Title :DARK MATTER AND DARK ENERGY
(v4)

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Abstract:

The article proposes a new model of dark matter. According to this new model, dark matter is a substance, that is a new physical element not constituted of classical particles, called *dark substance*, filling the universe and constituting what is called *emptiness*. Assuming some very simple physical properties to this dark substance, we theoretically justify the flat rotation curve of galaxies and the baryonic Tully-Fisher's law. We then study according to our new theory of dark matter the different possible distributions of dark matter in galaxies and in galaxy clusters, and the velocities of galaxies in galaxy clusters.

Then using the new model of dark matter we are naturally led to propose a new cosmological model of Universe, finite. This new cosmological model is divided in 2 different mathematical models. The first one leads to a model very close to SCM, but giving the nature of dark matter and dark energy and interpreting the CMB rest frame. The 2nd proposed mathematical model is mathematically much simpler than the SCM but we will see that its theoretical predictions agree with astronomical observations with a very good approximation for z<12. Moreover, this 2nd mathematical model of expansion does not need to introduce the existence of a dark energy contrary to the mathematical model of expansion of the SCM. But we will see that according to observations of the CMB, it is the 1st model, mathematically very close to SCM, that is the good one.

Key words: Tully-Fisher's law, dark matter, dark halo, CMB, galaxy clusters, gravitational lensing, galaxy rotation curve, velocity of galaxies, dark energy, structure formation, ΛCDM model.

1.INTRODUCTION

The objective of this study is to propose a general theory of dark matter and dark energy. As first section of this article, a theory of dark matter is proposed. In this section, we propose that a new physical element, called *dark substance*, constitutes the dark matter and constitutes what is called *emptiness*. According to the proposed model of dark matter, this dark substance fills all the Universe and has physical properties close to the physical properties of an ideal gas. Using those properties, we justify theoretically the flat rotation curve from observation of some galaxies, in a new way, with density of dark substance in $1/r^2$. A simple mathematical expression of the density of dark matter (in $1/r^2$) permitting to obtain the flat rotation curve which has already been proposed, but the model of dark matter that permits to justify theoretically this mathematical expression (in $1/r^2$) has never been proposed. Moreover the study hypothesizes simple thermal properties to this dark substance which exist in the theory of dark matter that permit to justify theoretically the baryonic Tully-Fisher's law. The theory called MOND (MILGROM 1983) also proposes a theoretical justification of the flat rotation curve of some galaxies, but this theory is contrary to Newton's attraction law and moreover it is contradicted by some astronomical observations (PINA et al. 2021). Theory of dark matter with different models of distribution of dark matter in galaxies is proposed in this study. We will show that the proposed theory of dark matter gives theoretical predictions concerning the velocities of galaxies inside clusters and the masses of galaxy clusters in agreement with astronomical observations. The new theory permits to obtain theoretical predictions of the dark radius of galaxies, in agreement with observations. Our model of dark matter permits to define a very simple geometrical model of Universe: Spherical.

In the 2nd section of the article, the new theory of dark matter and dark energy proposes a new Cosmological model. The proposal of the theory is based on the new geometrical form of the Universe introduced in the 1st part of the article, and also on the physical interpretation of the CMB Rest Frame (CRF), that we will name *local Cosmological frame*, because of the importance of the CRF in the new Cosmological model. The new Cosmological model permits to re-define distances in Cosmology that are completely analogous to distances in Cosmology according to SCM. The new Cosmological model is compatible with Special Relativity and General Relativity (locally) because according to this new Cosmological model the CRF cannot be detected using usual laboratory experiments but only by observation of the CMB. The new Cosmological model proposes 2 possible mathematical models of expansion of the Universe. The 1st mathematical model of expansion is based as the SCM on the equations of General Relativity but it gives the nature of dark matter and dark energy that are necessary in the SCM. It also interprets the CMB rest frame. So this 1st mathematical model gives theoretical predictions of distances used in Cosmology, of the Cosmological redshift and of the Hubble Constant that are identical to their theoretical predictions by the SCM.

The 2nd proposed mathematical model of expansion is not based on General Relativity but is mathematically much simpler. Nonetheless its theoretical predictions, in particular predictions of Hubble's Constant and of distances used in Cosmology, agree with astronomical observations with a very good approximation for $z<12$. Moreover, this 2nd model does not need the existence of a dark energy (contrary to the 1st mathematical model and to the SCM). The observation of the anisotropies of CMB contradicts the 2nd mathematical model. For instance they give the Cosmological time of apparition of the CMB (400000 years) that is in agreement with the 1st mathematical model and contradicts the 2nd mathematical model. It is also the case for the prediction of the comobile distance to the last diffusion surface (43 billion y.l). Nonetheless, we will only study the 2nd mathematical model of the new Cosmological model, that we can easily generalize in order to obtain the properties of the 1st mathematical model of the new Cosmological model because according to the SCM the Universe is flat.

At the end we study according to proposed theory of dark matter and dark energy the evolution of the dark substance temperature in the Universe.

We remind that for many astrophysicists and physicists, the enigmas in the SCM, in particular the enigmas concerning dark matter and dark energy, make necessary a new paradigm for the SCM (KROUPA, PAWLOWSKI&MILGROM 2012) . Our article proposes such a new evolutionary paradigm.

We will see that the theory of dark matter and dark energy exposed in this article remains compatible with the SCM (RAINE&THOMAS 2001; LIDDLE 2003; DODELSON&SCOTT 2008), inside the first mathematical model, in order to interpret most astronomical observations not directly linked to dark matter or dark energy, for instance primordial elements abundance, the apparition of baryonic particles (for the same Cosmological redshift z as in the SCM), structure formation), apparition of the CMB (for the same z as in the SCM), evolution of the temperature of the CMB (in 1/(1+z)), anisotropies of the CMB….

2. THEORY OF DARK MATTER

2.1 Physical properties of the dark substance.

As we have seen in 1.INTRODUCTION, we stated the Postulate 1 expressing the physical properties of the dark substance:

Postulate 1:

a)A substance, called *dark substance*, fills all the Universe.

b)This substance does not interact with photons crossing it.

c)This substance owns a mass and obeys to the Boyle's law, to the Charles'law , and to the following law that is their synthesis:
An element of dark substance with a mass m, a volume V, a pressure P and a temperature T verifies, k_0 being a constant:
$PV=k_0mT$

The preceding law is a valid statement for a given ideal gas G_0, replacing k_0 by a constant $k(G_0)$, and this is a consequence of the *universal gas equation*, which is also obtained using Boyle and Charles'laws.

We have 2 remarks consequences of this Postulate1:

-First, the dark substance is not really dark but translucent despite of its name. Indeed, because of the preceding Postulate 1b) it does not interact with photons crossing it.

-Secondly because of the Postulate 1a), what is usually called "emptiness" is not empty in reality: It is filled with dark substance.

2.2 Flat rotation curves of galaxies.

Using the fact that the dark substance behaves as an ideal gas (Postulate 1c) (See Wikipedia, "ideal gas law"), we are going to show that a spherical concentration of dark substance in gravitational equilibrium can constitute the dark matter in a galaxy with a flat rotation curve.

According to Postulate 1c) an element of dark substance with a mass m, a volume V, a pressure P and a temperature T verifies the law, k_0 being a constant:

$$PV=k_0mT \qquad (1)$$

Which means, setting $k_1=k_0T$:

$$PV=k_1m \qquad (2)$$

Or equivalently, ρ being the mass density of the element:

$P=k_1\rho$ (3a)

We hypothesized that the galaxy can be modeled as a concentration of dark substance with a spherical symmetry, at an homogeneous temperature T, in gravitational equilibrium.

We considered the spherical surface S(r) (resp. the spherical surface S(r+dr)) that is the spherical surface with a radius r (resp. r+dr) and whose the centre is the center O of the galaxy. S(O,r) is the sphere filled with dark substance with a radius r and the centre O.

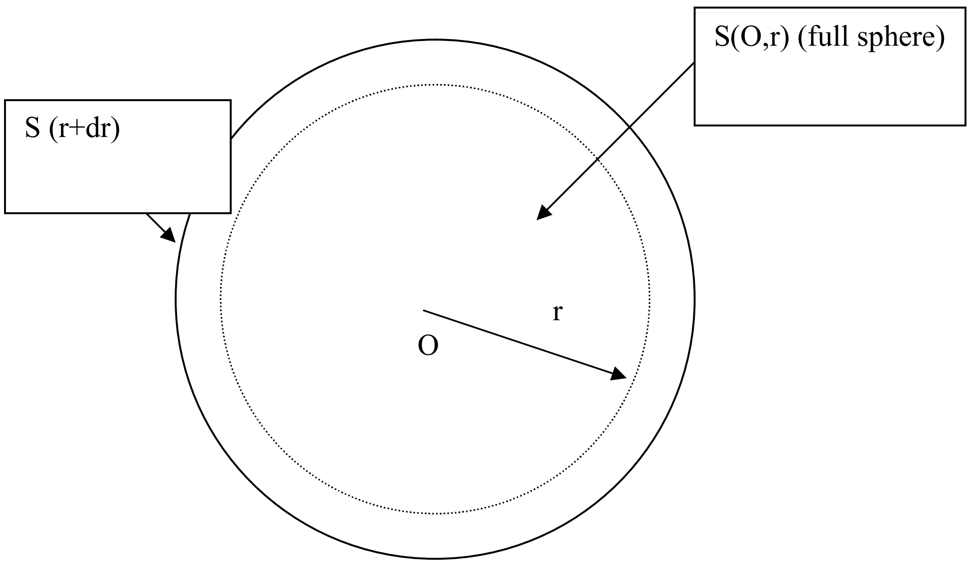

Figure 1:The spherical concentration of dark substance

The mass M(r) of the sphere S(O,r)is given by:

$$M(r) = \int_0^r \rho(x) 4\pi x^2 dx \qquad (3b)$$

Assuming a spherical symmetry for the density of dark substance, using Newton's law (Σ**F**=**0** for a material element in equilibrium with a mass m, $\mathbf{F}_G(r)=m\mathbf{G}(r)$, $\mathbf{F}_G(r)$ gravitational force acting on the element, **G**(r) gravitational field defined by Newton's universal law of gravitation) and Gauss theorem in order to obtain **G**(r), we obtain the following equation (4) of equilibrium of forces on an element dark substance with a surface dS, a width dr, situated between S(O,r) and S(r+dr):

$$dSP(r+dr) + \frac{G}{r^2}(\rho(r)dSdr)(\int_0^r \rho(x)4\pi x^2 dx) - dSP(r) = 0 \qquad (4)$$

Eliminating dS, we obtain the equation:

$$\frac{dP}{dr} = -\frac{G}{r^2}(\rho(r))(\int_0^r \rho(x) 4\pi x^2 dx) \qquad (5)$$

And using the equation (3) obtained using the Boyle-Charles'law assumed in the Postulate 1, we obtain the equation:

$$k_1 \frac{d\rho}{dr} = -\frac{G}{r^2}(\rho(r))(\int_0^r \rho(x) 4\pi x^2 dx) \qquad (6)$$

We then verify that the density of the dark substance ρ(r) satisfying the preceding equation of equilibrium is the evident solution:

$$\rho(r) = \frac{k_2}{4\pi r^2} \qquad (7)$$

(A density of dark matter expressed as in Equation (7) has already been proposed to explain the flat rotation curve of spiral galaxies, but it has not been proposed a model of dark matter permitting to justify theoretically this density in $1/r^2$ or to obtain the constant k_2. Here we give a theoretical justification of this density in $1/r^2$ and we are going to give the expression of the constant k_2 (Equation (8)). This is the consequence of the model of dark substance as an ideal gas, Postulate 1).

In order to obtain k_2 , we replace ρ(r) given by the expression (7) inside the equation (6), and we obtain immediately that this equation is verified for the following expression of k_2:

$$k_2 = \frac{2k_1}{G} = \frac{2k_0 T}{G} \qquad (8)$$

Using the preceding equation (7), we obtain that the mass M(r) of the sphere S(O,r) is given by the expression:

$$M(r) = \int_0^r 4\pi x^2 \rho(x) dx = k_2 r \qquad (9)$$

We then obtain, neglecting the mass of stars in the galaxy, that the velocity v(r) of a star of a galaxy situated at a distance r from the center O of the galaxy is given by $v(r)^2/r=GM(r)/r^2$ and consequently :

$$v(r)^2=Gk_2=2k_1=2k_0T \qquad (10)$$

So we obtain in the previous equality (10) that the velocity of a star in a galaxy is independent of its distance to the centre O of the galaxy.

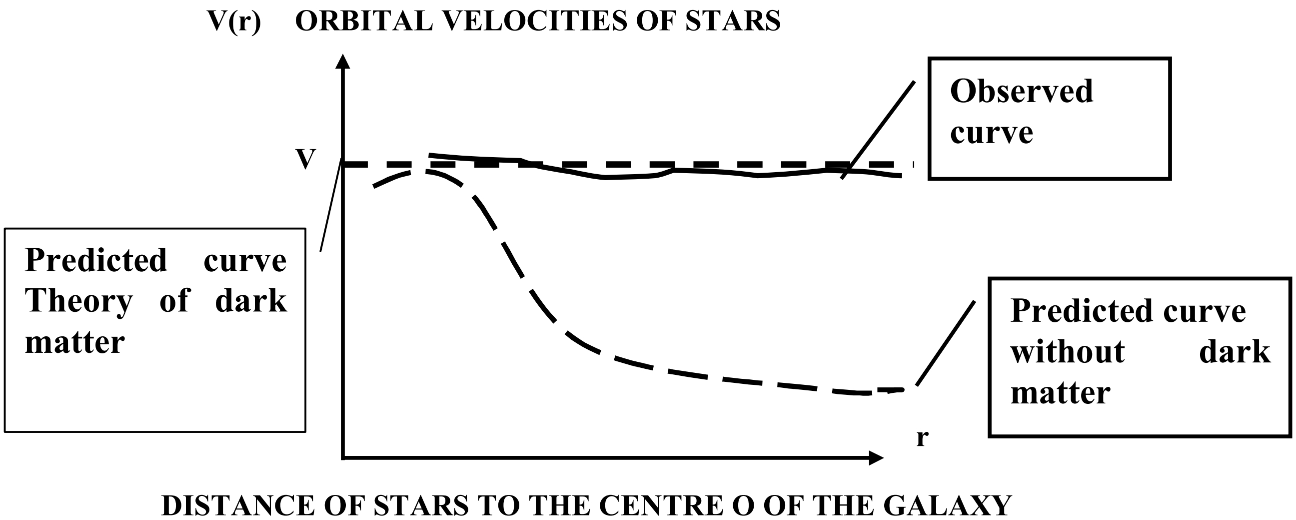

Figure 2 :Rotation curve of galaxies

2.3 Baryonic Tully-Fisher's law.

2.3.1 Recall.

Tully and Fisher realized some observations on spiral galaxies with a flat rotation curve. They obtained that the luminosity L of such a spiral galaxy is proportional to the 4th power of the velocity v of stars in this galaxy (TULLY&FISHER 1977). So we have the Tully-Fisher's law for spiral galaxies, K_1 being a constant:

$$L=K_1v^4 \qquad (11)$$

But in the cases studied by Tully and Fisher, the baryonic mass M of a spiral galaxy is usually proportional to its luminosity L. So we have also the law for such a spiral galaxy, K_2 being a constant:

$$M=K_2v^4 \qquad (12)$$

This 2nd form of Tully-Fisher's law is known as the *baryonic Tully-Fisher's law*.

The more recent observations of Mc Gaugh (McGAUGH 2011) show that the baryonic Tully-Fisher's law (equation (12)) seems to be true for all galaxies with a flat rotation curve, including the galaxies with a luminosity not proportional to their baryonic mass.

We are going to demonstrate that using the Postulate 1 and a Postulate 2 expressing very simple thermal properties of the dark substance, (in particular its thermal interaction with baryonic particles), we can justify this baryonic law of Tully-Fisher.

2.3.2 Theory of quantified loss of calorific energy (by nuclei).

We saw in the previous equation (10) that according to our model of dark substance the square of the velocity of stars in a galaxy with a flat rotation curve is proportional to the temperature of the concentration of dark substance constituting this galaxy. So we need to determinate T:
-A first possible idea is that the temperature T refers on CMB. But this is impossible because it would imply all the stars of all galaxies with a flat rotation curve be driven with the same velocity and we know that it is not the case.

-The second possibility is that in the considered galaxy, each baryon interacts with the dark substance constituting the galaxy, transmitting to a thermal energy. We can expect that this thermal energy is very low otherwise it would already have been observed, but because of the expected very low density of the dark substance and of the considered times (we remind that the baryonic diameter of galaxies can reach 100000 light-years), it can lead to appreciable temperatures of dark substance. A priori we could expect that this loss of thermal energy for each baryon (transmitted to the dark substance) depends on the temperature of this baryon and of the temperature T of the dark substance in which the baryon is immerged, but if it was the case, the total lost thermal energy by all the baryons would be extremely difficult to calculate and moreover it should be very probable that we would then be unable to obtain the very simple baryonic Tully-Fisher's law.

