Design & Implementation of Fuzzy Parallel Distributed Compensation Controller for Magnetic Levitation System

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Abstract: This study applies technique parallel distributed compensation (PDC) for position control of a Magnetic levitation system. PDC method is based on nonlinear Takagi-Sugeno (T-S) fuzzy model. It is shown that this technique can be successfully used to stabilize any chosen operating point of the system. All derived results are validated by experimental and computer simulation of a nonlinear mathematical model of the system. The controllers which introduced have big range for control the system.

Keywords: Parallel Distributed Compensation, Magnetic Levitation System, Implementation

I. Introduction

There are different ways to provide non-contact surfaces on of which is through magnetic suspension, usually known as (Maglev). They utilize it in many critical ways and various engineering systems such as magnetic high speed passenger trains, low friction ball bearings, suspension wind tunnel models. Based on the source of suspension, Maglev systems can be classified as attractive or repulsive branches. These systems usually function as open or transitory loops and are defined through complicated differential equations showing the difficulties of handling the system. Therefore designing a feedback controller to decide the precise location of the magnetic levitation is an important and complex task [1,2]. Controlling magnetic levitation has been a matter of great importance and continual concern of many researchers particularly in the current decade. Furthermore, they have utilized the linearization techniques to design control rules for levitation systems [3,4]. In 2004, Z. J. Yang, consider Robust position control of a magnetic levitation system via dynamic surface control technique [5]. Reference [6] explain Robust Output Feedback Control of a Class of Nonlinear Systems Using a Disturbance Observer. Yang et al. introduced adaptive robust output-feedback control method in 2008 [7]. In 2009, Lin proposed a robust dynamic sliding-mode controller utilizing adaptive recurrent neural network [8]. Rafael Morales planned Nonlinear Control for Magnetic Levitation Systems Based on Fast Online Algebraic Identification of the Input Gain in 2011[9]. References [10-12] depict other kinds of nonlinear controllers based on nonlinear methods. Other approaches used to control magnetic levitation system are Robust output feedback stabilization of a field-sensed magnetic suspension system [13], linear controller design[14], as well as Simulation and control design of an uniaxial magnetic levitation system [15]. In 2011, Chih-Min Lin designed SoPC-Based Adaptive PID Control System for Magnetic Levitation System [16]. Mamdani and logical type Nero-Fuzzy based on sliding mode separately and two types of FLEXNFIS designed for control magnetic levitation systems [17]. Also, RBF sliding mode controller has been designed by Aliasghary and his colleagues in 2008 [18]. In this paper, we consider a magnetic levitation system and propose a parallel distributed compensation (PDC) controller for magnetic levitation system. Recently, the Takagi–Sugeno (T–S) fuzzy model based controllers have been applied to several applications, such as Design and Implementation of Fuzzy Control on a Two-Wheel Inverted Pendulum [19], T-S Fuzzy Model-Based Adaptive Dynamic Surface Control for Ball and Beam System [20] and T-S Fuzzy Maximum Power Point Tracking Control of Solar Power Generation Systems [21]. The fuzzy controllers have been widely and successfully utilized for controlling many nonlinear systems. In General, a nonlinear system can be transformed into a T–S fuzzy model, and then, using linear matrix inequality (LMI) approaches the parallel distributed compensation (PDC) fuzzy controller design is accomplished [22-24]. More recently, authors in [25] utilized the neural adaptive controller for controlling magnetic levitation system. Their work is inspired from the method presented in [26]. Parallel distributed compensation have been utilized in [27] for speed control of digital servo system. This work is the extended version of authors from their early work in [28]. The rest of the paper is organized as follows. Section II, contains the mathematical model of the magnetic levitation system. Section III deals with the parallel distributed compensation controller in detail. Section IV discusses the simulation results of the proposed control schemes. In section V, we design and implement the controller for a magnetic levitation system model of Feedback company series (33-210). Finally, the conclusion is given in Section VI.

