momentum flow in the two opposite directions of each axis is equal to 2/3rds of the average kinetic energy of the particles

$$Q_{pTL_2} = 2Q_{pTL_1} = Nm_1q^2/3 = mq^2/3 = (2/3)(mq^2/2) = (2/3)K_T$$

This means that the thermal kinetic energy of the molecules of an ideal monatomic gas is only equal to 2/3rds of their kinetic energy, because it represents the total amount of momentum flow in the two directions of each of the 3 axis.

$$\text{For one molecule the temperature is expressed as: } kT = (2/3)(m_1v^2/2) = m_1v^2/3$$

$$\text{And for one mole the temperature is expressed as: } RT = (2/3)(N_A m_1 v^2/2) = N_A m_1 v^2/3$$

4.2 The definition of temperature from the ideal gas law

The physical nature of “temperature” can also be approached by means of to the ideal gas law: $pV = kNT = RnT$

This so-called “ideal gas law” has been established on the basis of experiments on the variation of the temperature of a diluted monatomic gas with its pressure (at constant volume) and with its volume (at constant pressure)^[5]. This led to the definition of (the absolute)

temperature (T) as proportional to the product of pressure (p) times the volume (V), divided by the number of moles (n), or the number of atoms (N):

$$T = pV/Rn = pV/kN$$

This simply means that the so-called ‘ideal gas law’ is in reality a definition of “temperature” in which the universal gas constant ‘R’ as well as Boltzmann’s constant ‘k’, are conversion factors ^[6] that adapt the natural units of ‘temperature’ (in Nm per mole or in kgm²/s² per mole) to the arbitrarily chosen degrees Kelvin.

4.3 Temperature as a calibrated pressure

In the light of our definition of the nature of (thermal) pressure (section 4.3), this means that the temperature of an ideal monatomic gas is a mathematical expression of the average two-sided momentum flow per unit area in each direction, for a unit number density of its molecules (per molecule or per mole), which is independent of the given reference frame.

When the ideal gas law is written as:

$$kT = p/(N/V) \quad \text{or} \quad RT = p/(n/V)$$

it becomes clear that what is actually defined as the ‘temperature’ of an ideal gas is in fact the pressure of that gas for a unit number density (per molecule or per mole) of its molecules.

This means that when we measure the “temperature” of a gas, we measure in fact the “pressure” for a unit (number) density of its molecules, which is a measure for the intensity of the collisions with the external walls.

These two appearances of momentum flow per unit area, in the form of ‘pressure’ and in the form of temperature can be explained by the following analyses of the pressure on a wall. The pressure on a wall is caused by the continuous impacts of the molecules on the surface of that wall. These impacts have a double effect:

1. When the considered wall is a movable piston, the impacts have the same repetitive effect as a driving force. In that case the consecutive impacts of the atoms push against that wall with a constant average “pressure” which is defined as the momentum transfer to that wall per unit time and per unit area:

$$p = mv^2/3V$$
 and which increases proportionally with the density (n/V or N/V) of the gas. This is exactly how a manometer works. In that case the movable piston compresses a calibrated spring of which the displacement is an indication of the pressure of the gas.
2. When the considered wall is an immobile, perfectly elastic partition wall, like e.g. a thin glass wall, the impacts have an individual effect. In that case the perfect elastic wall will serve as a perfect transmitter between the impacts it receives from the external molecules and the impacts it receives from the enclosed molecules. This is exactly how a thermometer works. In that case the thermal expansion of a calibrated fluid inside a glass tube is a direct indication for the temperature variation of an external medium.