The hypothesis of the study is defining the thermal transfer between dark substance and baryons, expressed in the following Postulate 2a) (Postulate 2 gives the thermal properties of the dark substance):

Postulate 2a):

-Each nucleus of atom in a galaxy is submitted to a loss of thermal energy, transmitted to the dark substance in which it is immerged.

-This thermal transfer depends only on the number n of nucleons constituting the nucleus (So it is independent of the temperature of the nucleus). So if p is the thermal power dissipated by the nucleus, it exists a constant p_0 (thermal power dissipated by nucleon) such that:

$$p=np_0 \qquad (13)$$

According to the equation (13), the total thermal power transmitted by all the atoms of a galaxy towards the spherical concentration of dark matter constituting the galaxy is proportional to the total number of nucleons of the galaxy and consequently to the baryonic mass of this galaxy. So if m_0 is the mass of one nucleon, M being the baryonic mass of the galaxy, we obtained according to the equation (13) that the total thermal power P_r received by the spherical concentration of dark substance constituting the galaxy from all the atoms is given by the following equation, K_3 being the constant p_0/m_0:

$$P_r=(M/m_0)p_0=K_3M \qquad (14)$$

Concerning the preceding Postulate 2a):

-It is possible (but not compulsory) that it be true only for atoms whose temperature is superior to the temperature T of the concentration of dark substance.

-It permits to obtain the very simple Equation (14). We will see that this equation is essential to obtain the baryonic Tully-Fisher's law.

2.3.3 Obtainment of the baryonic Tully-Fisher's law.

In agreement with the previous model of galaxy (Section 2.2), we modeled a galaxy with a flat rotation curve as a spherical concentration of dark substance, at a temperature T and surrounded itself by a medium constituted of dark substance (called "intergalactic dark substance") with a temperature T_0 and a density ρ_0.

It is natural to make the hypothesis of the continuity of ρ(r): R is the radius for which the density ρ(r) of the concentration of dark substance is equal to ρ_0 to obtain the radius R of the concentration of dark matter constituting the galaxy. We will call R the *dark radius* of the galaxy. So we have the equation:

$$\rho(R)=\rho_0 \qquad (15)$$

The equation according to (7) and (8):

$$\frac{k_2}{4\pi R^2} = \rho_0 \qquad (16)$$

$$\frac{2k_0 T}{G} \times \frac{1}{4\pi R^2} = \rho_0 \qquad (17)$$

So we obtain that the radius R of the concentration of dark substance constituting the galaxy is given approximately by the equation:

$$R = (\frac{2k_0 T}{4\pi G \rho_0})^{1/2} = K_4 T^{1/2} \qquad (18)$$

The constant K_4 being given by :

$$K_4 = (\frac{2k_0}{4\pi G \rho_0})^{1/2} \qquad (19)$$

Then we consider that the sphere with a radius R of dark substance at the temperature T is in thermal interaction with the medium constituted of intergalactic dark substance at the temperature T_0 surrounding this sphere. The simplest and most natural thermal transfer is the convective transfer. We stated this in the Postulate 2b):

Postulate 2b):

The thermal interaction between the spherical concentration of dark substance constituting the galaxy (with a density of dark substance in $1/r^2$ and a homogeneous temperature T) and the surrounding intergalactic dark substance (at the temperature T_0) can be modeled as a convective thermal transfer.

We know that if φ is the thermal flow of thermal energy on the borders of the spherical concentration of dark substance with a radius R, P_l being the total power lost by the spherical concentration of dark substance constituting the galaxy is given by the equation:

$$P_l=4\pi R^2\varphi \qquad (20)$$

But we know that according to the definition a convective thermal transfer between a medium at a temperature T and a medium at a temperature T_0 and according to the previous Postulate 2b) the flow φ between the 2 media is given by the expression, h being a constant depending only on ρ_0:

$$\varphi=h(T-T_0) \qquad (21)$$

The total power lost by the concentration of dark substance is:

$$P_l=4\pi R^2 h(T-T_0) \qquad (22)$$

We can consider that at the equilibrium, the total thermal power P_r received by the spherical concentration of dark substance constituting the galaxy is equal to the thermal power P_l lost by this spherical concentration. According to the equations (14) and (22), (M being the baryonic mass of the galaxy), we have:

$$K_3M=4\pi R^2 h(T-T_0) \qquad (23)$$

Using then the equation (18) :

$$K_3M=4\pi K_4^2 hT(T-T_0) \qquad (24)$$

Making the approximation $T_0<<T$:

$$M=4\pi\frac{K_4^2}{K_3}hT^2 \qquad (25)$$

Consequently we obtain the expression of T, defining the constant K_5 :

$$T=(\frac{K_3}{4\pi K_4^2 h})^{1/2}M^{1/2}=K_5M^{1/2} \qquad (26)$$

And then according to the equation (10) :

$$v^2=2k_0T=2k_0K_5M^{1/2} \qquad (27)$$

So :

$$M=(\frac{1}{2k_0K_5})^2v^4 \qquad (28)$$

So we finally obtain :

$$M=K_6v^4 \qquad (29)$$

The constant K_6 being defined by:

$$K_6 = (\frac{1}{2k_0 K_5})^2 = \frac{4\pi K_4^2 h}{4k_0^2 K_3} \qquad (30)$$

$$K_6 = \frac{4\pi h}{4k_0^2 K_3} \times \frac{2k_0}{4\pi G \rho_0} \qquad (31)$$

$$K_6 = \frac{m_0 h}{2k_0 G \rho_0 p_0} \qquad (32)$$

We obtain the baryonic Tully-Fisher's law (12), with K_2=K_6. It is natural to assume that h depends on ρ_0. The simplest expression of h is h=$C_1\rho_0$, C_1 being a constant. With this relation, K_6 is independent of ρ_0, and we can use the baryonic Tully-Fisher's law to define candles used to evaluate distances in the Universe.

2.4 Temperature of the intergalactic dark substance.

We introduced the temperature T_0 of the intergalactic dark substance. We could make the hypothesis that this temperature is the temperature of the CMB but we remind that in order to get the baryonic Tully-Fisher's law we supposed $T_0 \ll T$ (T temperature of the spherical concentration of dark substance in a galaxy). The previous hypothesis would lead to high temperatures of spherical concentrations of dark substance constituting galaxies. We presume further that according to the theory of dark matter exposed here, the temperature T_0 of the intergalactic dark substance is not equal to the temperature of the CMB, except for a particular cosmological redshift z.

We could be in the following cases:

a)The temperature T_0 of the intergalactic dark substance at the present age of the Universe (equation (21)) is far less than the temperature of the CMB.

b)Baryons can emit thermal power towards dark substance as assumed in the Postulate 2a) even if their temperature is inferior to the one of dark substance.

We keep in mind that according to the Postulate 1b), the dark substance does not interact with photons and in particular with the photons of the CMB. Dark substance does not receive radiated energy.

2.5 Form of the Universe

The basis of the new Theory of dark matter were exposed previously. As a result, the obtainment of the flat rotation curve of galaxies and of the baryonic Tully-Fisher's law, are compatible with the Standard Cosmological Model. We will observe that it is also the case for the full new Theory of dark matter. The proposed Theory of dark matter is compatible with the different possible topological models of the Universe predicted by the SCM. Nonetheless, the model of dark matter proposed by the new Theory permits the possibility of a new and very simple geometrical model of Universe:

This new geometrical model is a sphere filled of dark substance (called *Universal sphere*) and surrounded by a medium that we will call "nothingness", which was the medium before the Big-Bang. $R_U(t)$ being the radius of the Universal sphere (defined further) at a Cosmological time t, and 1+z being the factor of expansion of the Universe between the Cosmological times t_1 and t_2:

$$R_U(t_2)=(1+z)R_U(t_1) \qquad (33)$$

2.6 Superposed sphere.

We consider a spherical concentration of dark substance with a density in $1/r^2$ (that we defined in previous sections) moving in the space. We could expect that its velocity or its mass be modified because of its motion, of the Archimedes's force or the absorption or the loss of dark substance by the moving concentration of dark substance. This effect could be negligible, but we have a justification that it is nil much more interesting:

Indeed according to proposed new theory the dark substance has 2 possible behaviors: It can behave as a substance owning a mass or as absolute emptiness. For baryonic particles immerged inside dark substance, it always behaves as absolute emptiness and consequently the velocity of baryonic particles is never modified due to an Archimedes's force generated by the motion of baryonic particles through the dark substance. According to the new theory of dark matter, the intergalactic dark substance in which the spherical concentration of dark substance is immerged also behaves as it was absolute emptiness concerning the displacement of this spherical concentration of dark substance: Neither the velocity nor the mass of the spherical concentration of dark substance are modified by its motion through the intergalactic dark substance. We will say that the spherical concentration of dark substance is a *superposed sphere* on the intergalactic dark substance surrounding it to interpret this phenomenon.

We know that in the Newton's theory of gravitation, it is assumed that only baryonic density exists, which is not the case in the new theory of dark matter, and it is also assumed that the Universe is static, which is also not the case in the MSC nor in our theory of dark matter that as the MSC admits the expansion of the Universe. The equations of the Newtonian mechanics must be adapted to our theory of dark matter, and we are going to see 3 very simple examples of the adaptation of those equations to this theory of dark matter.

In section 2.2, we assumed that we had a spherical symmetry around the centre of the galaxy O_{GA} to obtain our model of a superposed sphere with a density in $1/r^2$. But we will see that usually this spherical symmetry does not exist if the galaxy is inside a cluster. The study proposes the following first rule of adaptation of Newton's law due to the fact that dark matter can behave as absolute emptiness:

The rule of adaptation is the following:

In the case of a galaxy G_A constituted of a superposed sphere S_{CDM} with a centre O_{GA} and a radius R_{GA}:

O_{GA} is accelerated by an acceleration $\mathbf{G}(O_{GA})$, $\mathbf{G}(O_{GA})$ is defined by $\mathbf{F}_G(S_{CDM})=m(S_{CDM})\mathbf{G}(O_{GA})$, with $\mathbf{F}_G(S_{CDM})$ is the gravitational force generated on S_{CDM} by the dark substance in which S_{CDM} is immerged and baryonic matter, $m(S_{CDM})$ mass of S_{CDM}. The dark substance in which S_{CDM} is immerged and baryonic matter acts on the spherical concentration of dark matter S_{CDM} as S_{CDM} was a solid.

A consequence of the preceding law is that baryonic matter has none action on the density of dark matter in S_{CDM}.

The preceding rule of a adaptation is equivalent to the hypothesis that the dark substance in which S_{CDM} is immerged and generates a field uniform and equal to $\mathbf{G}(O_{GA})$ (defined previously) in all S_{CDM}. The preceding rule of adaptation involves that the model that we used to obtain a superposed sphere with a density of dark substance in $1/r^2$ is always valid, by assuming as a spherical symmetry.

So this is a possible 1st example of adaptation of the equations of Newtonian dynamics to our theory of dark matter.

We have seen in the section 2.3 a model of convective thermal transfer between the superposed sphere at a temperature T and the dark substance in which it is immerged at the temperature T_0. The thermal flow was:

$$\varphi=h(T-T_0) \qquad (34)$$

It is possible that the dark substance in which the superposed sphere is immerged behaves as absolute emptiness not only from a gravitational point of view, but also from a thermal point of view. This brings us to propose a 2nd model of thermal transfer between the superposed sphere and the dark substance in which it is immerged, with a thermal flow not given by the equation (34) but by the following equation:

$$\varphi=hT \qquad (35)$$

The previous flow remains analogous to a convective thermal transfer. We notice that it has the same expression of a flow of a convective thermal transfer between a medium at a temperature T and a medium at a temperature $T_0=0$.

The 2nd model of thermal transfer is very interesting because it involves the baryonic Tully-Fisher's law that the study established in Section 2.3 remains valid for any temperature T_0 of the dark substance in which the superposed sphere is immerged. It is true only with the condition $T_0 \ll T$ in the 1st model of thermal transfer.

We saw that dark substance has the remarkable property of being able to behave sometimes as absolute emptiness, without any mass, and sometimes as ordinary matter with a mass. A 2nd fundamental property of dark substance that we will admit is that sometimes it can tend to be homogeneous, its density not obeying to Newton's Law and sometimes its density obeys to Newton's Laws. This 2nd fundamental property is important because if we admit that at the scale of a star or of a black hole the tendency to homogeneity of dark substance predominates, then there is no concentration of dark substance around stars constituting a galaxy, consequently it exists 2 main kinds of distribution of dark matter in galaxies: Galaxies immerged in dark substance with a density of dark substance in $1/r^2$ and galaxies immerged in intergalactic dark substance with a density of dark substance that is constant.

2.7 Baryonic and dark radius of a galaxy.

We observe in the Section 2.1 that if r is the distance to the centre O of a spherical concentration of dark substance constituting a galaxy, then the expression of the density of dark substance ρ(r) is given by, k_3 being a constant (See section 2.2, equation (7) $k_3=k_2/4\pi$):

$$\rho(r) = \frac{k_3}{r^2} \qquad (36)$$

So we obtain, M(r) being the mass of the sphere having its center in O and a radius r (See equation (9)):

$$M(r)=4\pi k_3 r \qquad (37)$$

Consequently, v being the velocity of a star at a distance r of O (see equation (10)):

$$v^2 = \frac{GM}{r} = 4\pi k_3 G \qquad (38)$$

Consequently:

$$k_3 = \frac{v^2}{4\pi G} \qquad (39)$$

We know also that if ρ_0 is the local density of the intergalactic dark substance surrounding the spherical concentration of dark substance constituting the galaxy, then the radius R of this concentration of dark substance is given by the expression (See equation (15)):

$$\rho(R) = \frac{k_3}{R^2} = \rho_0 \qquad (40)$$

Consequently:

$$R = \sqrt{\frac{k_3}{\rho_0}} = v\sqrt{\frac{1}{4\pi G\rho_0}} \qquad (41)$$

In a previous section, we called R the *dark radius* of the considered galaxy.

So in a galaxy for which it exists a spherical concentration of dark substance with a density in $1/r^2$, we have 2 different kinds of radius:

The 1^{st} kind of radius, called *dark radius*, is the radius of the spherical concentration of dark substance. The 2^{nd} kind of radius is the radius of the smallest sphere containing all the stars of the galaxy. We will call *baryonic radius* this second kind of radius.

2.8 Other models of distribution of dark matter in the Universe.

The analysis found that dark substance is not ordinary matter and a priori does not compulsory own the physical properties of ordinary matter. For instance, according to our model of dark substance, it can behave as absolute emptiness. In this section and also in the following section of interpreting dynamics of galaxy clusters, the study will propose some new physical properties from proposed model of dark substance, those properties being simple but different from the physical properties of ordinary matter, and permitting to interpret the astronomical observations linked to dark matter.

2.8.1 The double possible behavior of dark substance.

In addition to the 1^{st} model exposed in the section 2.2 of distribution of dark substance with a density in $1/r^2$, obtained for galaxies with a flat rotation curve, we must also consider the 2nd model of distribution of dark substance with a constant density $\rho(r)=\rho_0$, ρ_0 density of dark substance in which the galaxy is immerged. Generally, ρ_0 is the density of the intergalactic dark substance that we assumed to be homogeneous in temperature and in density in section 2.2.

The 2nd model of distribution of dark substance is the consequence of a possible behavior of the dark substance that is to be homogeneous in density, in violation of the equation of the equilibrium of the forces.

Therefore, we observed that dark substance can behave in 2 different ways: Either it is homogeneous in density (in a given volume) in violation of the equations of equilibrium (as the intergalactic dark substance), either its density obeys to the equations of the equilibrium of forces (As in our model of galaxies with a flat rotation curve).

The study defines, according to our model of dark substance, in which case dark substance behaves according to the first way and in which case it behaves according to the 2nd way. The study also showed that the dark halo of a galaxy with a flat rotation curve was constituted of a superposed sphere of dark substance. This brings the following hypothesis a) for our model of dark substance:

Hypothesis a):

Dark substance owns a constant density everywhere in the Universe outside the superposed spheres.

It is attractive to assume that inside a superposed sphere S, dark substance keeps the main properties that it owns in the Universe out of any superposed sphere. Eventually, we generalize our model of dark substance of the hypothesis a) by the hypothesis b):

Hypothesis b):

A local concentration of dark substance inside a volume dV belonging to a superposed sphere S (dV being small relative to the volume of S) can exist only if dV belongs to a sphere of dark substance S' superposed to S.

The preceding hypothesis a) and b) bring to obtain a very simple density of dark matter at any point of the Universe.