DOI: 10.9790/1676-1204022028 www.iosrjournals.org
II. Mathematical Model Of The System

In this paper the magnetic levitation system is considered. It consists of a ferromagnetic ball suspended in a voltage-controlled magnetic field. The vertical motion is only considered. Keeping the ball at a prescribed reference level is the objective. The schematic diagram of the system is shown in Figure 1. The dynamic model of the system can be written as [3]:

\[
\frac{dp}{dt} = v
\]

\[
m \frac{dv}{dt} = mg_c - \frac{1}{p}
\]

\[
R_i \frac{d(L_p)}{dt} = e
\]

\[
L_p = L_1 + \frac{2C}{p}
\]

Where
- \(P\) the ball position
- \(v\) the ball's velocity
- \(R\) the coil's resistance
- \(i\) the current through the electromagnet
- \(e\) the applied voltage
- \(m\) the mass of the levitated object

\[
x_1 = p, \quad x_2 = v, \quad x_3 = i \quad \text{and} \quad u = e.
\]

Thus, the state space equations of the magnetic levitation system are as follows:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \left( \frac{c_l}{m} \right) x_2 - g_c \left( \frac{x_2}{x_1} \right) \\
\dot{x}_3 &= -\frac{R}{L_1} x_3 + \frac{2C}{L} \left( \frac{x_2 x_2}{x_1} \right) + \frac{u}{L} \\
y &= x_1
\end{align*}
\]

The parameters of the magnetic levitation system are as follows [3]. The coil’s resistance \(R=28.7\Omega\), the inductance \(L_1=0.65\text{H}\), the gravitational constant \(g_c=9.81\text{m/s}^2\), the magnetic force constant \(C=1.24 \times 10^{-2}\) and the mass of the ball \(m=11.87\text{g}\) and \(x_{1d}=0.01\text{m}\) the desired value of \(x_1\).

![Figure 1. Magnetic levitation system](image)

III. Parallel Distributed Compensation

Initially, a given nonlinear plant is represented by the Takagi-Sugeno fuzzy model. Expressing the joint dynamics of each fuzzy inference (rule) by a linear system model is the main characteristics of the T-S fuzzy model. Particularly, description of the Takagi-Sugeno fuzzy systems is done by fuzzy IF-THEN rules, which linear input-output relations of a system is locally represented by. The fuzzy system is of the following form [29, 30]:

\[
\text{Rule } i: \quad \text{IF } q_i(t) \text{ is } M_{i1} \ldots \text{and } q_i(t) \text{ is } M_{ir} \text{ THEN} \\
\dot{x}(t) = A_i x(t) + B_i u(t) \\
\text{FOR } i=1,2,\ldots,r
\]
Where as
\[ q^T(t) = [q_1(t), q_2(t), ..., q_r(t)] \]
\[ u^T(t) = [u_1(t), u_2(t), ..., u_r(t)] \]

Where \( x(t), u(t) \) and \( q(t) \) are the state vector, the input vector and the assumptive variables’ vector, respectively. 

\( r \) is the number of model rules, \( M_i \) is the fuzzy set and \( \dot{x}(t) = A_i x(t) + B_i u(t) \) is the output for every \( i \) rule where \( A_i \) and \( B_i \) are the state vector matrix, respectively.

\[ A \in R^{n \times n}, B \in R^{n \times m} \] (6)

Parallel Distributed Compensation theory (PDC) is a compensation for every fuzzy model rule. The resulting overall fuzzy controller, which is nonlinear in general, is a fuzzy blending of each individual linear controller shares the same fuzzy sets with the fuzzy system (4). PDC controller is often shown as below: Where \( F_i \) are the feedback gains. The fuzzy control rules have a linear controller (state feedback laws in this case) in the consequent parts. We can use other controllers, for example, output feedback controllers and dynamic output feedback controllers, instead of the state feedback. Thus, fuzzy controller is as follow:

\[ u_i(t) = \sum_{j=1}^{r} w_{ij}(t) F_j x(t) \]

\[ w_{ij}(t) = \prod_{j=1}^{r} M_j(q_j(t)) \] (9)