The impact of a gas molecule on the elastic wall can however only be transmitted to the calibrated fluid, when a molecule of that fluid hits the wall. This means that the impacts can only be transmitted for an equal amount of molecules on both sides, so that the amount of momentum transfer between both fluids will be

independent from their densities (N/V or n/V):
 $T = p/(n/V) = N_A m_1 v^2/3$

In the former section we have seen that, the thermal pressure of an ideal gas is equal to twice its momentum flow per unit area:

$$p_T = 2Q_{pT}/A = Nm v^2/3V$$

so that in this case the kinetic equation for the temperature of an ideal gas is equal to:

$$kT = p_T \cdot V/N = m_1 v^2/3 \quad \text{or} \quad RT = p_T V/n = N_A m_1 v^2/3$$

which means that in this case the temperature of an ideal gas (expressed in kg.m²/s² per molecule) is equal to twice the momentum flow for a unit number density (one molecule or one mol per unit volume) of that gas, so that the temperature of an ideal monatomic gas represents in fact the ‘total amount of momentum flow’ of one molecule (or one mol) in both opposite sides along one direction.

When the temperature is expressed in degrees Kelvin, this gives us the classic kinetic equations:

$$T = m_1 v^2/3k \quad \text{or} \quad T = N_A m_1 v^2/3R$$

It follows from this equation that the temperature of a monatomic gas:

- is proportional to the linear momentum “mv” of that gas, which (for a given gas) is proportional to the speed of the molecules;
- is proportional to the return period (or the frequency ‘v’) of one molecule, which depends on the speed “v” of the molecules;
- is independent of the density (N/V or n/V) of the gas.

This means that if we should change the frequency of the collisions independently from their average speed, e.g. if we should make the motions of the molecules anisotropic, the temperature would still be proportional to the frequency of the collisions, but it would no longer be proportional to the square of the average speed of the molecules any more, which explains why a group of monatomic particles, all moving in a perfectly congruent way, that is with the same speed in the same direction, has no “temperature” at all (because in that case the frequency of the internal collisions is equal to zero), but it surely has kinetic energy.

Since the temperature of a particle system is an absolute physical phenomenon, that produces thermal radiation and phase transitions, such as the melting and even the vaporization of hard solid rocks, our definition of the true nature of ‘temperature’ demonstrates that the internal (or ‘thermal’) velocities, and consequently the thermal linear momentums, the thermal kinetic energy and the thermal pressure of the particles of a particle system, have an absolute, physical nature (v).

5. The thermal speed distribution

5.1 Maxwell’s speed distribution function

(v) The nature of velocity will be analyzed in my paper on the physical of extent, velocity and length contraction.

The Maxwell's speed distribution ^[7] is a probability distribution for describing particle speeds in idealized gases, which are gases where the particles can move freely inside a stationary container without interacting with one another, except for very brief, perfectly elastic collisions. It was first derived by Maxwell in 1860. Maxwell's approach ^[8] is based on general assumptions, such as:

- the motions of the molecules are completely isotropic and symmetric, which leads to the conclusion that only an exponential function satisfies these conditions.
- The self-evident condition that each molecule must have a velocity in the range between $-\infty$ and $+\infty$
- A definition of the root mean square speed $v_{\text{RMS}} = (3RT/N_A m_1)^{1/2}$ of the particles
- The probability $f(v)dv$ that the molecules have a speed in the range v to $v + dv$ regardless of direction is the sum of the probabilities that the velocity lies in any of the volume elements $dv_x dv_y dv_z$ forming a spherical shell of radius v , with a volume $4\pi v^2$.

This leads to the well-known Maxwell speed distribution function:

$$f(v) = 4\pi \left(\frac{m}{2\pi RT} \right)^{3/2} v^2 e^{-mv^2/2RT}$$

which gives 'v' as a fraction of the most probable speed ' v_{MP} ' (Fig. 5.1)