Researchers can wonder if it can exist several levels of superposed sphere, meaning if it is possible that a sphere full of dark substance S' can be superposed to a sphere full of dark substance S, as in the case of the hypothesis b). The simplest hypothesis would be that this is not possible, and this hypothesis seems to agree with observations. As a result, the following hypothesis c) is acceptable in our model of dark substance:

Hypothesis c):

It cannot exist several levels of superposed sphere.

The hypothesis a) implies that if in the Universe a star does not belong to a superposed sphere, there is not concentration of dark substance locally around it. The hypothesis b) and c) imply that inside a superposed sphere S constituting the dark halo of a galaxy with a flat rotation curve, there are no local concentrations of dark substance, not locally around stars nor locally around dwarf galaxies.

If the hypothesis b) and c) are true there are no concentrations of dark substance locally around the Magellanic clouds. Nonetheless, if the study discover using astronomical observations that the Magellanic clouds are galaxies with a flat rotation curve and obeying to the baryonic law of Tully-Fisher, this would predict that the Hypothesis c) is wrong (keeping the hypothesis b). But the hypothesis c) is not necessary to our theory of dark matter, and our justification of the baryonic law of Tully-Fisher can be applied to a sphere S' superposed to a superposed sphere S. Nonetheless, according to most recent observations, neither the Large Magellanic cloud nor the Small Magellanic cloud are galaxies with a flat rotation curve obeying to the baryonic law of Tully-Fisher.

We have a last fundamental hypothesis concerning the dark substance and explaining many observations:

Hypothesis d):

-Baryonic matter has no effect on the density of dark substance, and consequently we must take everywhere a nil value of baryonic matter in order to get the density of dark substance.

-Neither the intergalactic dark substance neither a superposed sphere S_A have any effect on the density of a superposed sphere S_B different from S_A.

We will predict that ordinary baryonic matter and the superposed sphere S_A have a *global gravitational effect* on the superposed sphere S_B. This will mean that despite neither ordinary matter nor S_A have any effect on the density of the dark substance constituting S_B, the gravitational force that they generate on S_B is obtained but its application point is the centre of S_B. An alternative to Hypothesis d) would be that at the scale of stars, the tendency of homogeneity of dark substance predominates.

Our theory of dark matter permits to obtain an estimation of the mass of the Milky Way in agreement with its estimation through astronomical observations.

Indeed further observations linked to the dynamical model of galaxy clusters according to our theory of dark matter permit to obtain an estimation of the density of the intergalactic dark substance ρ_0 and consequently using the equation (41) to obtain an estimation of the radius of the halo of dark matter of the Milky Way R_H equal to 550000 l.y. Then the study can obtain an estimation of the mass of the Milky Way $M_{M.W}$, v being the orbital velocity at a distance R_H of the centre of the Milky Way using the equation:

$$GM_{M.W}/R_H=v^2 \qquad (41A)$$

Taking $v \approx 205$ km/s we obtain $M_{M.W} \approx 1540\ 10^9$ S.M, that is exactly its very recent estimation by teams of NASA and ESA (WATKINS et al. 2019) .

2.8.2 The generation of the superposed spheres.

An interesting research gap is to determine the way the superposed spheres of dark substance appear in the Universe. We found that we do not observe concentrations of dark matter locally around stars nor around black holes with a low mass. This means according to our preceding hypothesis a) and b) that there are none superposed spheres locally around stars nor around black holes with a low mass, and consequently we will admit the following hypothesis e):

Hypothesis e):

No planets, nor stars nor black holes with weak masses generate superposed sphere.

Nonetheless it is possible that superposed sphere be generated by super-massive black holes. If it is the case, it should exist a super-massive black hole at the centre of each galaxy with a flat rotation curve and reciprocally any galaxy which the central point is the super-massive black hole should be a galaxy with a flat rotation curve. It is also possible that superposed sphere be generated by primordial black holes (meaning appeared in the primordial very dense Universe), but disappeared today.

So, we have 2 main possibilities for the formation of superposed sphere: Either they are generated by some celestial objects, as for instance the super-massive black holes, either they are generated by some phenomena in the primordial Universe.

2.8.3 The rotation curve of galaxies with a flat rotation curve close to the centre of those galaxies.

We obtained in our model of galaxies with a flat rotation curve a density in $1/r^2$ (r distance to the centre of the galaxy). Nonetheless the astronomical observations show that close to the centre the rotation curve is not flat, and that we have $v(r)=0$ for $r=0$.

We have the following simple explanation to justify this:

We have previously seen that dark substance had 2 possible behaviors: It was homogeneous in density, violating the equations of equilibrium of forces, either its density obeyed to the equations of the equilibrium of forces. We propose the simple following explanation, Hypothesis f), for our model of dark substance to justify the aspect of the rotation curve of galaxies close to r=0.

Hypothesis f):

T being any temperature, it exists a maximal density $\rho_M(T)$ for which dark substance can behave in agreement with the equation of the equilibrium of forces. For a density superior or equal to $\rho_M(T)$, dark substance behaves as a substance homogeneous in density.

With the previous hypothesis f), we obtain that for a galaxy with a flat rotation it exist a distance d_0 such that for $0<r<d_0$ the density of dark substance is equal to $\rho_M(T)$ and for $d_0<r$ $\rho(r)$ decreases till ρ_0, density of the intergalactic dark substance. For r sufficiently great, we obtain that the curve $\rho(r)$ is asymptotic to the curve in $1/r^2$ obtained in our first model without the hypothesis f). So, we obtain a rotation curve in agreement with observation. We could improve our model considering the baryonic matter.

To determine $\rho(r)$ with the hypothesis f) we proceed as follows (without taking into account the lower limit of $\rho(r)$ that is equal to ρ_0):

a being a positive reel we define the function $\rho_{Sa}(r)$ by:

(i) For $0\leq r\leq a$: $\rho_{Sa}(r)=\rho_M(T)$

(ii)For $a<r$:$\rho_{Sa}(r)$ is solution is solution of the equation of the equilibrium of forces and is consequently asymptotic to the curve in $1/r^2$ obtained in the model without the Hypothesis f)

We then define the function $\rho_{Sam}(r)$ as the (unique) function among the previously defined functions $\rho_{Sa}(r)$ verifying:

(i)For any r, $\rho_{Sa}(r)\leq\rho_M(T)$
(ii)a is minimal.

Then the solution of the density of dark matter in a galaxy with a flat rotation curve considering the hypothesis f) is $\rho_{Sam}(r)$. Moreover $d_0=a_m$. We can easily adapt what precedes considering the lower limit of the density of dark substance that is equal to ρ_0.

2.8.4 The inter cluster medium and the baryonic law of Tully-Fisher.

The astronomical observations have showed the existence inside galaxy clusters of a plasma constituted of baryonic matter; this plasma being called *inter cluster medium*. This plasma constitutes an important part of the mass of a cluster, generally more important than the mass of all the galaxies belonging to this cluster.

But to obtain the baryonic law of Tully-Fisher for a galaxy according to our theory of dark matter, we considered that all the baryonic particles inside the halo of the considered galaxy transmit thermal energy to the dark substance constituting this dark halo. And if we considered the plasma, then we would not obtain the baryonic law of Tully-Fisher taking into account only the mass of the stars and the mass of visible gas of the considered galaxy, which was what we did.

We propose the following explanation: The plasma is constituted of ionized particles, generally helium or hydrogen. We obtain the baryonic law of Tully-Fisher taking as baryonic mass only the mass of stars and visible gas of galaxies if we state that if a baryonic particle is charged as for instance a ionized particle, then it does not transmit thermal energy to the dark substance in which it is immerged.

The astronomical observations show that the particles of the plasma do not cool down.

2.8.5 Collisions between dark matter and baryonic matter.

None astronomical observations proved the existence of collisions between dark matter and baryonic matter. This is very well explained in our Theory of dark matter. According to this theory, dark substance is a substance filling all the space and that can behave as absolute emptiness. It is evident that collisions between absolute vacuum and baryonic matter are impossible. According to our Theory of dark matter it does not exist Archimedes's pressure acting on a particle moving inside dark substance for the same reason. The Theory of dark matter does not predict any possible collision between baryonic matter and dark substance.

2.9 Other observations of dark matter.

It exists a priori 2 possible main models that concerning distribution of dark substance inside galaxy clusters. In the first model, the study demonstrates in details, the observed mass of dark substance in a galaxy cluster is much greater than the total mass of dark halos of galaxies contained by the considered galaxy cluster. On the contrary in the 2nd model of distribution of dark matter inside galaxy clusters the observed mass of dark substance of a galaxy cluster is equal to the total mass of dark halos of superposed spheres belonging to the considered galaxy cluster. We will observe that in the 1st model we must consider the mass of intergalactic dark substance, that is the dark substance outside dark halos that we assumed to be at a homogeneous density. Then it is necessary to admit a double possible gravitational behavior for the intergalactic dark substance depending on its localization inside or outside a concentration of baryonic matter. So in our first model of dark matter in galaxy clusters we will admit the fundamental property:

-If a point P belongs to a concentration of baryonic matter (galaxy cluster, concentration due to anisotropies of baryonic matter in the early Universe), then we must take the real density of dark matter at P in Newton's equations. If P does not belong to any concentration of baryonic matter nor to any dark halo, then we must take at point P a nil density in Newton's equations.

We remind that models of formation of galaxies (structure formation) need dark matter. The previous property could be the origin of the effect of dark matter in structure formation.

We are now going to interpret using our new theory of dark matter experimental data linked to the velocities of galaxies in galaxy clusters. We will only study the 1st model.

In the 1st model of distribution of dark matter we take into account all the mass of dark substance contained by the galaxy cluster.

According to what precedes, the velocity of a galaxy in a cluster is determined by:

-The baryonic mass inside the cluster (stars, gas..)
-The mass of the dark halos of galaxies.
-The mass of the intergalactic dark substance.

We admit using the preceding section that the galaxy cluster contains only either galaxies with a density of dark substance in $1/r^2$ as defined in the section 2.1 (1st model of distribution of dark matter around galaxy) or galaxies with a homogeneous density of dark matter equal to ρ_0, density of the intergalactic dark substance (2nd model of distribution of dark matter around galaxy).

We obtain a very interesting result concerning the mean density of galaxies corresponding to the 1st model of distribution (density of dark substance in $1/r^2$):

Indeed, according to the equation (18), for those galaxies the dark radius is:

$$R_S=(2k_0T/4\pi G\rho_0)^{1/2} \qquad (42)$$

According to the equation (8) :

$$k_2=2k_0T/G \qquad (43)$$

Consequently :

$R_S=(k_2/4\pi\rho_0)^{1/2}$ (44)

So according to the equation (9) the total mass of the dark halo is:

$$M_S(R_S)=\frac{k_2^{3/2}}{(4\pi\rho_0)^{1/2}} \qquad (45)$$

Let us now calculate the mass of a sphere with the same radius R_S and a density equal to the density of the intergalactic dark substance ρ_0 :

$$M_I(R_S)=\rho_0\frac{4}{3}\pi\left(\frac{k_2}{4\pi\rho_0}\right)^{3/2}=\frac{1}{3}\frac{k_2^{3/2}}{(4\pi\rho_0)^{1/2}} \qquad (46)$$

Consequently :

$M_I(R_S)=M_S(R_S)/3$ (47)

So the mean density of the halos of galaxies belonging to the 1st model of distribution of dark matter is equal to $3\rho_0$, whatever be the radius and the temperature of the considered halo, and consequently whatever be the orbital velocity of stars in the considered galaxy.

According to the previous equation (47) we can assume that the dark mass of a cluster be much greater than the baryonic matter in the galaxies of this cluster. Indeed, according to the theory of dark matter, for a galaxy corresponding to the 1st model of distribution of dark substance, R_B being the baryonic radius of the galaxy, then the mass $M_B(R_B)$ of baryonic matter contained in the sphere with a radius R_B (centre O, centre of the galaxy) was much lower than the mass $M_S(R_B)$ of the dark substance contained in the same sphere. Because $R_B<R_S$, the total mass of the dark halo $M_S(R_S)$ is much greater than the total mass of baryonic matter contained by the galaxy . But according to the equation (47), the mean density of the halo is only 3 times of the minimum density of dark matter inside the cluster. (Because we supposed that only the 1st and the 2nd model of distribution of dark matter existed for galaxies). The study also assumes that the dark mass of clusters be much greater than the baryonic mass of the galaxies belonging to this cluster.

So for a cluster A with a mean density ρ_{mA}, we obtain if we neglect the baryonic density :

$\rho_0<\rho_{mA}<3\rho_0$ (48)

The mean densities of clusters permit to obtain an estimation of the density ρ_0 of the intergalactic dark substance. Moreover if A1 and A2 are 2 clusters with mean densities ρ_{mA1} and ρ_{mA2} with for instance $\rho_{mA1}<\rho_{mA2}$, then according to the previous relation :

$\rho_{mA2}<3\rho_{mA1}$ (49)

We will see that the preceding theoretical prediction agrees with astronomical observations.

It is interesting to introduce the mean volume of dark halo corresponding to the 1st model of distribution of dark substance per galaxy Vol_{SG}. Then if clusters contain the same kind of galaxies in the same proportions (which is not always the case), we can express the mean density of dark substance ρ_{mA} as a function of N_A the number of galaxies inside the cluster A, and Vol_{SG}. Indeed we immediately obtain, using that the mean density of dark halos corresponding to the 1st model of distribution of dark substance is equal to $3\rho_0$ (Equation (47)) and that elsewhere the density of dark substance is equal to ρ_0 ,Vol_A being the volume of the cluster:

$$\rho_{mA} = \frac{1}{Vol_A}[3\rho_0 N_A Vol_{SG} + \rho_0(Vol_A - N_A Vol_{SG})] \quad (50)$$

So we obtain, ρ_{mAG} being the mean density of the number of galaxies in the cluster, $\rho_{mAG}=N_A/Vol_A$:

$$\rho_{mA}=\rho_{mAG}(2\rho_0 Vol_{SG})+\rho_0 \quad (51)$$

Moreover, $Vol_A(H)$ being the volume of dark halo of galaxies belonging to the 1st model in the cluster A, we have always, still using that the mean density of dark halos corresponding to the 1st model of distribution of dark substance is equal to $3\rho_0$ (Equation (47)) and that elsewhere the density of dark substance is equal to ρ_0:

$$\rho_{mA} = \frac{1}{Vol_A}[3\rho_0 Vol_A(H) + \rho_0(Vol_A - Vol_A(H))] \quad (52)$$

$$\rho_{mA} = 2\rho_0 \frac{Vol_A(H)}{Vol_A} + \rho_0 \quad (53)$$

An important case is the case in which we have $Vol_A(H)/Vol_A<<1$ for all clusters. Then we have for all clusters ρ_{mA} very close to ρ_0 for all clusters. This implies, ρ_0 depending on the Cosmological redshift z, that clusters corresponding to the same z have approximately the same mean density ρ_{mA} very close to $\rho_0(z)$.

We remind that we assumed that we could neglect the contribution of baryonic matter to obtain the mean density of the cluster ρ_{mA}. In what follows, always according to the 1st model of distribution of dark substance, we will assume that we have generally for clusters $Vol_A(H)/Vol_A<<1$ and consequently $\rho_{mA}\approx\rho_0$. We remind that ρ_0 depends on t, age of the Universe. We will see further that the previous assumption is in agreement with astronomical observations.

In the 2nd model of distribution of dark substance in galaxy clusters, the density of dark substance interacting gravitationally is the one of the mass of dark halos:

$$\rho_{mA}=3\rho_0 Vol_A(H)/Vol_A \quad (54)$$

Despite that in the 2nd model density of dark substance interacting gravitationally is not homogeneous, it presents approximately a spherical symmetry by assumption. Because of this spherical symmetry it will be possible to make the approximation of a homogeneous density equal to ρ_{mA} to obtain a relation between the mass of a galaxy cluster, its radius and the maximal velocity of the galaxies that it contains, using the 3 dynamical model of galaxies, with a homogeneous density, that we are going to expose. We could also expect in this 2nd model that ρ_{mA} be of the order of ρ_0.

We have 3 dynamical models of clusters permitting to obtain some relations between the mass of clusters and the velocities of galaxies belonging to those clusters were studied. Only the 3rd model is new and the 2nd model is generally admitted in the SCM, but without model of dark matter. We will observe that the 3 models have theoretical predictions that are close one another concerning the relations for a given cluster A between the mass of this cluster, its radius, and the dispersion velocity of the galaxies or the maximal recession velocity of galaxies of this cluster A. Nonetheless, we will observe that the 1st dynamical model is not compatible with astronomical observations, and the 3rd dynamical model is based on our model of dark matter and moreover permits to interpret some astronomical observations not interpreted by the 2nd dynamical model. In what follows we will study the 1st model of distribution of dark substance in clusters and we will observe that its theoretical predictions are in good agreement with astronomical observations.