The term \( M_j(q_j(t)) \) is the grade of membership \( (q_j(t)) \) in \( M_j \). Where the feedback gains with state feedback laws (Ackerman) will be attained as below:

\[ F_i = q_{mi}^T \times \alpha(A_i), \]

\[ i = 1, 2, ..., n. \] (10)

Where \( q_{mi} \) is as follow:

\[ q_{mi} = [0, 0, ..., 0, 1]^T \times \phi_i \]

\[ \phi_i = [B_i A_i B_i ..., A_i^{-1} B_i] \]

\[ i = 1, 2, ..., n. \] (11)

The equilibrium of a fuzzy control system (4) is globally asymptotically stable if there exists a common positive definite matrix \( P \) the following two conditions are satisfied:

\[ A_i^T P A_i - P = 0 \]

\[ i = 1, 2, ..., r. \] (12)

A. PDC Controller for Magnetic Levitation System

In section we used Takagi-Sugeno fuzzy model based on PDC for Magnetic levitation system. It should be noted that the ball equilibrium points is between 1 to 2 centimeters. The state space (3) can be model by Sugeno rule as follow:

\[ \dot{x}(t) = A_i x(t) + B_i u(t) \]

\[ i = 1, 2, ..., r. \] (13)

Matrixes \( A_i \) and \( B_i \) computed by linearization equilibrium points as follow:

\[ A_i = \begin{bmatrix} 0 & 1 & 0 \\ \frac{c_{m1} \cdot i}{m_{c1}} & 0 & -2 \frac{c_{m1}}{m_{c1}} \frac{R}{l_1 + 2 \frac{R}{\omega}} & 0 \\ 0 & 2 \frac{c_{m2}}{k_{m2}} & \frac{R}{l_1} & 0 \\ 0 & 0 & \frac{R}{l_1 + 2 \frac{R}{\omega}} & 0 \end{bmatrix} \]

\[ B_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, i = 1, 2, ..., 5. \] (14)
Where equilibrium points as follow:
\[ x_{01} = 0.010, x_{02} = 0, x_{03} = 0.3064 \\
 x_{02} = 0.012, x_{03} = 1 \times 10^{-3}, x_{04} = 0.3064 \\
 x_{03} = 0.014, x_{04} = 1 \times 10^{-4}, x_{05} = 0.3677 \\
 x_{04} = 0.016, x_{05} = 1 \times 10^{-5}, x_{06} = 0.4290 \\
 x_{06} = 0.018, x_{07} = 1 \times 10^{-6}, x_{08} = 0.4903 \]  
(15)

Considering the fact that equilibrium point of \( X_i \) is between 1 and 2 centimeters, the membership function of the system was determined as such a distance. Figure 2 shows the membership functions.

![Membership functions](image)

**Figure 2.** Membership functions

In PDC method for each rule model a proper control rule is considered where is as follow:

\[ \text{Rule } i \] 
\[ \text{IF } x_1(t) \text{ is } M_i, \text{ THEN } \]
\[ u(t) = -F_i x(t) \]
\[ \text{FOR } i = 1, 2, \ldots, 5. \]  
(16)

The feedback gains with state feedback laws will be attained as below:
\[ F_1 = \begin{bmatrix} -1.30 \times 10^4 & -2.75 \times 10^2 & -9.8 \times 10^2 \end{bmatrix} \]
\[ F_2 = \begin{bmatrix} -1.46 \times 10^4 & -3.24 \times 10^2 & -9.9 \times 10^2 \end{bmatrix} \]
\[ F_3 = \begin{bmatrix} -1.60 \times 10^4 & -3.73 \times 10^2 & -1.00 \times 10^2 \end{bmatrix} \]
\[ F_4 = \begin{bmatrix} -1.75 \times 10^4 & -4.22 \times 10^2 & -1.01 \times 10^2 \end{bmatrix} \]
\[ F_5 = \begin{bmatrix} -1.89 \times 10^4 & -4.74 \times 10^2 & -1.01 \times 10^2 \end{bmatrix} \]  
(17)

