Exploring Novel Cyclic Extensions of Hamilton's Dual-Quaternion Algebra

RICHARD L. AMOROSO

Physics Lab, Noetic Advanced Studies Institute
902 W 5400 N, Beryl, UT 84714-5104 USA
amoroso@noeticadvancedstudies.us

PETER ROWLANDS

Department of Physics
University of Liverpool, Liverpool, L69 7ZE UK
p.rowlands@liverpool.ac.uk

LOUIS H. KAUFFMAN

Department of Mathematics, Statistics & Computer Science
University of Illinois at Chicago, Chicago, IL, 60607-704 USA
Kauffman@uic.edu

We make a preliminary exploratory study of higher dimensional (HD) orthogonal forms of the quaternion algebra in order to explore putative novel Nilpotent/Idempotent/Dirac symmetry properties. Stage-1 transforms the dual quaternion algebra in a manner that extends the standard anticommutative 3-form, *i, j, k* into a 5D/6D triplet. Each is a copy of the others and each is self-commutative and believed to represent spin or different orientations of a 3-cube. The triplet represents a copy of the original that contains no new information other than rotational perspective and maps back to the original quaternion vertex or to a second point in a line element. In Stage-2 we attempt to break the inherent quaterniónic property of algebraic closure by stereographic projection of the Argand plane onto rotating Riemann 4-spheres. Finally, we explore the properties of various topological symmetries in order to study anticommutative - commutative cycles in the periodic rotational motions of the quaternion algebra in additional HD dualities.

Keywords: Algebraic closure, Antispace, Dirac algebra, Dual quaternions, Idempotent, Nilpotent, Quaternions

There is a world of strange beauty to be explored here... John C. Baez [1].

1. Introduction - Brief Quaternion Review

Since quaternions are not ubiquitously appreciated by physicists we review them briefly here. For many years prior to discovering the quaternions William Rowan Hamilton tried unsuccessfully to extend the common system of complex numbers, $a + bi$ where the real numbers, a are represented by the x-axis and the imaginary numbers, bi are represented orthogonally on the y-axis. Hamilton considered this a 2-dimensional (2D) space and desired to extend the system to 3D by utilizing an additional set of orthogonal complex numbers, cj. This attempt failed because he found that this system of 'triplets', $a + bi + cj$ did not exhibit the necessary algebraic property called 'closure' that would complete the algebra, i.e. the product ij had no meaning within the algebra and therefore had to represent something else. In 1843 Hamilton finally realized he needed a 3rd system of imaginary numbers, dk to define an algebraic system properly exhibiting the needed property of closure [2-4].

$$i^2 = j^2 = k^2 = ijk = -1 \qquad (1)$$

with additional anticommutative multiplication rules

$$ij = -ji = k,\ \ jk = -kj = i,\ \ ki = -ik = j\,. \qquad (2)$$

which can be represented graphically as

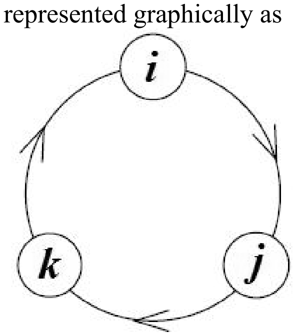

Figure 1. Graphic summary of the quaternion cyclic permutation rules. Adapted from Baez. [1].

Or represented in tabular form the dual quaternions, $\pm 1, \pm i, \pm j, \pm k$ form a non-Abelian group of order 8.

TABLE 1. Quaternion Cyclic Multiplication

	1	***i***	***j***	***k***
1	**1**	***i***	***j***	***k***
i	***i***	**-1**	***k***	***-j***
j	***j***	***-k***	**-1**	***i***
k	***k***	***j***	***-i***	**-1**

But by introducing the property of closure Hamilton sacrificed the principle of commutivity, *ij* = -*ji* = *k*. Frobenius proved in 1878 that no other system of complex algebra was possible with the same algebraic rules. This convinced him that he found a true explanation of the 3-dimensionality of space; with the 4th or real component in this case representing time as the real component (the 1 in quaternions, Tbl. 1). The Quaternion, Q_4 and Octonion, O_8 algebras arise naturally from restrictions inherent in the properties producing the discreteness of 3-dimensionality, i.e. the property of anticommutivity as realized by Hamilton as the key requirement for achieving the property of 'closure' when he discovered the *i, j, k* basis for the quaternion algebra or that in complex representation, $H = a \cdot 1 + bi + cj + dk$ is not commutative.