According to a 1st dynamical model of clusters, galaxies turn around the centre of a cluster the same way planets turn around the sun or stars turn around the centre of the Milky Way. So we will call the *planetary dynamical model* of clusters this 1st model. We stated that this model is contradicted by astronomical observations.

Some astronomical observations that are very important to study the validity of the study's different dynamical models of clusters have been realized concerning the Coma cluster that we will name A4 (COLESS 1996; BIVIANO 1997). Using some astronomical observations of the Coma cluster, some astrophysicists realized a graph giving for some galaxies G belonging to the Coma cluster the recession velocity $V_R(G)$ observed from a point O_T close to the earth and being the origin of an inertial frame R_T in which the velocity of the earth is small relative to c, as a function of the angle a(G) between the lines (O_T,O_4) and (O_T,O_G), with O_4 the centre of the Coma cluster and O_G the centre of the galaxy G (Or equivalently as a function of $d(G)=a(G)O_TO_4$, O_TO_4 angular distance between O_T and O_4).

According to this graph, the gap between the maximal recession velocity and the minimal recession velocity is maximal for an angle a(G)=0 . Then it decreases.

We will observe that those astronomical observations can be interpreted by our 3rd dynamical model of galaxy clusters, as for instance the symmetries of the previous graph relative to the axis O_TO_4 and relative to the horizontal axis containing O_4, and also the maximal and minimal velocities for d(G)=0 and $d(G)=R_{A4}$, R_{A4} radius of the galaxy cluster.

A 2nd possible dynamical model of clusters is the model generally used in the Standard Cosmological Model (SCM) (NARLIKAR 2002) based on the Virial's theorem. So we will name this model the *Virial's dynamical model* of clusters.

According to this model, if σ_A is the velocity dispersion inside a cluster A, M_A being the mass of the cluster and R_A its radius:

$$\frac{GM_A}{R_A} \approx \alpha_A \sigma_A^2 \qquad (56)$$

In the previous expression, α_A is of the order of the unity and depends on the cluster A. Very often we take it equal to 1 or 2. We can also replace in the preceding expression R_A by the Abel radius (RAINE&THOMAS 2001).

We remind that the equation (56) obtained by the Virial's model seem to be approximately in agreement with astronomical observations. We will see that it will be also the case for the 3rd dynamical model of cluster.

We are now going to propose a 3rd dynamical model of clusters based on our model of dark matter. In this model, G_A being a galaxy of a cluster A situated at a point P of the cluster, we consider only the gravitational potential generated in P by the dark substance. So we will name this 3rd model the *dynamical model of the dark potential* of clusters.

In order to obtain in this 3rd model the gravitational potential generated by the dark substance at any point of the cluster, it is necessary to expose the elements of our theory of dark matter permitting to calculate the gravitational field **G** and the gravitational potential U at any point of the Universe. We have already seen 2 examples of adaptation of the equations of Newtonian mechanics to our theory of dark matter (Section 2.6 and 2.8). We have seen that those adaptations are necessary because in the Newton's Theory of Gravitation, only baryonic matter exists and moreover, there is no expansion, which is not the case in our theory of dark matter. In order to obtain **G**(Q) and U(Q) at a point Q of the Universe using the equations of Newtonian mechanics, in order to take into account the density of dark substance at a point P, we remind that we must distinguish the cases in which P is inside a concentration of baryonic matter or if it is not the case. Indeed, we have seen the fundamental property:

a)Let us suppose that P is a point of the Universe belonging to none concentration of baryonic matter or of dark substance, but belonging to the intergalactic dark substance. We know that the density of dark substance in P is equal to ρ_0 (Section 2.3 and 2.8). Because of the expansion of the Universe and of the properties of dark substance, we will admit in our theory of dark matter that there is a symmetry for all points P with the preceding properties, involving that we must take $\rho(P)=0$ in the equations of Newtonian mechanics in order to obtain **G**(Q) and U(Q) at a point Q. This means that dark substance behaves as it was absolute emptiness in P, the same way as in Section 2.8.

So the previous rule a) justifies that between clusters, dark matter behaves as absolute emptiness, in agreement with astronomical observations.

b)If P belongs to an important concentration of baryonic matter (cluster, galaxy, star, concentration due to anisotropies of baryonic matter in the early Universe...), then the symmetry in P is broken: We must take $\rho(P)=\rho_0$ (or $\rho(P)$ is equal to the density of dark substance in P) in the equations of Newtonian mechanics in order to obtain **G**(Q) and U(Q).

So we have a 3rd example of adaptation of the equations of Newtonian mechanics to our theory of dark matter that is due to the expansion of the Universe, that did not exist in the Newton's Theory of Gravitation.

In this 3rd dynamical model of cluster, we model a cluster as a system (ideal cluster) with the following properties:

a)The cluster is a sphere with a radius R_A, containing galaxies and dark substance, presenting a spherical symmetry.

b)In order to obtain **G** and U in the cluster, permitting to obtain the velocities, accelerations and energies of the galaxies of the cluster, those galaxies being modeled as punctual masses (coinciding with their centre of mass), we can consider that inside the cluster, the density is homogeneous and equal to ρ_{mA}. (Because of the equation (53), assuming $Vol_A(H)/Vol_A<<1$ and neglecting the baryonic matter of the cluster).

Concerning the galaxies of the cluster, the velocities and energies are calculated in the frame from the origin is O_A centre of the cluster. Galaxies of the cluster are modeled the following way :

c)We define for a galaxy G_A the ratio $r(G_A)$ defined by $r(G_A)=E_T(G_A)/m(G_A)$ ($E_T(G_A)$ total energy of the galaxy G_A and $m(G_A)$ mass of G_A) and r_{AMax} as being the maximal value of this ratio. Then according to our model of galaxy cluster:
(i)The radius R_A of the cluster is the maximal possible distance between a galaxy G_A of the cluster and O_A centre of the cluster (with the condition $r(G_A)\leq r_{AMax}$).

(ii)The galaxies G_A with $r(G_A)=r_{AMax}$ have a great density in the cluster (not compulsory homogeneous). This means that at any point Q of the cluster, it exists a galaxy G_A close to Q such that $r(G_A)=r_{AMax}$. Moreover in the case in which $Q=O_A$ centre of the cluster, because of the spherical symmetry if **u** is any unitary vector, it exists a galaxy G_{A0} close to O_A with $r(G_{A0})=r_{AMax}$ such that, $\mathbf{V}(G_{A0})$ being the vector velocity of G_{A0}: $\mathbf{V}(G_{A0}).\mathbf{u} \approx V(G_{A0})$, with $V(G_{A0})$ norm of $\mathbf{V}(G_{A0})$. (This means that the vector $V(G_{A0})$ is approximately collinear to **u**).

d)The galaxies G_A such that $r(G_A)=r_{AMax}$ keep their energy and their mass, and consequently r_{AMax} is constant.

Therefore,according to the preceding property a) of our model of cluster and also to our adaptation of the equations of the Newtonian mechanics (Preceding example):

$$U(R_A)=-GM_A/R_A \qquad (57a)$$

$$\mathbf{G}(R_A)=-GM_A/R_A^2\ \mathbf{u} \qquad (57b)$$

Moreover, G_A being a galaxy situated at a distance r from O_A, $m(G_A)$ and $V(G_A)$ being the mass and the velocity of G_A the total energy $E_T(G_A)$ of G_A is therefore, U(r) being the gravitational potential at a distance r from O_A:

$$E_T(G_A)=(1/2)m(G_A)V(G_A)^2+m(G_A)U(r) \qquad (58)$$

Using the spherical symmetry of our model of cluster, applying the Gauss theorem, M(r) being the mass of the sphere with the centre O_A and the radius r, the gravitational field **G**(r) is then:

$$\mathbf{G}(r) = -G\frac{M(r)}{r^2}\mathbf{u} \qquad (59)$$

According to the property b) of our model of cluster, M(r)=(4/3)πr3ρmA and consequently :

$$\mathbf{G}(r) = -G\frac{4}{3}\pi r\rho_{mA}\mathbf{u} \qquad (60)$$

By definition **G**=-**Grad**(U), so we obtain, C_{AU} being a positive constant at a given age of the Universe:

$U(r)=G(4/6)\pi r^2\rho_{mA}-C_{AU}$ (61)

This equation can also be written, in the approximation that the density of dark matter in the cluster is approximately constant an equal to ρ_{mA}, M(r) being the mass of the sphere with the centre O_A and a radius r :

$U(r)=GM(r)/2r-C_{AU}$ (62)

Consequently we have, $M_A=M(R_A)$ being the mass of the cluster, using the equation (57a) :

$$\frac{GM_A}{2R_A}-C_{AU}=-\frac{GM_A}{R_A} \quad (63)$$

So we finally obtain, with M_A and R_A depending a priori on t, age of the Universe:

$$C_{AU}=\frac{3}{2}\frac{GM_A(t)}{R_A(t)} \quad (64)$$

Therefore, using the equation (58), for a galaxy at a distance r from O_A :

$$\frac{1}{2}m(G_A)V(G_A)^2+Gm(G_A)\frac{M(r)}{2r}=E_T(G_A)+m(G_A)C_{AU} \quad (65a)$$

Moreover we have defined, in the property c) of our model of cluster, r_{AMax} as being the maximal value of $r(G_A)=E_T(G_A)/m(G_A)$. So we have for any galaxy G_A:

$$\frac{1}{2}V(G_A)+G\frac{M(r)}{2r}\leq r_{AMax}+C_{AU} \quad (65b)$$

We are now going to consider a galaxy G_{Al} at the limits of the cluster ($r=R_A$) and a galaxy G_{A0} in O_A (r=0).

According to the property c)(i) of our model of cluster, the radius R_A of the cluster is the maximal possible distance between a galaxy G_A of the cluster and O_A the centre of the cluster with the condition $r(G_A)\leq r_{AMax}$. Considering the previous inequality (65b) we have therefore for a galaxy G_{Al} at the limit of the cluster, $V(G_{Al})=0$ and:

$$G\frac{M(R_A)}{2R_A}=r_{AMax}+C_{AU} \quad (66)$$

For a galaxy G_{A0} situated at the centre of the cluster (r=0), such that $r(G_{A0})=r_{AMax}$, according to the equation (65a):

$$\frac{1}{2}V(G_{A0})^2 = r_{AMax} + C_{AU} \qquad (67)$$

Therefore, because of the equation (65b), $V(G_{A0})$ is equal to the maximal velocity of the galaxies in the cluster V_{MA}. Consequently, using the equations (66) (67) we obtain:

$$V_{MA}^2 = \frac{GM_A}{R_A} \qquad (68a)$$

Moreover according to the property c) of our model of cluster, **u** being any unitary vector, it exists a galaxy G_{A0} close to O_A such that $r(G_{A0})=r_{AMax}$ and $\mathbf{V}(G_{A0}).\mathbf{u} \approx V(G_{A0})$ ($\mathbf{V}(G_{A0})$ vector velocity of G_{A0} and $V(G_{A0})$ its norm). Consequently if we define $V_{MA}(\mathbf{u})$ as the maximal value of $\mathbf{V}(_{GA}).\mathbf{u}$, considering all galaxies G_A of the cluster, then $V_{MA}(\mathbf{u}) \approx V_{MA}$.

In the astronomical observations, G_A being a galaxy of the cluster, **u** being the unitary vector of the direction of observation, we measure $V_T(G_A)(\mathbf{u}) = \mathbf{V}_T(G_A).\mathbf{u}$, component on **u** of the vector velocity $\mathbf{V}_T(G_A)$, velocity of G_A in an inertial frame R_T whose the origin is a point O_T close to the earth, and in which the velocity of the earth is small relative to c. We then obtain $V_{MA}(\mathbf{u})$ by the following expression, with evident notations:

$$V_{MA}(\mathbf{u})=(1/2)[Max_A(V_T(G_A)(\mathbf{u}))-min_A(V_T(G_A)(\mathbf{u}))] \qquad (68b)$$

Considering that the validity of our model of cluster described by the properties a)b)c)d) is only an approximation, we introduce a constant β_A, depending on the cluster and on the vector **u**, such that, $V_{MA}(\mathbf{u})$ being defined by the previous expression (68b):

$$V_{MA}(\mathbf{u})^2 = \beta_A \frac{GM_A}{R_A} \qquad (69)$$

So we obtain in our 3rd model of the dark potential an equation analogous to the equations (55)(56). Nonetheless, this 3rd model predicts that the velocity of galaxies is maximal for galaxies close to the centre of the cluster, in agreement with astronomical observations (RAINE&THOMAS 2001), which is not the case for the 2nd Virial's model.

Moreover, A_i and A_j being 2 clusters, using $M_{Ai}=(4/3)\pi\rho_{mAi}R_{Ai}^3$, we obtain immediately, using the equation (68a) :

$$\frac{\rho_{mAj}}{\rho_{mAi}} = (\frac{V_{MAj}}{V_{MAi}})^2 (\frac{R_{Ai}}{R_{Aj}})^2 \qquad (70a)$$

But we have seen in the equation (53) that if Ai and Aj are 2 galaxy clusters corresponding to the same Cosmological redshift z, if moreover $Vol_{Ai}(H)/Vol_{Ai} \ll 1$ and $Vol_{Aj}(H)/Vol_{Aj} \ll 1$, then ρ_{mAj}/ρ_{mAi} should be close to the unity.

We have not enough data in order to validate or invalidate the previous model of dark matter in galaxy clusters, and previous equation (70a). Moreover, real clusters can only approximately modeled as ideal clusters. But the few data we have, relative to Coma's and Virgo's clusters are in agreement with this model.

Consider for instance the Virgo cluster A2 (z_2<0,01) and the Coma cluster A4 (z_4<0,03). According to astronomical observations considering the galaxies NGC4388 and IC3258 and also galaxies with greatest velocity relative to the centre of the considered cluster we can take $V_{MA2}(\mathbf{u}_2)$=1600 km/s (SEDS MESSIER DATABASE 2006). Moreover we can take R_{A2}=7,3 millions l.y (FOUQUE et al. 2001). (The values of V_{MA2} and R_{A2} are also those given by Wikipedia, "Virgo cluster"). For the Coma cluster, we can take V_{MA4}=2300 km/s (COLESS&DUNN 1996; BIVIANO 1997) and we take the presently admitted value, given by Wikipedia, "Coma cluster", R_{A4}=10 million l.y=3Mpc. Then we obtain using the previous experimental data and the equation (70a) ρ_{mA4}/ρ_{mA2}=1,1. The gap of the previous ratio and 1 could be explained by the fact that the validity of our model is only approximate. We did not consider that the proportion of the mass of baryonic matter and of the dark halos of spiral galaxies is not compulsory the same in the 2 clusters. Moreover, those 2 clusters are not ideal clusters, only Coma cluster is approximately spherical (regular cluster), the Virgo cluster being an irregular cluster, and none of them are homogeneous, because of the heterogeneity of baryonic matter and of dark halos of spiral galaxies.

Taking into account of the approximate validity of our model, we can expect that the ratio given by the previous equation (70a) be of the order of the unity which is the case.

According to the property d) of our model of cluster, r_{AMax} keeps itself to obtain the evolution of the mass and the radius of a galaxy cluster. According to the equation (64), replacing the Cosmological time t by the corresponding Cosmological redshift z, $C_{AU}(z)=(3/2)GM_A(z)/R_A(z)$. So using the equation (66) we obtain:

$$r_{AMax} = -G\frac{M_A(z)}{R_A(z)} \qquad (70b)$$

Therefore, because according to the property d) of our model of galaxy cluster r_{AMax} keeps itself, $M_A(z)/R_A(z)$ also keeps itself. Moreover $M_A(z)=(4/3)\pi R_A(z)^3\rho_{mA}(z)$, and according to the equation (53), with $Vol_A(H)/Vol_A<<1$, $\rho_{mA}(z)\approx\rho_0(z)$, $\rho_0(z)$ being the density of the intergalactic dark substance for the Universe corresponding to a Cosmological redshift z. Therefore, according to the previous equation (70b), the evolution of $M_A(z)$ and $R_A(z)$ is in $1/\rho_0(z)^{1/2}$. But we will see further in this section that $\rho_0(z)\approx\rho_0(0)(1+z)^3$. Consequently we have:

$$M_A(z)\approx M_A(0)/(1+z)^{3/2}$$

$$R_A(z)\approx R_A(0)/(1+z)^{3/2} \qquad (70c)$$

For instance we obtain $M_A(2)\approx M_A(0)/5$, $M_A(1)\approx M_A(0)/3$. Which means that for instance the Coma cluster was approximately 5 times lighter for a Universe corresponding to a Cosmological redshift z=2. Nonetheless, it is possible that r_{AMax} depend on z, permitting to obtain $M_A(z)$ constant, and therefore a constant mean density of dark matter in the Universe.