Considering relation (12) matrix \( P \) is a definite positive matrix which is shown as follow:
\[ P = \begin{bmatrix} 97.85 & -2.5 & 55.19 \\ -2.5 & 0.0771 & -2.632 \\ 55.19 & -2.632 & 544.50 \end{bmatrix} \]  
(18)

**IV. Simulation Results**

In this section, the results of simulation are shown. The Figure 3 shows the position of ball. Figures 4 and 5 show the states system and Figure 6 shows the signal control (the applied voltage) for system. In Figures 3, 4, 5 and 6 we compare PDC controller result with PID controller and Mamdani Neuro-Fuzzy controller.

![Position of Ball](image)

**Figure 3.** Position of Ball for \( x_{1d} = 0.01 \) m

![Simulation result](image)

**Figure 4.** Simulation result \( x_2 \) for \( x_{2d} = 0 \) rad/s
Results show that PDC controller delivers the ball to the desired position much more quickly than Mamdani Neuro-Fuzzy and PID controllers and has shorter settling time than other controllers. In Table I we compare PDC controller result with other controllers.

<table>
<thead>
<tr>
<th>Controller</th>
<th>Maximum signal control (volt)</th>
<th>Settling time (Sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDC[purpose]</td>
<td>13</td>
<td>0.11</td>
</tr>
<tr>
<td>PID[18]</td>
<td>-800</td>
<td>0.9</td>
</tr>
<tr>
<td>SLIDING MODE[12]</td>
<td>25</td>
<td>0.5</td>
</tr>
<tr>
<td>FEL[18]</td>
<td>25</td>
<td>0.3</td>
</tr>
<tr>
<td>RBF[18]</td>
<td>100</td>
<td>0.25</td>
</tr>
<tr>
<td>MAMDANI NEURO FUZZY[17]</td>
<td>250</td>
<td>0.2</td>
</tr>
<tr>
<td>ANFIS[17]</td>
<td>55</td>
<td>0.28</td>
</tr>
<tr>
<td>Basic flexible OR-type NFIS[17]</td>
<td>54</td>
<td>0.25</td>
</tr>
<tr>
<td>Parallel OR-type FLEXNFIS[17]</td>
<td>55</td>
<td>0.6</td>
</tr>
<tr>
<td>Basic flexible AND-type NFIS[17]</td>
<td>58</td>
<td>0.32</td>
</tr>
<tr>
<td>Parallel AND-type FLEXNFIS[17]</td>
<td>65</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table I show that PDC controller has shorter settling time than other controllers and it has less Maximum signal control competed with other controllers. PDC controller as compared with sliding mode, FEL, RBF, Mamdani Neuro fuzzy, ANFIS and flexible fuzzy controller, has got much easier planning and mathematical calculation.

V. Design And Implementation

In this section, we introduced a magnetic levitation system model of Feedback company series (33-210) and we have design and implemented a PDC controller for it. Here, we inspired from the presented approaches in [31, 32, and 33] in our experimental evaluation of PDC controller for magnetic levitation system. Figure 7 shows the system. The dynamic model of the system can be written as follow:
\begin{equation}
\begin{aligned}
\epsilon &= Ri + L \frac{dx}{dt} - L_e x_0 + \frac{i}{x} \\
\frac{dx}{dt} &= v \\
m \frac{dv}{dt} &= mg - c(\frac{i}{x})^2
\end{aligned}
\tag{19}
\end{equation}