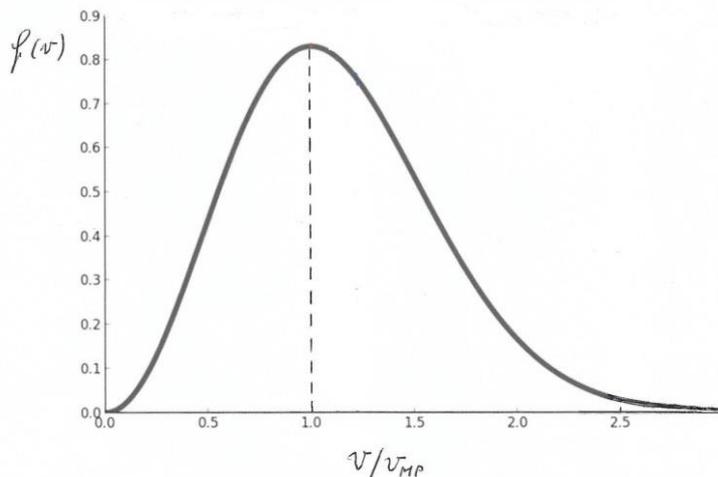


Fig 5.1

In his book "Kinetic Theory and Entropy" ^[9] C. H. Collie writes that "*Many students on first reading Maxwell's arguments feel that they have been tricked. The most unsatisfactory part of the derivation of Maxwell's distribution function is that it has been deduced without any reference to the molecular collisions through which the molecular distribution is maintained. In a perfect gas in which the molecules do not interact with each other, their velocities will never change and any distribution will be maintained indefinitely.*"

So it is rather strange that Maxwell's distribution function leads to correct results. According to Collie "*Scrutiny of the derivation of the distribution function shows that it is based upon a concealed assumption, of which Maxwell was, of course, well aware.*"

According to Collie, Maxwell's treatment assumes that the molecules selected to have velocities between v and $v + dv$ will have the same distribution function as molecules selected at random and that this is a special and not very plausible assumption.

5.2 The Maxwell-Boltzmann speed distribution

Ludwig Boltzmann was the first physicist who established the physical connection between the entropy of an ideal gas and the velocities of its particles ^[10].

Boltzmann made a mathematical study of the way in which a gas composed of perfectly elastic spheres reached and maintained a Maxwell distribution. He did this by adopting ideas from the laws of probability developed by Pascal, who was the first mathematician to give a precise meaning of the term 'probability'. In that way ^[11] Boltzmann treated the problem of the distribution function as a complicated example of the well-known problem of the distribution of heads and tails in a pile of coins.

By doing so he demonstrated that in a perfect gas in which all the molecules start with the same scalar velocity ' v ', this initial uniformity would soon be destroyed by the repetitive energy transfers between the molecules, which would lead to successively more complicated distributions. He could thereby demonstrate that the successively more and more complicated distributions lead to a macroscopic configuration that maximizes the number of microstates. He thereby showed that these increasing chaotic states were states of increasing probability, so that the final state was the one with the greatest probability and the greatest entropy. He thereby showed that this most probable state must be self-reproducing and that Maxwell's speed distribution and no other has just this self-reproducing property.

Boltzmann also stressed the fact that in this final steady state of complete thermal equilibrium where the distribution function remains constant, the velocities of the individual molecules keep continuously changing.

5.3 The physical nature of the thermal speed distribution

The Maxwell-Boltzmann speed distribution is an extremely accurate work of mathematical ingenuity.

But although his deduction is based on the energy exchanges between molecular systems, the unsatisfactory feeling expressed by C. H. Collie, namely that it has not been deduced from the actual collisions between the particles, remains.

To obtain a little more insight in this matter, we consider the case of consecutive collisions between the perfectly elastic pointlike particles and we will see that these perfectly elastic collisions will gradually increase the divergence of the directions of velocities of these particles and will lead to a completely isotropic velocity distribution. This means that Maxwell-Boltzmann's speed thermal speed distribution is in fact an intrinsic property of isotropic collisions (vi).

To demonstrate this, we consider a particle system that initially consists of perfectly elastic unit mass particles that initially move with exactly the same congruent velocity ' v ' in the same direction (e.g. because they are at rest in a spherical container with perfectly elastic walls, that moves with a steady velocity ' v ' in a given direction). When this moving globe is then suddenly stopped, the particles in it will collide with the curved wall of the front side and will rebound different directions.

(vi) In my paper on the physical nature of 'entropy' I will demonstrate the direct link between the entropy and the degree of isotropy of particle systems.