The fact that it seems that there is more dark matter close to the centre of clusters could be explained by the fact that the most massive galaxies with a flat rotation curve are close to the centre of clusters.

The density of the intergalactic dark substance depends on the age of the Universe. We will use as previously the notation $\rho_0(0)$ to represent the density of dark matter at the present age of the Universe (z=0) and $\rho_0(z)$ in order to represent the density of the intergalactic dark substance at the age of the Universe corresponding to a cosmological redshift z. The estimation of the intergalactic density $\rho_0(0)$ obtained using the previous 3[rd] dynamical models of clusters permits other theoretical predictions confirming the validity of our model of dark matter.

Indeed, according to the equation (18), for a galaxy corresponding to the 1[st] model (density of dark substance in $1/r^2$) immerged in the intergalactic dark substance, the radius R_S of this galaxy is given by, at the present age of the Universe:

$$R_S = \left(\frac{2k_0T}{4\pi G\rho_0(0)}\right)^{1/2} \qquad (70d)$$

Therefore, v being the orbital velocity of stars in this galaxy, according to the equation (10):

$$R_S = \frac{v}{(4\pi G\rho_0(0))^{1/2}} \qquad (70e)$$

But the dynamical model of the dark potential exposed previously permits to obtain an estimation of $\rho_0(0)$. Let us for instance consider the case of the Milky Way. To get $\rho_0(0)$, we apply the dynamical model of the dark potential to the Virgo cluster A2(z_{A2}<0,01). According to the equation (68) we obtain, ρ_{mA} being the mean density of the cluster A, and using $M_A=\rho_{mA}(4/3)\pi R_A^3$:

$$\rho_{mA} = \frac{1}{(4/3)\pi G}\frac{V_{MA}^2}{R_A^2} \qquad (70f)$$

If A is a cluster with z_A very close to 0, and assuming $Vol_A(H) << Vol_A$ in the equation (53), then $\rho_{mA} \approx \rho_0(0)$. Therefore, replacing $\rho_0(0)$ in the equation (70e) by ρ_{mA} given by the equation (70f):

$$R_S = \frac{v}{\sqrt{3}}\frac{R_A}{V_{MA}} \qquad (70g)$$

Taking as the cluster A the Virgo cluster A2, with the preceding experimental data , z_{A2}<0,01, R_2=7,3 million l.y, $V_{M2} \approx 1600$ km/s and v≈205 km/s, we find the dark radius of the Milky Way $R_{SM.W} \approx 540000$ l.y. With the data given previously of Coma cluster we obtain $R_{SM.W} \approx 510000$ l.y. Those results are not only coherent, but they both also give a dark radius of the Milky Way superior to the distance between the centre of the Milky Way and the Magellanic clouds (approximately 250000 l.y) (ALVES&NELSON 2000). So this is also a new and remarkable prediction of our model of dark matter. The difference of 5% between the 2 obtained values has already be justified by the approximation of the validity of our models. Researchers can expect that the values of R_S obtained by different data be of the same order which is the case by considering the approximate validity of our model. The fact that Coma cluster is approximately spherical brings us to retain the value using the data of Coma cluster. Moreover if we take into account the difference between ρ_{mA4} and $\rho_0(0)$, for instance if we have $\rho_{mA}\approx 1,2\rho_0(0)$ we obtain $R_S\approx 550000$ l.y. We used this value to predict the mass of the Milky Way, in good agreement with most recent estimations.

It exists observation of an effect called *gravitational lensing*, predicted by General Relativity, that consists in a deviation of luminous rays due to the mass of clusters. According to the 3rd example of adaptation of the equations of Newtonian mechanics, the dark substance between clusters behaved as it was absolute vacuum in the equations of Newtonian mechanics. Consequently, generalizing this to the equations of General Relativity, to obtain the deviation of a luminous ray by a cluster, we can apply the equations of General Relativity as if the cluster was surrounded by absolute vacuum. It would be interesting to compare the mass of a cluster obtained by gravitational lensing with the mass obtained using the previous 3rd dynamical model of cluster.

Moreover investigators aware that the study of the CMB shows the existence of anisotropies due to the density of dark substance in the Universe. We can distinguish 2 kinds of density of dark matter: The 1st kind of density is the density of dark matter with a gravitational effect. Then to obtain the mean density of dark matter in the Universe corresponding to this 1st kind of density, we must only take into account the dark matter inside clusters. We easily obtain this density $\rho m_{UG}(z)$ as a function of the volume of the Universe $Vol_U(z)$, of the total volume of clusters $Vol_U(A)(z)$ and of the intergalactic density $\rho_0(z)$ (corresponding to a Cosmological redshift z). We assume that the mean densities of clusters is approximately equal to the intergalactic density $\rho_0(z)$:

$$\rho_{mUG}(z) = \rho_0(z)\frac{Vol_U(A)(z)}{Vol_U(z)} \qquad (70h)$$

The 2nd kind of density of dark matter considers all the dark substance in the Universe. We are now going to obtain this last density $\rho_{mU}(z)$.

As in the case of clusters, it is interesting to introduce $Vol_U(z)$ volume of the Universe corresponding to a Cosmological redshift z and $Vol_U(H)(z)$ the volume of dark halos corresponding to distributions of dark substance with a density in $1/r^2$ in this Universe. We then obtain the same way we obtained the equation (53), neglecting baryonic matter, $\rho_{mU}(z)$ being the mean density of dark substance in a Universe corresponding to a Cosmological redshift z:

$$\rho_{mU}(z)=2\rho_0(z)(Vol_U(H)(z)/Vol_U(z))+\rho_0(z) \qquad (70i)$$

(If we take consider the dark substance on which are superposed the dark halos, we must replace in the previous equation the factor 2 by the factor 3).

With the approximation $Vol_U(H)(z)/Vol_U(z)<<1$ we obtain:

$$\rho_{mU}(z)=\rho_0(z) \qquad (70j)$$

We also remark that if we assume that the dark mass of the Universe keeps itself, 1+z being the factor of expansion of the Universe between the age of the Universe corresponding to the redshift z and the present age of the Universe:

$\rho_{mU}(z)=\rho_{mU}(0)(1+z)^3$ (70k)

Therefore, according to the equation (70j):

$\rho_0(z)=\rho_0(0)(1+z)^3$ (70l)

We have seen that we could obtain an estimation of $\rho_0(0)$, consequently we can obtain a prediction of $\rho_0(z)$, that we used previously in the study of the evolution of clusters.

In what precedes we assumed a finite Universe, but it is evident that we can generalize the previous relations to the case of an infinite Universe.

2.10 Formation of the large structures in the Universe.

According to the SCM galaxies, stars and more generally the large structures of the Universe observed today have appeared because of heterogeneities of the density of the primordial Universe. Nonetheless, if we estimate the heterogeneities of baryonic matter in the primordial Universe, they are by far insufficient to explain the large structures observed today. It is allowed in the SCM that those heterogeneities were due to dark matter.

According to our Theory of dark matter, those heterogeneities are explained generalizing our hypothesis introduced in the previous section:

Because of the expansion of the Universe and of the properties of dark substance, in the primordial Universe, if a point P does not belong to a concentration of baryonic matter (In the early Universe the density of dark substance is assumed to be constant and the density of baryonic matter is supposed also to be constant in nearly all the Universe), then we must take in P in the Newtonian equations of gravitation $\rho_{SN}(P)=0$ for the density of dark substance in P and $\rho_{BN}(P)=0$ for the density of baryonic matter in P.

We must take in those equations $\rho_{SN}(P)=\rho_0$, ρ_0 being the real density of dark substance and $\rho_{BN}(P)=\rho_G(P)$, $\rho_G(P)$ being the real density of baryonic matter in P if P belongs to a concentration of baryonic matter due to anisotropies of baryonic matter.

So the previous hypothesis amplifies the gravitational effect of the heterogeneities of baryonic matter and could be the origin of the large structures of the Universe observed today.

3.NEW COSMOLOGICAL MODEL AND DARK ENERGY

3.1 Introduction

In the preceding Part 2. we exposed a theory interpreting the whole of astronomical observations linked to dark matter. The concept of dark substance filling all the Universe led to propose a spherical geometrical form for the Universe. In the Part 3, the study proposes a new Cosmological model based on this spherical form of the Universe and also on the physical interpretation of the CMB Rest Frame (CRF). The study can define distances that are completely analogous to distances used in Cosmology in the Standard Cosmological Model (SCM), (angular distance, luminosity distance, commoving distance, light-travel distance) and also a Hubble constant analogous to the Hubble constant defined in the SCM. The new Cosmological model is physically much simpler and much more understandable than the SCM. The study also proposes inside the new Cosmological model 2 possible mathematical models of expansion (permitting to obtain the factor of expansion 1+z and the Cosmological redshift z). The 1st mathematical model of expansion of the Universe is based as the model of expansion of the SCM on the equations of General Relativity. As the SCM it needs the existence of a dark energy, and it predicts the same values as the SCM for the Cosmological distances used in Cosmology and the same Hubble's constant. But it gives the nature of dark matter and dark energy used in the SCM. The 2nd mathematical model of expansion is much simpler but despite of its simplicity, it predicts values of the Hubble's constant and of Cosmological distances that are in good agreement with astronomical observations for $z<12$. Moreover this 2nd mathematical model of expansion has the remarkable property of not needing the existence of dark energy, contrary to the 1st mathematical model of expansion and to the mathematical model of expansion of the SCM. It will appear in this Part 3. that the new Cosmological model remains compatible with Special Relativity and General Relativity, because according to this new Cosmological model the CMB Rest Frame (CRF) cannot be detected by usual physical experiments in laboratory but only by the observation of the CMB. So the study assumes the validity of Special Relativity and General Relativity, even if it exists another possibility (DELORT 2000; DELORT 2020).

We have seen in 1.INTRODUCTION that the observations of the anisotropies of the CMB were in agreement with the 1st mathematical model and contradicted the 2nd mathematical model. Nonetheless, we will only study the 2nd mathematical model that can easily be generalized in order to obtain the properties of the 1st mathematical model because according to the SCM, the Universe is flat. Moreover, the 2nd mathematical model permits to give good predictions for $z<12$. We will also expose in the 1St model the interpretation of dark energy and of the Cosmological parameters of the ΛCDM model. Dark energy will be different from the internal energy of the dark substance considered as an ideal gas but will be analogous to it.

3.2 Physical Interpretation of the CRF. Local and Universal Cosmological frames.

The CMB presents a Doppler effect that is canceled in a frame called for this reason the CMB Rest Frame (CRF). But this CRF has no physical interpretation in the SCM. We are going to give in our theory of dark matter and dark energy a physical interpretation of this frame, which will permit to define a new model of expansion of the Universe that is also based on the geometrical model of the Universe (spherical), admitted in our theory. This new model of expansion of the Universe permits to define Cosmological variables (Cosmological time, distances used in Cosmology, Hubble Constant) completely analogous to their definition in the SCM. In order to obtain the Cosmological redshift z, which is fundamental in the new model of expansion of the Universe as it was in the SCM, our theory of dark matter and of dark energy proposes 2 mathematical models of expansion. The 1^{st} mathematical model is based on the equations of General Relativity as the SCM. According to this 1^{st} mathematical model of expansion, Cosmological variables, and in particular the Cosmological redshift z, are given by the same mathematical expressions as in the SCM, but for a flat Universe because according to the new model of expansion of the Universe, the Universe is flat. The 2^{nd} mathematical model of expansion of the Universe is much simpler. Despite of this its theoretical predictions are in excellent agreement with astronomical observations for $z<12$.

Concerning the physical interpretation of the CRF:

-First it is natural that in each point of the Universe (and not only on the earth), we can define a CRF. We then can suppose that all CRF have parallel corresponding axis.

-Second the CRF permits to define very easily the Cosmological time, identified to the age of the Universe. The simplest definition of the Cosmological time would be that the time of the CRF (meaning the time given by the clocks at rest in the CRF) be precisely the Cosmological time and the hypothesis agrees with astronomical observations. Indeed, this hypothesis implies that the Cosmological time is also with a very good approximation the time of our earth. With this hypothesis, we will name the CRF *local Cosmological frame*, and we will designate it as R_{LC}. Let H_S be a clock linked to the sun and giving the time of the inertial frame R_S linked to the sun, and V_S the velocity of R_S relative to R_{LC}. According to Special Relativity the transformations between R_S and R_{LC} are Lorentz transformations, and if T_S is a time measured by H_S corresponding to a Cosmological time T_C of R_{LC}, then:

$T_S=T_C(1-V_S^2/c^2)^{1/2}$.

If $V_S<<c$, which is the case (V_S is the velocity of the sun relative to the local CMB rest frame and observation of the CMB gives $V_S\approx$ 300km/s) we get $T_S\approx T_C$. We state that it is completely impossible that locally all the inertial frames (with Lorentz transformations between themselves) give the Cosmological time (Age of the Universe) and consequently it was not at all evident that the time of our sun be approximately the Cosmological time.

-Third we know that according to Special Relativity (We remind that we admit it as in the SCM) the velocity of a photon relative to the CRF in which it is situated keeps itself, as a vector or as a norm. We will call *local velocity* this velocity **c**. The problem is the evolution of this local velocity, the photon traveling in the Universe. The simplest hypothesis would be that the local velocity of the photon keeps itself the photon traveling in all the Universe, and consequently being situated in many different CRF. Here also we will see that this simple hypothesis involves theoretical predictions that are in agreement with observation. It permits to justify very simply the effect of the expansion of the Universe on the lengths of wave of photons and on the distances between 2 photons following one another. (This effect is also predicted by the SCM).

So we express the preceding hypothesis in the following Postulate 3:

Postulate 3:

a)At each point of the Universe, we can define a CRF. We will assume that all CRF have parallel corresponding axis.

b)The Cosmological time (identified with the age of the Universe) is the time of all the CRF, meaning given by clocks at rest in any CRF.

c)The *local velocity* of a photon, meaning measured in the CRF in which it is situated, keeps itself, the photon traveling in all the Universe.

Considering its important in Cosmology, according to our theory of dark matter and dark energy, we will also call the CRF *local Cosmological frame*.

We remind that because of the Postulate 3b), and since we know that the inertial frame R_S linked to the sun is driven with a velocity v_S<<c relative to the local CRF, the time of this frame R_S is very close to the time of the CRF, that is the Cosmological time, which is an agreement with observation. So the Postulate 3b) justifies that the time of R_S can be identified to the Cosmological time which was not at all evident. We stressed that according to astronomical observations, locally (meaning close to the Milky Way) all galaxies have a local velocity (meaning relative to the local CRF) very small relative to c. According to the Postulate 3b) the time of any star of any galaxy close to the Milky Way is very close to the Cosmological time.

It is natural to assume that the previous property can be generalized to all the Universe, then we obtain that the time of any star (and consequently of any planet) of the Universe is approximately the Cosmological time.

We know need to define completely all the CRF. We have seen previously that according to our theory of dark matter the Universe was finite with borders and we will assume that it is spherical, with a centre O. We remind that it is possible to generalize what follows for many other geometrical models of finite Universes, with borders. So we assume that the Universe is modeled as a sphere in expansion with a centre O, and with a radius $R_E(t)$, t being the Cosmological time. We have seen in Section 2.5 that $R_E(t_0)=R_E(t)(1+z)$, t and t_0 being any Cosmological times ($t<t_0$), with 1+z factor of expansion of the Universe between t and t_0. We will see further how we can get 1+z, using mathematical models of expansion.

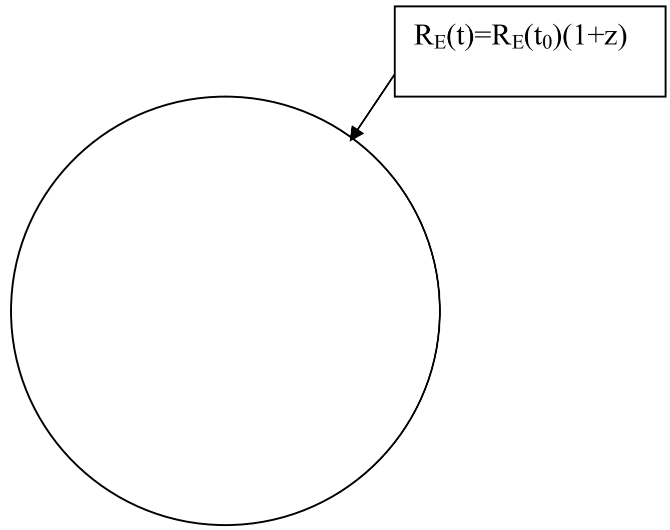

Figure 3:The spherical model of the Universe in expansion.