The where $x$ denotes the ball position, $v$ is the ball’s velocity, $R$ is the coil’s resistance, $i$ is the current through the electromagnet, $e$ is the applied voltage, $m$ is the mass of the levitated object, $g$ denotes the gravity and $c$ is the magnetic force constant. $L$ is the coil’s inductance that is a nonlinear function of ball’s position ($x$) and $L_e$ is a parameter of system $x$, the desired value of $x$. In table II shows Parameter values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_e$</td>
<td>0.65H</td>
</tr>
<tr>
<td>$R$</td>
<td>28.7Ω</td>
</tr>
<tr>
<td>$c$</td>
<td>$1.477 \times 10^4$</td>
</tr>
<tr>
<td>$g$</td>
<td>9.81 m/sec²</td>
</tr>
<tr>
<td>$m$</td>
<td>0.021 kg</td>
</tr>
<tr>
<td>$x_e$</td>
<td>$[-1\ldots 2]$ cm</td>
</tr>
</tbody>
</table>

Obtaining and fitting frequency response data to estimate a plant model is a common practice in control design. Closed-loop frequency response data for the plant were obtained and plotted in the bode diagram shown in Figure 8.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{frequency_response_data.png}
\caption{Frequency response data for the plant}
\end{figure}

From the diagram, the experimental model of the maglev system was approximated as:

\begin{equation}
G_{\text{exp}}(s) = \frac{1.6}{\frac{x}{30.5} + 1} \frac{x}{30.5} - 1
\end{equation}

It should be noted that the ball equilibrium points is between -1 to -2 centimeters. The dynamic model of the system linearized around 5 equilibrium points. Therefore there would be 5 subsystems. Each subsystem was approximated to a second degree system.

\begin{align*}
x_{e_1} &= -1\text{cm} \\
x_{e_2} &= -1.2\text{cm} \\
x_{e_3} &= -1.4\text{cm} \\
x_{e_4} &= -1.6\text{cm} \\
x_{e_5} &= -1.8\text{cm}
\end{align*}

The state space equations (19) can be model by Sugeno rule as follow:

\begin{equation}
\text{Rule }i \quad \text{IF } X(t) \text{ is } M, \text{ THEN } A_i x(t) + B_i u(t)
\end{equation}

FOR $i=1,2,\ldots,r$

Matrices $A_i$ and $B_i$ computed by linearization equilibrium points as follow:
Considering the fact that equilibrium point of $X$ is between -1 and -2 centimeters, the membership function of the system was determined as such a distance. Figure 9 shows the membership functions. The control rule is as follow:

$$\text{Rule } i \quad \text{IF} \quad X(t) \text{ is } M_i \text{, THEN} \quad u(t) = -F_i x(t)$$

FOR $i=1, 2, \ldots, 5$. \hspace{1cm} (23)

The feedback gains with state feedback laws will be attained as below:

$$F_1 = [2.6405 \ 0.073904]$$
$$F_2 = [2.5530 \ 0.073904]$$
$$F_3 = [2.4858 \ 0.073904]$$
$$F_4 = [2.4187 \ 0.073904]$$
$$F_5 = [2.3515 \ 0.073904]$$

The figure 9 shows simulation of PDC controller. \hspace{1cm} (24)

The figures 10 and 11 show results of experimental and figures 12 and 13 show results computer simulation. Figures 10 and 12 show the position of ball. Figures 10 and 12 show signal control (the applied voltage) for system.
Results show that PDC controller delivers the ball to the desired position. The experimental and simulating results are similar with regards to the settling time and the control signal quantity.

VI. Conclusions

In this paper, the intelligent Fuzzy controller for the Magnetic Levitation system has been designed implemented. This approach has been developed based on Takagi Sugeno Fuzzy model and Fuzzy parallel distributed system. Designing this technique is to change a nonlinear system into a series of linear subsystems. Each linear subsystem will be controlled independently and separate sub controllers will be included in a Fuzzy function basis. The Fuzzy function basis under the control of the linear controllers applies to the system a controlling signal relevant to the current system condition based on linear modeling.

In this paper, suggested method maintains system stability for each equilibrium point. Results show that a PDC controller has shorter settling time than other controllers and it has less Maximum signal control competed with other controllers. PDC controller as compared with other controllers has got much easier planning and mathematical calculation.

References

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IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE) is UGC approved Journal with Sl. No. 4198, Journal no. 45125.