2. Background and Utility of New Quaternion Extensions

Our problem is similar to the one originally facing Hamilton. We also wish to add additional complex dimensionality but in a manner that also reintroduces commutivity in a periodic manner. Our procedure in this exercise is to manipulate the inherent cyclicality of the normative quaternion algebra (anticommutative) in a manner that the property of closure is broken periodically during a transformation through a Large Scale Additional Dimensional (LSXD) 'continuous-state' mirror symmetric space evolving through periodic anticommutative and commutative modes [5,6]. This requires a new form of transformation of the line element. The usual representation of the line element, *ds* in an *n*-dimensional metric space is $ds^2 = d\mathrm{q} \cdot d\mathrm{q} = g\left(d\mathrm{q}, d\mathrm{q}\right)$ where g is the metric tensor, and dq an infinitesimal displacement. In *n*-dimensional coordinates q = $(q^1, q^2, q^3 \ldots q^n)$ and the square of arc length is $ds^2 = \sum g_{ij} dq^i dq^j$. Indices *i* and *j* take values 1, 2, 3 ... *n*. The metric chosen determines the line element. We will continue this development in an ensuing section in terms of stereographically projected Riemann manifolds where a Riemann surface does not entail an inherent particular metric.

According to Rowlands [4,7-10] quaternion cyclicity reduces the number of operators by a factor of 2 and prevents the possibility of defining further complex terms. To overcome this generalization of term limitations we explore utilizing shadow dualities employing additional copies of the quaternion algebra mapped into a 5D and 6D space to transform a new dualed 'trivector' quaternion algebra. As far as we know this can only occur under the auspices of a continuous-state cosmology within a dual metric where one coordinate system is fixed and then the other is fixed in a sort of leap-frog baton passing fashion [5]. We will explain this in more detail later with as much rigor currently at our disposal and conceptually in terms of a wonderfully illustrative metaphor in the 'Walking of the Moai' [11,12]. Correspondence is made to the usual 3D quaternion algebra as the resultant or re-closure that occurs at the end of the cycle repeats over and over continuously.

According to Rowlands' the nilpotent duality or a zero totality universe model requires an anti-commutative system that has this property of closure, whereas a commutative system remains open and degenerate to infinity [4]. This duality is fundamental to the continuous-state (like an internalized gravitational free-fall) postulate of Amoroso's Holographic Anthropic Multiverse cosmology [5] which is compatible with Calabi-Yau mirror symmetry properties of M-Theory suggesting a putative relationship to physicality. We suspect also that in the future correspondences will be found between the continuous-state hypothesis and the Rowlands' nilpotent universal rewrite system [4,5].

3. A New Concept in Quaternion Algebra - The 1st Triplet

We start with a standard quaternion in italicized notation *i,j,k* which physically represents a fermionic vertex in space and therefore has a copy as a vacuum anti-space *Zitterberwegung* partner [10], ***i,j,k*** (bold italic notation) which together can form a nilpotent [4,6-10]. Multiplying the two (space-antispace) nilpotent components together we get 3 copies (3 sets of three) of the original quaternion (uppercase notation) at some new level of postulated higher space; but still with no new information such that each of the three copies commute with themselves internally. Our initial consideration is that these three copies are spin-like objects that conceptually represent 3 different views of a cube such as: **I**) planar face, **J**) edge and **K**) corner (as if collapsed to a hexagon). See Fig. 2.

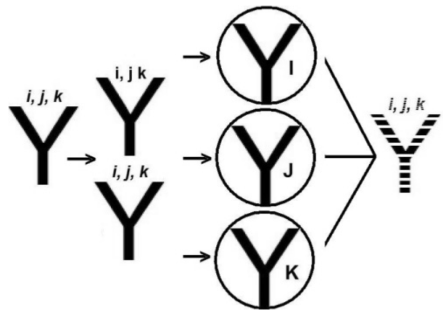

Figure 2. This figure and next (Fig. 3) represent different aspects of the usual quaternion algebra containing no new information beyond the original fermionic vertex. We postulate they represent conceptually 3 different ways (orientations) of viewing a cube. The first, $\mathbf{Y_I}$ a usual planar face, second, $\mathbf{Y_J}$ perhaps like view onto an edge and the third, $\mathbf{Y_K}$ from a corner which would make the cube look like it was collapsed to a 2D hexagon. In summary: Fig. 2a) Usual *i,j,k* quaternion. b) Space-antispace dual quaternion, *i,j,k and* ***i,j,k***. c) The new **IJK** nilpotent triplet. d) Reduced back to the original quaternion in 2a).