In order to define completely the CRF (or equivalently the local Cosmological frames) we introduce a new kind of frame R_C, called *Universal Cosmological frame*, which the origin is O centre of the Universe. The time of the Universal Cosmological frame R_C is defined as being the Cosmological time of the CRF (See Postulate 3b)). Moreover the axis of R_C are defined as being parallel to the corresponding axis of the CRF (Postulate 3a)), and as giving locally the same distances as the CRF.

The Universal Cosmological frame R_C permits to define distances between any couple of points (A,B) of the Universe, contrary to local Cosmological frames (CRF) that give only local distances. We will see that we can express all the classical Cosmological distances used in the SCM (luminosity distance, angular distance, commoving distance and light-travel distance) as functions of the distances measured in R_C, of the Cosmological time and of the Cosmological redshift z.

The study defines very important points of the Universal Cosmological frame R_C, called *commoving points* of the sphere in expansion.

We assume that P(t) is any point belonging to the border of the sphere in expansion, t being the Cosmological time, with **OP**(t) (O is the centre of the sphere in expansion) remaining in the same direction **u**, fixed vector of R_C.

A *commoving point* A(t) of the sphere in expansion is defined by :

-A(t) remains on the segment [O,P(t)]
-OA(t)=aOP(t), a being a constant belonging to [0,1]. (71)

So O and P(t) are particular commoving points of the sphere in expansion. Moreover if A(t) and B(t) are 2 commoving points of the sphere in expansion, belonging both to a radius [O,P(t)], and if t_1 and t_2 are 2 ages of the Universe, if $1+z=OP(t_2)/OP(t_1)$, (Here 1+z is the factor of expansion of the Universe between t_1 and t_2) then we have the 2 relations:

$A(t_2)B(t_2)=(1+z)A(t_1)B(t_1)$ (72)

And :

$[A(t_2),B(t_2)]//[A(t_1),B(t_1)]$ (73)

(We classically note, P,Q being 2 points of R_C, PQ is the distance between P and Q measured in R_C, [P,Q] is the segment with extremities P and Q, (P,Q) is the straight line containing P and Q).

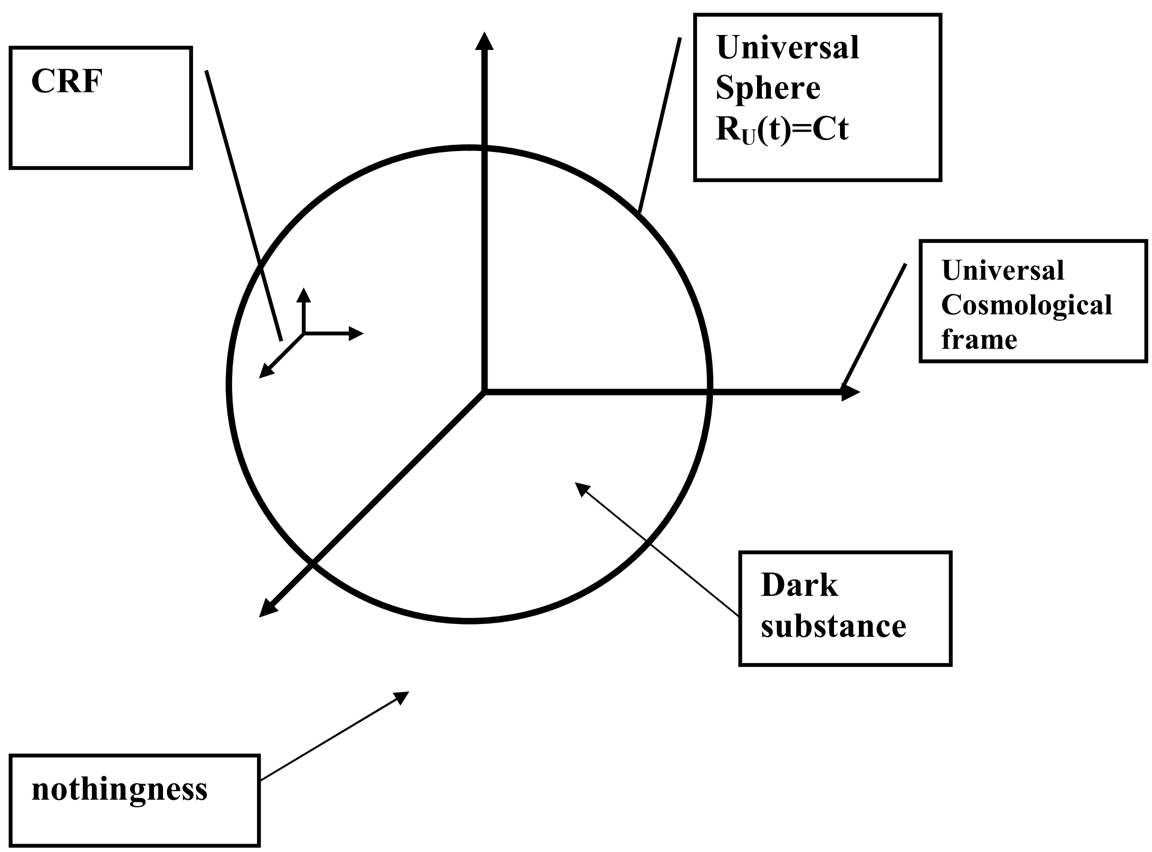

Figure 4 :New Cosmological model

The study shows using Thales Theorem that the previous relations (72)(73) remain valid, A(t), B(t) being any couple of commoving points of the sphere in expansion (defined by relations (71)), not compulsory belonging to the same segment [O,P(t)].

We consider 2 commoving points (different from O) $A(t_1)$ and $B(t_1)$ at a Cosmological time t_1. We assume that A(t) belongs to the segment [O,P(t)], P(t) point belonging to the border of the sphere in expansion, and in the same way B(t) belongs to the segment [O,Q(t)].

t_2 being a Cosmological time strictly superior to t_1, according to the relations (71), $O,B(t_1)$ and $B(t_2)$ belong to the same straight line, and it is also the case for $O,A(t_1),A(t_2)$. We then consider the triangle $(O,A(t_2),B(t_2))$. In this triangle, according to the relations (71), 1+z being the factor of expansion of the Universe between t_1 and t_2:

$$OA(t_2)/OA(t_1)=OP(t_2)/OP(t_1)=1+z \quad (74)$$

And in the same way:

$$OB(t_2)/OB(t_1)=1+z \quad (75)$$

Therefore:

$$OA(t_2)/OA(t_1)=OB(t_2)/OB(t_1)=1+z \quad (76)$$

Essentially applying the converse of Thales Theorem to the triangle $(O,A(t_2),B(t_2))$ we obtain the same relations as the relations (72)(73):

$$A(t_2)B(t_2)=(1+z)A(t_1)B(t_1) \quad (77)$$

And :

$$[A(t_2),B(t_2)]//[A(t_1),B(t_1)] \quad (78)$$

The preceding properties, valid A(t), B(t) being any couple of commoving points, are very remarkable and very important in the model of expansion of the Universe proposed by our theory of dark matter and dark energy.

We remark that if A(t) is a commoving point of a segment [O,P(t)], according to the relations (71), if $V_P(t)$ and $V_A(t)$ are respectively the velocities of P(t) and A(t) measured in the Universal Cosmological frame R_C, we obtain, a being a constant:

$$V_A(t)=aV_P(t) \quad (79a)$$

The previous definition of the commoving points of the sphere in expansion permits us to complete the definition of the local Cosmological frames (CRF), in the following Postulate 4:

Postulate 4:

a)The Universe is a sphere in expansion.

b) The origins of the local Cosmological frames (CRF) are the comoving points of this sphere in expansion.

The study stated the factor of expansion 1+z in our new Cosmological model of expansion of the Universe. It proposes 2 possible mathematical models of expansion inside our new Cosmological model of expansion of the Universe, permitting to obtain 1+z. Both mathematical models are not equivalent and do not give the same expression of 1+z. Nonetheless we will see that both models give theoretical predictions in good agreement with astronomical observations for $z<12$. Determining the mathematical model which has the best theoretical predictions should be an important element to know which is the best model.

According to the 1^{st} mathematical model of expansion, 1+z is obtained as it is obtained in the SCM, with a flat Universe: We apply locally the equations of General Relativity, assuming the same values as in the SCM for the densities of dark substance, baryonic matter and dark energy and assuming that those densities and that the Universe is flat. And therefore in this 1^{st} mathematical model, the factor of expansion 1+z can be mathematically expressed the same way as in the SCM for a flat Universe. A consequence of this is that the 1^{st} mathematical model of expansion predicts distances used in Cosmology and a Hubble constant that have the same mathematical expression as their expression in the SCM, for an observer sufficiently far from the borders of the Universe. The new Cosmological model with the 1^{st} mathematical model is very close to the SCM, but it gives the nature of dark matter and dark energy used in the SCM and moreover interprets the CMB rest frame.

Nonetheless, a priori, it is possible that the factor of expansion 1+z be not obtained by the equations of General Relativity. It is possible that as the local velocity of light, the velocity $V_E(t)$ of the borders of the Universe measured in R_C (defined by $V_E(t)=d(R_E(t))/dt$, t Cosmological time) be equal to a constant C. There is no reason for which C should be equal to the local velocity of light c. So in our 2^{nd} mathematical model of expansion, we assume that the velocity of the borders of the spherical Universe measured in the Universal Cosmological frame R_C is equal to a constant C. We will see further that it is possible to obtain an inferior limit to this constant C. And we will also see that despite of this great simplicity, the theoretical predictions of this 2^{nd} mathematical model agree with all astronomical observations for z<12. Then if P(t) is a point belonging to the border of the sphere OP(t)=Ct. And we have a very simple expression of the factor of expansion 1+z: Between t and t_0 (t_0>t), the factor of expansion 1+z is given by:

$$1+z=(Ct_0)/(Ct)=t_0/t \qquad (79b)$$

In the new cosmological model with the 1^{st} mathematical model, very close to SCM, we should have $OP(t_0)/OP(t)=1+z(t,t_0)$, $z(t,t_0)$ Cosmological redshift between t and t_0, obtained by the same equations as in the SCM. We remind that this 1^{st} mathematical model should be true, according to the observations of the anisotropies of the CMB.

In our model of expansion of the Universe we can prove that as in the model of expansion of the SCM, if 2 photons move on the same straight line towards the origin O of R_C, then between t_1 and t_2 2 cosmological times (with $t_2>t_1$), then the distance between the 2 photons and the lengths of wave of the 2 photons are increased by the factor of expansion of the Universe between t_1 and t_2 1+z .This is true for both mathematical models of expansion. We will see further that it is possible to replace O by any commoving point O' of the sphere in expansion.

2 photons ph1 and ph2 are considered. We take the following notations: At the Cosmological time t ph1 is situated at the point ph1(t) of R_C, and ph2 is situated in the point ph2(t) of R_C. Let us suppose that at a given Cosmological time t_1, ph1(t_1) coincides with a commoving point $A_1(t_1)$ and ph2(t_1) with a commoving point $A_2(t_1)$. We also assume that it exists a unitary vector **u** of R_C, such that $A_1(t_1),A_2(t_1)$ belong to the same segment [O,P(t_1)], with (O,P(t)) parallel to **u**, and that the local velocities of ph1 and ph2 are identical and equal to **c**=c**u**. We remind that according to the Postulate 3, those local velocities keep themselves. Let 1+dz the factor of expansion of the Universe between t_1 and t_1+dt. Then we have according to the properties (77) of commoving points:

$A_1(t_1+dt)A_2(t_1+dt)=(1+dz)A_1(t_1)A_2(t_1)=(1+dz)ph1(t_1)ph2(t_1)$ (79c)

Moreover, the local velocity of photons being equal to c:

$A_1(t_1+dt)ph1(t_1+dt)=A_2(t_1+dt)ph2(t_1+dt)=cdt$ (79d)

According to properties (relations (77)) of commoving points, and the local velocities of ph1 and ph2 being parallel to **u**, O, $A_1(t_1+dt)$, $ph1(t_1+dt)$,$A_2(t_1+dt)$,$ph2(t_1+dt)$ are aligned on the same straight line as O, $A_1(t_1)$ and $A_2(t_1)$ (with the direction **u**) and moreover we assume that they are ranked in this order. Therefore:

$ph1(t_1+dt)ph2(t_1+dt)=A_1(t_1+dt)ph2(t_1+dt)-A_1(t_1+dt)ph1(t_1+dt)$ (79e)

$ph1(t_1+dt)ph2(t_1+dt)=A_1(t_1+dt)A_2(t_1+dt)+ A_2(t_1+dt)ph2(t_1+dt)-A_1(t_1+dt)ph1(t_1+dt)$

Consequently according to the equation (79d) :

$ph1(t_1+dt)ph2(t_1+dt)=A_1(t_1+dt)A_2(t_1+dt)$ (79f)

Therefore, according to the equation (79c) :

$ph1(t_1+dt)ph2(t_1+dt)=(1+dz)ph1(t_1)ph2(t_1)$ (80a)

So between t_1 and t_1+dt, the distance between $ph1(t_1)$ and $ph2(t_1)$ is increased by the factor of expansion between t_1 and t_1+dt 1+dz. Consequently between t_1 and t_2 the distance between $ph1(t_1)$ and $ph2(t_2)$ is increased by the factor of expansion of the Universe between t_1 and t_2 1+z :

$ph1(t_2)ph2(t_2)=(1+z)ph1(t_1)ph2(t_1)$ (80b)

In order to show the previous effect on the lengths of wave of ph1 and ph2, we proceed as previously : We model the photon ph1 as a system whose extremities are 2 mobile points a(t) and b(t), the length a(t)b(t) being the length of wave of the photon. ph1(t) belongs as previously to a segment [O,P(t)], with (O,P(t) parallel to the unitary vector **u** and ph1(t) driven with a local velocity **c**=c**u**. We assume that for any photon ph1(t) a(t) and b(t) are driven with the same local velocity **c**, and that a(t),b(t) belong also to [O,P(t)]. We proceed then with a(t) and b(t) the same way we proceeded with ph1(t) and ph2(t). So we obtain in our new model of expansion of the Universe, λ(t) being the length of wave of a photon, a relation analogous to (80b):

$\lambda(t_2)=\lambda(t_1)(1+z)$ (80c)

We stated that the relations (80b)(80c) were also valid in the model of expansion of the SCM. It is because of the previous relation (80c), valid for any photon according to our theory of dark matter and dark energy as it was in the SCM, that we use the notation 1+z in to represent the factor of expansion in the Universe. We remind that in the previous relation (80c), $\lambda(t_1)$ and $\lambda(t_2)$ must be measured in the local Cosmological frame (CMB rest frame) in which is situated the photon, that also gives the distances measured in the Universal Cosmological frame R_C according to the definition of R_C.

We can show more generally using an analogous way that if we only suppose that ph1 and ph2 own the same local velocity (ph1(t), ph2(t) not compulsory belonging to the same straight line containing O), then between 2 Cosmological times t_1 and t_2 the distance measured in R_C between ph1 and ph2 increases by the factor of expansion of the Universe between t_1 and t_2 1+z (as in the equation (80b)), and moreover we have the relation $(ph1(t_2),ph2(t_2))//(ph1(t_1),ph2(t_1))$.

We remark that for any commoving point of the swelling sphere O'(t) we can define a Cosmological frame R_C' whose the origin is O'(t), the time is the Cosmological time (time of R_C), the axis are parallel to the corresponding axis of R_C and defining the same distances between 2 points, at a given Cosmological time t, as the distances defined by R_C. We will call R_C' *secondary Universal Cosmological frame*.

Then if A(t) is any commoving point of the swelling sphere defined previously, t_1 and t_2 being 2 Cosmological times, according to the properties of commoving points (72)(73), if 1+z is the factor of expansion of the Universe between t_1 and t_2:

$$O'(t_2)A(t_2)=(1+z)O'(t_1)A(t_1)$$
$$(O'(t_2),A(t_2))//(O'(t_1),A(t_1)) \qquad (81)$$

And consequently $(O'(t_1),A(t_1))$ et $(O'(t_2),A(t_2))$ are in the same direction **u** of R_C'.

The relations (71)(72)(73) remain valid, replacing R_C by R_C'and O by O'. P(t) is still defined as a point belonging to the borders of the sphere in expansion, but we have no more $OP(t)=R_E(t)$, $R_E(t)$ radius of the sphere in expansion at a Cosmological time t.

Therefore it should have been possible to define commoving points in R_C' the same way we defined them in R_C. The expressions of the distances used in Cosmology and of the Hubble constant obtained in R_C are also valid in R_C'.

We will see that generally it is not possible to observe all the Universe from any commoving point O'(Which was also the case in the SCM: According to SCM it is not possible to observe all the Universe from our planet), but if O' is sufficiently far from the borders of the Universe, then the Universe observed from O' is approximately identical to the Universe observed from O.