So we then have a particle system in which all the particles have the same speed 'v', who collide with each other under all possible angles. We can represent such a collision in a plane formed by the velocities of the colliding particles, in which (Fig. 5.2)

- in which the initial speed 'v' of particle 1 points in the direction of the x-axis (v_{1i}), and
- in which the initial speed 'v' of particle 2 can make any angle ' α ' (v_{2i}) with the velocity of particle 1. Because of the angular symmetry we can suppose that the angle ' α ' varies from 0° to 180° (Fig. 5.2)

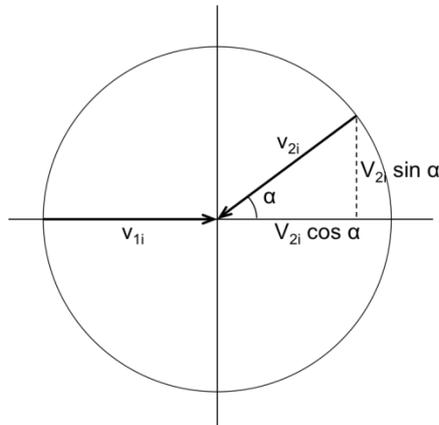


Fig. 5.2

When these particles collide, we can apply the solution of the final velocities for an elastic collision between two identical particles (vii):

- Particle 1 receives the x-component of particle 2, so that its speed is reduced from $v_{1i} = v$ to $v_{1f} = v_{2i} \cdot \cos \alpha = v \cdot \cos \alpha$.
- Particle 2 receives the velocity of particle 1 (v) and conserves its own y-component ' $v \cdot \sin \alpha$ ', so that its speed increases from ' v ' to $v_{2f} = v \sqrt{1 + \sin^2 \alpha}$

This collision analyses demonstrates that Maxwell Boltzmann's speed distribution

$$f(v) = \left[4\pi \left(\frac{m}{2\pi RT} \right)^{3/2} \right] v^2 e^{-v^2 [m/2RT]} = C_1 v^2 e^{-v^2 C_2}$$

is composed of two distinct dominant mathematical functions:

- the left side, with relative speeds $0 \leq v/v_{MP} \leq 1$, which is mainly formed by powers of the cosine of the collision angle: $v \cdot \cos^n \alpha$ (and which is in Maxwell's speed distribution represented by the factor v^2)
- the right side, with relative speeds $v/v_{MP} \geq 1$, which is mainly formed by powers of the mathematical equation of the form $v(1 + \sin^2 \alpha)^{n/2}$ (and which is in Maxwell's speed distribution represented by the factor e^{-v^2})

Particle 1 will have a minimum final speed for $\alpha = 90^\circ$ where $\cos \alpha = 0$ and $v/v_{MP} = \cos \alpha = 0$

Particle 2 will have a maximum final speed for $\alpha = 90^\circ$,

So that $\sin \alpha = 1$ and

$$v/v_{MP} = (1 + \sin^2 \alpha)^{1/2} = \sqrt{2} = 1,414$$

(vii) See section 2.3 of my paper Part 4 on the conservation of kinetic energy in elastic collisions.

If this particle with a speed of $1,414v$ then collides orthogonally with a particle with the same speed, it will obtain a speed of $v/v_{MP} = 2$, and a third collision of this kind can even produce a speed of nearly 2,828. This physically explains that the Maxwell- Boltzmann's speed distribution curve is elongated to the right side because of the repetitive collisions between the particles.

Both curves join each other at $v = v_{MP}$:

- For $\alpha = 0^\circ$, $v/v_{MP} = \cos\alpha = 1$
- For $\alpha = 90^\circ$, $\sin\alpha = 1$ and $v/v_{MP} = (1 + \sin^2\alpha)^{1/2} = \sqrt{2} = 1,414$

In that way we have demonstrated a direct relation between the actual elastic collisions between pointlike particles which initially have isotropic velocities with the same speed and the speed range of the Maxwell – Boltzmann speed distribution (viii).

For a real gas consisting of perfectly elastic spheres, the dispersion will proceed much faster because of the possibility of eccentric collisions.

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(viii) The reader who finds a more general approach to arrive to the Maxwell speed distribution on the basis of elastic collisions between identical particles, may always contact me.