The spherical form of the Universe could be confirmed if some celestial bodies would not own a homogeneous distribution in the Universe, but a distribution presenting a spherical symmetry relative to a point O. According to our models, O would be then the centre of the spherical Universe.

3.3 Hubble's law-Distances used in Cosmology.

We keep the notations of the previous section, R_C is the Universal Cosmological frame, O is the origin of R_C centre of the Universe. (We remind that we can generalize what follows replacing O by any commoving point O' (sufficiently far from the borders of the Universe, and R_C by a secondary Universal Cosmological frame R_C', with O' as origin). Let us suppose that a photon is emitted from a star S at a point $Q(t_E)$ of R_C (Q(t) being commoving point of the sphere in expansion) at a Cosmological time t_E towards O. We assume that the photon reaches O at the present Cosmological time t_0. We assume that between t_E and t_0 the factor of expansion of the Universe is $1+z_0$.

Between t and t+dt, we know that the photon covers the local distance cdt. Consequently between t_E and t_0 the sum of the local distances covered by the photon will be :

$D_T=c(t_0-t_E)$ (82)

We will call this distance, which is completely identical to the *light- travel distance* in the SCM, by the same name. We can also call it *time-back distance* because it permits to obtain the Cosmological time between the emission of the photon at the point $Q(t_E)$ and the reception of the photon in O, at the Cosmological time t_0.

According to the 1st mathematical model of expansion of the Universe, the theoretical prediction of the distance D_T, between 2 points at rest relative to their associate Local Cosmological Frame, given by the equation (82), as a function of Cosmological variables z_0, t_0…, is identical to the theoretical prediction of the SCM, because the equations giving D_T are identical in those both models (equations of the General Relativity).

But in the 2nd mathematical model of expansion of the Universe, we obtain very easily the Hubble's Constant using the light-travel distance defined previously:

Indeed according to this 2nd mathematical model and the equation (79b), $1+z_0$ being the factor of expansion of the Universe between t_E and t_0:

$1+z_0=(Ct_0)/(Ct_E)=t_0/(t_0-D_T/c)$ (83a)

When $D_T/ct_0<<1$ we obtain $z_0\approx D_T/ct_0$ and consequently the Hubble's constant is equal to $1/t_0$. The preceding equation (83a) is very simple and can easily be verified. For instance taking t_0=15 billion years, for z_0=0.5,we obtain D_T=5 billion light years and for z_0=9 we obtain D_T=13.5 billion years. These predicted values agree with the usual admitted experimental values for the light-travel distance D_T.

Up to date, 2 models exist in order to obtain the Hubble constant H: The 1st model, using standard candles that are supernovae, brings to obtain H=73km/sMpc^{-1} (WONG et al. 2020).The 2nd model, using CMB and the model ΛCDM, brings to obtain H=67 km/sMpc^{-1}(AGHANIM et al. 2020). The 2nd value of H brings to obtain t_0=14,4 billion years which is an acceptable value, but the 1st value of H brings to obtain t_0=13,4 billion years which is not an acceptable value considering the age of the oldest stars. Therefore if it is this last value of H that is valid, the 2nd mathematical model of expansion leads to an underestimation of the age of the Universe t_0 of 3% to 5%. Then we should have to replace in the 2nd mathematical model $R_U(t)=Ct$ by $R_U(t)=f(t)$, with f(t) very close to Ct, and independent of dark matter and dark energy (For instance $f(t)=Ct^\alpha$, then $t_0=\alpha/H$. For α=1,05, t_0=14 billion years). In what follows, we will take the simplest mathematical model $R_U(t)=Ct$, but it is clear that we could generalize the obtained predictions to other mathematical models, and that in the 1st mathematical model close to SCM those predictions are usually identical to predictions of the SCM, considering points at rest relative to their associate Local Cosmological Frame.

The 2 previous model of obtainment of H lead to incompatible results and moreover are not necessarily compatible with the new Cosmological model, especially the 1st model not using the Cosmological Frame (identified with the CMB rest frame) nor the new expression of commoving distance, given by equation (88a).

We still assume that a photon is emitted by a star S at a commoving point $Q(t_E)$, t_E age of the Universe when the photon is emitted and reaches the origin O of the Universal

Cosmological frame R_C at the present age of the Universe t_0. We have seen in section 3.2 that we could assume that the local velocity of S is small relative to c, the same way local velocities of stars close to our Milky Way (measured in the local CMB Rest frame) are small relative to c. If the photon emitted by S at a Cosmological time t_E owns the length of wave λ_0 measured in the inertial frame linked to S, if it reaches at time t_0 a planet T very close to O, with a local velocity very small relative to c, then if $\lambda_T(t_0)$ is the length of wave of the photon measured in the inertial frame linked to the planet T (at t_0), according to the equation (80c), $1+z_0$ being the factor of expansion of the Universe between t_E and t_0:

$$\lambda_T(t_0) \approx (1+z_0)\lambda_0 \qquad (83b)$$

We then can define in our model of spherical Universe in expansion other kinds of distances used in Cosmology in a completely analogous way to their definition in the SCM:

We have seen (Equation (82)) that we can express the light-travel distance as:

$$D_T = \int_{tE}^{t0} c\,dt \qquad (84)$$

The local distance covered by the photon between t and t+dt is, according to the Postulate 3 equal to cdt. This local distance, considered as a distance between 2 commoving points of the sphere in expansion, is increased by the factor of expansion of the Universe 1+z between t and t_0 (See equation (79b)).

In complete analogy with the SCM, we will call *commoving distance* between O and S the distance between $Q(t_0)$ and $O(t_0)$ measured in the Universal Cosmological frame R_C, which is the sum of all the local distances cdt covered by the photon, increased by the factor 1+z. Let D_C be this distance:

$$D_C = \int_{tE}^{t0} c(1+z)\,dt \qquad (85)$$

From this expression we define the *luminosity-distance* D_L between O and S (at the Cosmological time t_0) and the *angular-distance* D_A between O and S in complete analogy with their definition in the SCM:

$$D_L = (1+z_0)D_C \qquad (86a)$$

$$D_A = D_C/(1+z_0) \qquad (86b)$$

The distance D_A appears to be the distance measured in R_C between $Q(t_E)$ and O. In complete analogy with the SCM it permits to obtain some angles with a summit O in R_C.

The distance D_L, in complete analogy with its definition in the SCM, appears to be obtained measuring the luminous flow of a supernova considering the effect of the expansion of the Universe on the lengths of wave of the photons and on the distances between 2 photons (moving on the same axis). We saw in the section 3.2 (Equations (80b)(80c)) that this effect, predicted by the SCM, was also true in the model of expansion of the Universe proposed by our theory of dark matter and of dark energy.

The mathematical expressions of the different kinds of distances used in Cosmology (85)(86a)(86b) are in agreement with their mathematical expression in the SCM, in which the commoving distance D_C is usually expressed as a function of the variable z, for a flat Universe.

In the 1st mathematical model of expansion, since 1+z has the same mathematical expression as in the SCM the mathematical expression of those distances used in Cosmology as a function of z_0 is identical to their mathematical expression in the SCM. We also obtain an identical Hubble's constant. In this model we obtain t(z) and z(t) using the differential equation $ct'(z)=-d_H/((1+z)E(z))$ (Conventional notations) with the initial condition $t(0)=t_0$.

In the 2nd model, the expressions of distances used in Cosmology are much simpler. Using $1+z=t_0/t$ we obtain (Equation (79b) and (85)):

$$D_C = \int_{tE}^{t0} c(1+z)dt = \int_{tE}^{t0} c(t_0/t)dt \qquad (87)$$

So we obtain finally the mathematical expression of the commoving distance, using $1+z_0=t_0/t_E$:

$$D_C=ct_0 Log(t_0/t_E)=ct_0 Log(1+z_0) \qquad (88a)$$

Here also this simple expression is in good agreement with the usual admitted experimental values for the commoving distance for z<12. We deduce very easily from this expression the expression of the luminosity distance and of the angular distance (86a)(86b). We remark that in this 2nd model, according with the previous equations we have as in the SCM for z_0<<1:

$$D_T \approx D_C \approx D_A \approx D_L \approx ct_0 z_0 \qquad (88b)$$

We know that according to the 2nd mathematical model, the velocity measured in R_C of any commoving point Q(t) is constant. (According to the equation (79a) with $V_P(t)=C$ according to the definition of the 2nd mathematical model of expansion of the Universe.) Let V_Q be this velocity. Then the distance in R_C between O and $Q(t_0)$, that we also called the commoving distance D_C is also equal to $V_Q t_0$. Therefore, according to the equation (88a):

$$V_Q=cLog(1+z_0) \qquad (89)$$

We can interpret in our new model of expansion of the Universe the observation of the explosion of a supernova the same way as in the SCM, taking into account the effect of the expansion of the Universe on the lengths of wave of photons and on the distances between photons moving on the same axis (Equations (80b)(80c)). So our new model of expansion of the Universe can interpret the astronomical observations concerning the explosion of a supernova (PERLMUTTER et al. 1998) the same was as the model of expansion of the SCM.

3.4 Cosmological limits of the observable Universe.

In our model of finite Universe in expansion we cannot, as it was also the case in the SCM, observe the Universe (through the observation of galaxies) before a given time t_{OU}. This implies that observing the Universe from a commoving point $O'(t_0)$ (t_0 present Cosmological time) sufficiently far from the borders of the Universe, the observable Universe is isotropic and that in many cases, the borders of the Universe cannot be observed from $O'(t_0)$. In this section we are going to see how we can obtain this time t_{OU} according to our model of finite Universe in expansion, and more precisely according to the 2nd mathematical model of expansion of the Universe, that is much simpler than the mathematical model of the SCM. We must proceed the same way, just modifying mathematical expressions, to obtain t_{OU} according to the 1st mathematical model of expansion of our theory of dark matter and dark energy.

We keep in our theory the hypothesis admitted in the SCM of the existence of a dark age in the Universe during which light cannot propagate in the Universe. Let t_D be the end of this dark age. It is evident that t_{OU} must be superior to t_D. Moreover, galaxies cannot be observed before the Cosmological time t_G, that is the time of the apparitions of the first galaxies. It exist another limit according to our model of spherical Universe in expansion. This is very clear in our 2nd model:

According to the equation (89), V_Q being compulsory inferior to C, we have:

$$C \geq c\mathrm{Log}(1+z_0) \qquad (90)$$

Consequently, with the notations of the previous section:

$$t_0/t_E=1+z_0=\leq \exp(C/c) \qquad (91)$$

Which implies that the Universe cannot be observed in $O(t_0)$ (We remind that t_0 is the present age of the Universe) before the time t_I defined by:

$$t_I=t_0\exp(-C/c) \qquad (92)$$

So according to our theory of dark matter and of dark energy, t_{OU}, minimal Cosmological time for which the Universe can be observed is the is the greatest time between t_I, t_G and t_D. Moreover if $t_{OU}>t_I$, we cannot observe the borders of the Universe from O.

We remark that the equation (90) permits to give an inferior limit to the constant C of the 2nd model: The fact that we have observed some redshift z equal to 10 implies that C>2,3c. If we take C=10c, we obtain t_I of the order of 1million years.

We must use analogous methods if our galaxy is situated not in O but in another commoving point O'(t). Then only t_I is modified, depending on the distance between $O'(t_0)$ and the borders of the spherical Universe.

3.5 The Cosmic Microwave Background.

As in the SCM, we admit the apparition of a CMB at a Cosmological time very close to the Big-Bang (We admit as in the SCM that the Big Bang occurs at a Cosmological time equal to 0). Proceeding exactly as in the SCM, taking into account the effect of the expansion of the Universe on the lengths of wave of photons and on photons moving on the same axis (effect obtained in section 3.2 (Equations (80b)(80c)) , we obtain in our theory of dark matter and dark energy that if the CMB appears at a Cosmological time t_{iCMB} corresponding to a temperature T_{iCMB}, then at a Cosmological time t superior to t_{iCMB}, if the factor of expansion between t_{iCMB} and t is 1+z, then the CMB at a Cosmological time t corresponds to a temperature $T_{CMB}(t)=T_{iCMB}/(1+z)$. (This is obtained exactly the same way as in SCM, because we have in both Cosmological models that with the same notations the density of photons is divided by $(1+z)^3$ (Because the radius of the Universe $R_E(t)$ increases by a factor 1+z) and the lengths of wave of photons are increased by a factor (1+z)(Equation (80c)). Therefore, our new model of expansion of the universe is in agreement with the observation of the CMB corresponding to a great redshift z_0 (RAINE&THOMAS 2001) .

If we admit that at the apparition of the CMB ($z\approx1100$), the temperature of the CMB was equal to the temperature of the dark substance filling the Universe, then we obtain the isotropy of the CMB observed today, without needing to introduce the phenomenon of inflation, because we admitted that the dark substance was homogeneous in temperature.

But now we have given a very complete physical interpretation of the CMB Rest Frame that did not exist in the SCM, permitting to define completely the CMB rest frame (Postulate 4) at any point of the Universe, and giving also fundamental physical properties of the CMB Rest Frame (Postulate 3). As we have seen in our 1.INTRODUCTION, our theory of dark matter and dark energy remains compatible with the SCM in order to interpret the anisotropies of the CMB .

It is important to know what happens to a photon reaching the borders of the spherical Universe. It could be absorbed but it is not the only possible hypothesis. The simplest hypothesis would be that the photon is reflected, taking exactly as new local velocity after reflection the opposite of its local velocity before reflection (as a vector). With this last hypothesis, we could expect to observe the images of galaxies reflected on the borders of the Universe, but we have several explanations that this effect is not observed. Indeed with the notations of the section 2.4, if $t_G>t_I$ or $t_I<t_D$ then an observer situated in O centre of the Universe cannot observe at the present time t_0 images of galaxies reflected on the borders of the Universe. In the 1st case, images of galaxies reflected on the borders of the Universe reach O after t_0, and in the 2nd case the reflected photons are absorbed during the dark age.

3.6 Dipole contribution of the CMB.

We know that according to the SCM we have the following fluctuations of temperature of the CMB [(7)]:

$$(\frac{\Delta T}{T}) = \frac{1}{4\pi}\sum_{l} l(2l+1)C_l \qquad (93)$$

We will keep this expression in our theory of dark matter and dark energy. But according to the preceding theory, l=1 is the dipole contribution, corresponding as in the SCM to the motion of the earth relative to the CRF (CMB Rest Frame). So this dipole contribution is completely interpreted by our theory of dark matter and dark energy, which was not the case in the SCM, in which the CMB rest frame has non physical interpretation.

3.7 Link between the CMB and the temperature of the intergalactic dark substance.

We have seen that according to the new Cosmological model, the Universe was a sphere filled with dark substance, surrounded by a medium called “nothingness” (See Section 2.5). In analogy with the spherical concentrations of dark substance defined in the Part 2., we could assume that it exists a convective thermal transfer between the intergalactic dark substance and the nothingness. The convective thermal flow F would then be given by the expression $F=h_nT_0(t)$, $T_0(t)$ being the temperature of the intergalactic dark substance at a

Cosmological time t. Generalizing the analogy with the case of spherical concentrations of dark substance, we obtain the equation of thermal equilibrium with K_3 constant (K_3 given by the Equation (14)) , M_B baryonic mass of the Universe, $R_E(t)$ radius of the Universe at a Cosmological time t:

$$K_3M_B=4\pi R_E(t)^2(h_nT_0(t)) \qquad (94a)$$

Nonetheless, to obtain the previous equation, we assumed the existence of a convective thermal transfer between the Universal sphere and the nothingness (And it is possible that this transfer be nil), and moreover we neglected the other energetic factors acting on the temperature of the intergalactic dark substance (Which could be a non valid approximation. We will study in the following section all those energetic factors).

We remark that if we had (in analogy with our hypothesis in the obtainment of the baryonic law of Tully-Fisher) a constant C_2 such that $h_n=C_2\rho(t)$, then we would obtain according to the equation (94a) that the temperature $T_0(t)$ would increase with t. This would be impossible with the 1st model of thermal transfer exposed in the Section 2.3, but would be possible with the 2nd model of thermal transfer exposed in the Section 2.7. But if we assume that h_n is constant, then we obtain according to the equation (94a) that $T_0(t)$ evolves in $1/(1+z)^2$, 1+z being the factor of expansion of the Universe. In our theory of dark matter and dark energy, we admit as in the SCM that the apparition of the CMB in the Universe corresponds to a redshift z approximately equal to 1100. If we assume in our new Cosmological model that for this value of z, the temperature of the intergalactic dark substance was equal to the temperature of the CMB, we obtain that presently (with an age of the Universe of 15 billion years), the temperature of the intergalactic dark substance is 1100 times lower than the temperature of the CMB, which is an acceptable value, justifying our approximation in Section 2.3 expressing that the temperature of the intergalactic dark substance can be neglected in comparison with the temperature of spherical concentrations of dark substance corresponding to galaxies with flat rotation curve.

Moreover the hypothesis of the initial temperature of the CMB and the temperature of the intergalactic dark substance implies because we assumed that the latter was homogeneous in all the universe, that the initial temperature of the CMB was also homogeneous in all the Universe. And so the previous hypothesis justifies the isotropy of the CMB relative to the CRF at the present age of the Universe (and at any age), without needing to introduce the phenomenon of inflation, as it was the case in the SCM.

3.8 Dark energy in the Universe.

We observe in the first part of our theory (**2.THEORY OF DARK MATTER**) that the Universe was filled with a dark substance that could be modeled as an ideal gas (Section 2.1). So it is natural to assume that as an ideal gas this dark substance owns an internal energy, that could be identified with a dark energy, existing in all the Universe. (But we are going to see further that this assumption is wrong).

We remind the equation (94a), with M_B baryonic mass of the Universe, $R_U(t)$ the radius of the Universe at a Cosmological time t, $T_0(t)$ temperature of the intergalactic dark substance at the Cosmological time t, K_3 being a constant defined by the equation (14):

$$K_3M_B=4\pi R_U(t)^2(h_nT_0(t)) \qquad (94b)$$

As we remarked in the previous section, taking h_n constant brings to obtain a temperature $T_0(t)$ evolving in $1/(1+z)^2$.

In order to obtain $T_0(t)$ in the previous equation, we did not take into account the evolution of the internal energy of the dark substance nor the internal energy lost because of the dilatation of the volume of the intergalactic dark substance, modeled as an ideal gas. We will call 1st *model of the evolution of the temperature* of the intergalactic dark substance the preceding model.

Let us consider a 2nd *model of the evolution of the temperature of the intergalactic dark substance* in which on the contrary we neglect the energy transferred from the baryons towards the dark substance (energy that is obviously nil before the apparition of baryons) and also the energy lost by the intergalactic dark substance at the borders of the Universe through the convective transfer defined previously in comparison with the variation of the internal energy of the intergalactic dark substance and also with the energy lost because of the variation of the volume of the intergalactic dark substance (modeled as an ideal gas). We assume that in this 2nd model, the dark substance is homogeneous in all the Universe. As a result the dark substance obeys to the Law of ideal gas (Postulate 1) and moreover we assume that it also obeys to Joule's law for ideal gas: It exists a constant K_{ES} such that T(t) being the temperature of the dark substance, M_S being the total mass of the dark substance and U(T(t)) being the total internal energy of the dark substance for an age of the Universe t:

$$U(T(t))=K_{ES}M_ST(t) \qquad (95).$$

Moreover the energy lost that is the work corresponding to a variation of the volume of the dark substance dV under the pressure P is equal to:

$$W=-PdV \qquad (96)$$

We assume in this 2nd model of the evolution of the temperature of the dark substance that the transformation is adiabatic reversible. We can apply the Laplace's law: It exists a constant γ such that, V being the volume of the Universe for a temperature T at an age of the Universe t, and V_1 its volume for a temperature T_1 at an age t_1:

$$TV^{\gamma-1}=T_1V_1^{\gamma-1} \qquad (97)$$

Consequently if 1+z is the factor of expansion of the Universe between t_1 and t, $V(t)=V(t_1)(1+z)^3$ and:

$$T(t)=T(t_1)/(1+z)^{3(\gamma-1)} \qquad (98)$$

In a 3rd *model of evolution of the temperature of the intergalactic dark substance* we consider every kind of energy received or lost by the dark substance. Nonetheless, we consider in this model that the dark substance is homogeneous in density and temperature in all the Universe, without considering the dark halos of galaxies with a flat rotation curve, and we have seen that this was justified because the total volume of those dark halos was very small relative to the total volume of the Universe. We will take the following notations:

dW(t,t+dt) is the energy received by the dark substance as a work (negative) due to the variation of volume of the dark substance between the ages of the Universe t and t+dt.

$dE_{TF}(t,t+dt)$ is the energy received by the dark substance (negative) due to the thermal transfer between the dark substance and the medium that we called “nothingness” between t and t+dt. $R_U(t)$ being the radius of the Universe at the age of the Universe t, we have seen (equation (94b)):

$dE_{TF}(t,t+dt)=(-h_nT(t))(4\pi R_U(t)^2)dt$ (99)

$dE_{TB}(t,t+dt)$ is the energy received by the dark substance (positive) received from the baryons , (Equation (14) and Equation (94b)) between t and t+dt. $M_B(t)$ being the mass of the baryons at the age t of the Universe we have:

$dE_{TB}(t,t+dt)=K_3M_B(t)dt$ (100)

Then the equation of equilibrium of the energy received and lost by the intergalactic dark substance between t and t+dt is:

$dU(t,t+dt)=dW(t,t+dt) + dE_{TF}(t,t+dt) + dE_{TB}(t,t+dt)$ (101)

We remind that according to the Boyle-Charles law, M_S being the total mass of the dark substance (assumed to be constant):

$P(t)V(t)=k_0M_ST(t)$ (102)

And, $R_U(t)$ being the radius of the Universe, $V(t)=(4/3)\pi R_U(t)^3$ and $d(R_U(t))=dzR_U(t)$ (1+dz being the factor of expansion of the Universe between t and t+dt), $dV(t)=4\pi R_U(t)^2dR_U(t)=4\pi R_U(t)^3dz$ and consequently $dV(t)/V(t)=3dz$. So we have:

$dW(t,t+dt)=-PdV(t)=-k_0M_ST(t)(dV(t)/V(t))$ (103a)

$dW(t,t+dt)=-3k_0M_ST(t)dz$ (103b)

So we obtain the following differential equation in T(t), because dz and $R_U(t)$ can be expressed as a function of t:

$d(K_{ES}M_ST(t))=-3k_0T(t)dz-h_nT(t)(4\pi R_U(t)^2)dt+K_3M_B(t)dt$ (104a)

$K_{ES}M_S(dT(t)/dt)=-3k_0M_ST(t)(dz/dt)-h_n(4\pi R_U(t)^2)T(t)+K_3M_B(t)$ (104b))

We can easily prove that with the previous notations, the parameter γ used in Laplace’s equation (97) can be expressed by:

$\gamma=1+k_0/K_{ES}$

k_0 should be of the order of K_{ES} in analogy with existing gas modeled as ideal gas. Using the previous equation (104b) we can express the conditions of validity of the 1st model of the evolution of the temperature of the dark substance, in which we neglected the variation of internal energy and the work received by the dark matter due to the variation of its volume. Those conditions are:

$-K_{ES}M_S(dT(t)/dt) \ll K_3M_B(t)$

$-K_{ES}M_S(dT(t)/dt) << h_n(4\pi R_U(t)^2)T(t)$

$3k_0M_ST(t)(dz/dt) << K_3M_B(t)$

$3k_0M_ST(t)(dz/dt) << h_n(4\pi R_U(t)^2)T(t)$ (106)

The conditions for which the 2nd model of the evolution of the temperature of dark substance be valid are the inverse conditions (replacing "<<" by ">>")

3.9 Evolution of the temperature of dark substance- 2nd model of expansion.

We consider the application of the preceding section 3.8 in the case of the 2nd mathematical model of expansion of the Universe, meaning with $R_U(t)=Ct$, (C constant, see Section 3.2), and consequently between t and t+dt, 1+dz=(t+dt)/t, so dz=dt/t.

We remark that in the 1st model of evolution of the temperature T(t) evolves in $1/(1+z)^2$, consequently for this 2nd model of expansion in $1/t^2$. In the 2nd model of the evolution of the temperature, T(t) evolves in $1/(1+z)^{3(\gamma-1)}$ with $\gamma>1$, consequently in this 2nd model of expansion in $1/t^{3(\gamma-1)}$. So in both cases T(t) evolves in $1/t^p$, with p>0. For such a function T(t), we obtain that for t tending towards the infinite both functions T(t) and (dT(t)/dt)/T(t) tend towards 0. So for t sufficiently great the relations (106) are valid and the 1st model of evolution of the temperature of dark substance is also valid.
On the contrary for t tending towards 0, the functions (dT(t)/dt)/T(t) and T(t) tend towards the infinite and consequently for t sufficiently small (for instance just after the Big-Bang), the inverse of the relations (106) are valid and consequently the 2nd model of the evolution of the temperature of dark substance is also valid.

3.10 Dark energy of baryonic particles.

We observed in Section 3.8 that according to our theory of dark matter and dark energy it existed in all the Universe a dark energy that could be identified with the internal energy of the dark substance. We are going to see in this section that it is also possible that baryonic particles also contain a dark energy, meaning an energy that cannot be detected using classical laboratory experiments. Nonetheless, this hypothesis, even if it is interesting and must be considered, is not necessary to our theory.

We defined in the Postulate 1 law of ideal gas for an element of dark substance with a pressure P, a volume V, a temperature T and a mass m, k_0 being a constant:

$PV=k_0mT$ (107)

Using the previous law and the Newton's Universal law of gravitation, we obtained the equation (10), valid for all galaxies with a flat rotation curve. For instance for the Milky Way, T_{MW} being the temperature of the dark halo of the Milky Way and v_{MW} being the orbital velocity of stars in Milky Way, we have the equation:

$v_{MW}^2 \approx 2k_0T_{MW}$ (108)

Taking $v_{MW} \approx 2.\ 10^5$m/s we obtain $k_0T_{MW} \approx 2.\ 10^{10}$ U.S.I .

Let us compare the equation (108) with the analogous equation valid for hydrogen modeled as an ideal gas. We know that it exists a constant k_H such that for a hydrogen element with a mass m_H, a volume V, at a temperature T and a pressure P:

$$PV=k_H m_H T \qquad (109)$$

We know that for a mole of hydrogen, for $T=T_K=273°K$, $V=20.\ 10^{-3}$, $P=10^5$ Pa, $m_H=10^{-3}$ kg, we have:

$$k_H T_K \approx PV/m_H=10^5\times 20.\ 10^{-3}\times 10^3= 2.\ 10^6 \text{ U.S.I} \qquad (110)$$

If we assume that dark substance and hydrogen obeys to Joule's law, therefore the internal energy of a kg of hydrogen at the temperature T_K is of the order of $k_H T_K$ meaning 2. 10^6 Joules despite that the internal energy of a kg of dark substance belonging to the halo of the Milky Way is of the order of $k_0 T_{MW}$ meaning 2. 10^{10} Joules, and therefore the latter energy is by far superior to the former (We use the equation (105), if as for all existing gas modeled as ideal gas, k_0/K_{ES} is of the order of the unity). Considering this important difference of energy, we must consider a 2nd possible model of energetic transfer from baryons towards the dark substance, permitting a transmitted power much greater than a power corresponding to a diminution quasi-imperceptible of the temperature of the baryonic matter. In this 2nd model of energetic transfer, the transferred energy is *dark energy*. In this 2nd model, baryonic particles contain a very important quantity of dark energy, but this dark energy must not be considered in the mass appearing in the classical equations $E=mc^2$ or $E_p=mU$. We cannot detect this dark energy using classical laboratory experiments. According to our theory of dark matter and dark energy, in order that the results of section 2.3 remain valid (permitting to obtain the baryonic Tully-Fisher's law), the power of dark energy transmitted from baryons towards dark substance has the same expression as in the 1st model of energetic transfer (thermal power):

$$P_r=K_{3S}M \qquad (111)$$

With M the mass of the considered baryonic particles and K_{3S} constant. p_{0S} being the power of dark energy lost by nucleus and m_0 being the mass of a nucleus we obtain $K_{3S}=p_{0S}/m_0$.

The hypothesis of a dark energy for baryonic particles is effective because not only it permits the transmission of an energy from baryonic particles to dark substance that could be much greater than thermal energy, but also because it justifies that this transmitted energy is independent of the temperature of those baryons and the temperature of this dark substance.

Nonetheless, the hypothesis of a dark energy for baryonic particles is not a hypothesis that is necessary to our theory of dark matter. Indeed according to our model of evolution of the temperature of dark matter (Section 2.8), we can expect that the initial temperature of the concentrations of dark substance be very high, equal to the temperature of the intergalactic dark substance, and then decreases till it reaches its final temperature. Consequently the variation of the internal energy of a spherical concentration of dark substance as defined in this article is very slow, and is therefore compatible with a very low thermal power emitted by baryonic particles towards the dark substance.

4.CONCLUSION

In the Theory of dark matter exposed in this article, we have modeled dark matter as a dark substance whose the physical properties, and in particular the fact that it can be modeled as an ideal gas, permitted to interpret all the astronomical observations linked to dark matter. For instance, those physical properties permitted us to justify theoretically the flat rotation curve of galaxies and the baryonic Tully-Fisher's law. To obtain this, we interpreted galaxies with flat rotation curve as spherical concentrations of dark substance in gravitational equilibrium. We have also seen that our concept of dark substance led naturally to propose a new geometrical form of the Universe, flat, finite and spherical.

We have studied according to our theory of dark matter the effects of the displacement of a concentration of dark substance on its mass and its velocity, and we have seen that those effects were nil. We saw that this theory permitted to define, in agreement with astronomical observations 2 kinds of radius for galaxies: The baryonic radius and the dark radius. We then exposed according to this theory the different models of distribution of dark matter in galaxies. Then we have seen that this theory predicted important relations between the masses of clusters and the velocities of galaxies in those clusters, and relations between the mean densities of some clusters corresponding to the same Cosmological redshift. It also modeled an action of dark matter in structure formation. Finally we saw that our theory of dark matter permitted to give an estimation of the dark radius of galaxies, and we gave this estimation for the Milky Way, and also the mean density of the Universe and the density of the intergalactic dark substance for any Cosmological redshift z.

We have seen that the new Theory of dark matter was compatible with the MSC.

In the 2nd Part of our article (3.DARK ENERGY IN THE UNIVERSE), we have proposed a new Cosmological model based on the geometrical form of the Universe obtained in the 1st Part (spherical), and also on the Physical Interpretation of the CMB Rest Frame (CRF) that we also called the *local Cosmological frame*. This new Cosmological model permitted to us to give a simple interpretation of the Cosmological time, in agreement with all astronomical observations. This new Cosmological model also led us to define a new and fundamental frame, called *Universal Cosmological frame*. Then we defined inside the new Cosmological model a first mathematical model of expansion of the Universe, based as the SCM on General Relativity with most theoretical predictions identical to the predictions of the SCM. But this first mathematical model gave the nature of dark matter and dark energy that are necessary in the SCM. We also have seen that a 2nd mathematical model of expansion, much simpler than the 1st one, led despite its great simplicity to theoretical predictions in good agreement with astronomical observations, for instance the theoretical predictions of luminosity distance, angular distance, light-travel distance, commoving distance and Hubble's constant for $z<12$. Moreover this 2nd mathematical model of expansion of the Universe did not need a dark energy, contrary to the SCM and to the first mathematical model of expansion of the Universe, and consequently as the first mathematical model brought a solution to the enigma of dark energy. Finally we studied according to our theory of dark matter and dark energy the evolution of the temperature of the dark substance from the Big-Bang till the present age of the Universe, and we have seen the existence in all the Universe of an energy that was the internal energy of the dark substance, identified with an ideal gas.

We have seen that the observation of the anisotropies of the CMB was in agreement with the 1st mathematical model and contradicted the 2nd mathematical model. For instance, they give a Cosmological time of apparition of the CMB (400000 years) that is in agreement with the prediction of the 1st mathematical model that is the same as SCM. Moreover, they are in agreement with a comobile distance of the last diffusion surface of 43 billions y.l, in agreement with the predictions of the SCM. But the dark energy could not be the internal energy of the dark substance considered as a gas. Indeed, then total dark energy would depend on the temperature of the dark substance and consequently would not remain the same. But we can assume that dark energy is an energy owned by the dark substance analogous to dark energy but proportional to its mass (And consequently total dark energy, and the Cosmological parameter Ω_Λ will remain the same). In the same way, the model of dark matter in galaxy clusters is a priori not satisfying because it implies that the density of dark substance does not evolves in $1/(1+z)^3$. To solve this problem, we first remark that dark substance being a special substance it does not necessarily contribute to the expansion of the Universe the same way as baryonic matter. So we will admit that the density used in order to obtain the Cosmological parameter Ω_C relative to the expansion of the Universe is not the density of dark substance but is proportional to it. With previous hypothesis, the Cosmological parameters Ω_C and Ω_Λ are constant and we will take for them their predicted values according to SCM.

We remarked that a very attractive element in favor of the geometrical model of the Universe proposed by our theory of dark matter and dark energy is that this geometrical model of Universe, finite, spherical and with borders, can be easily conceived by the human mind, which was not the case for models of Universe proposed by the SCM that were either infinite or finite but without borders. It is our model of dark substance that permitted to us to define easily such a Universe, flat and finite.

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