We pass through a stage of three commutative objects, each of which is constructed from *ijk* (original quaternion) and ***ijk*** (nilpotent vacuum copy) which are thus a set of dual quaternions. Knowing one of these gives us the other two automatically.

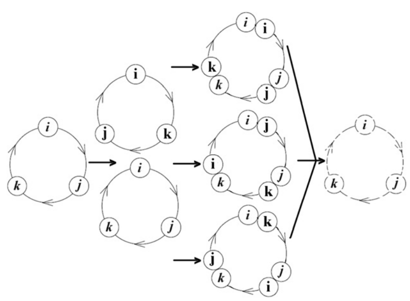

Figure 3. Varied view of Fig. 2 but highlighting the cyclicality of the algebraic elements. Reduces to original *i,j,k* quaternion fermionic vertex. Also in summary (same as Fig. 2): Fig. 3a) Usual *i,j,k* quaternion. b) Space-antispace dual quaternion, *i,j,k and* ***i,j,k***. c) The new nilpotent triplet. d) Reduced back to the original quaternion in 3a).

Here is the algebraic derivation of the geometric objects in Figs. 2 & 3. In the quaternion algebra below, the usual italicized anticommutative quaternion, *i,j,k* acquires a dual vacuum (bold italicized) quaternion, ***i,j,k.*** The product of these two quaternions becomes three commutative algebras (Figs. 2&3):

$$\begin{pmatrix} i\mathrm{i} \\ j\mathrm{j} \\ k\mathrm{k} \end{pmatrix}\begin{pmatrix} i\mathrm{j} \\ j\mathrm{k} \\ k\mathrm{i} \end{pmatrix}\begin{pmatrix} i\mathrm{k} \\ j\mathrm{i} \\ k\mathrm{j} \end{pmatrix} \qquad (3)$$

And their product becomes the new quaternion, **I J K** (Fig.2c). Each of these algebras is a copy of the others and each is commutative on itself. Selecting out the three versions makes an anticommutative set. In Eq. (3) Mathtype would not allow bold font so we used regular and italic to illustrate the dualing.

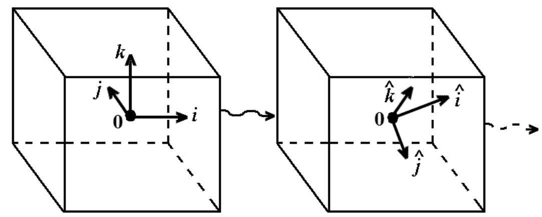

Figure 4. Different complex orientations of the Euclidean 3-cube projected from a quaternion space-antispace fermionic vertex or singularity, *i,j,k*.

4. Vectors, Scalars, Quaternions and Commutivity

In this section notation is as follows. Vector are in regular and bold font. Quaternions are in regular italic and bold italic font. For the trivector we add the half quote mark (').

i j k	*i***i** *i***j** *i***k**	*i'*	1
i j k	*i*i *i*j *i*k	*i'*	1
vector	*bivector*	*trivector*	*scalar*
	quaternions		

Vectors

i j k anticommute
i j k anticommute
ii **j**j **k**k commute

Quaternions

i j k anticommute
i j k anticommute

i**i* ***i**j* ***i**k* ***j *i'**k*** *nilpotent*
i**i* ***i**j* ***i**k* ***j - *nilpotent*
(*5th term automatic*)

*ii' **jj** k**k*** *commute*

ai bj ck 1d
***A**i* ***B**j* ***C**k* 1D

ai bj ck 1d
ai bj ck 1D

ii jj** k**k
ij jk** k**i
*i**k** **ji** k**j***

ijk anticommute SPACE
ijk *anticommute* VACUUM ANTISPACE
TOTAL 6 Degrees of freedom

This is Fig. 2 part b.

ii ***jj*** ***kk*** *commute* SPINOR SPACE I
ij ***jk*** ***ki*** *commute* SPINOR SPACE J
ik ***ji*** ***kj*** *commute* SPINOR SPACE K
TOTAL 9 Degrees of freedom

The Above (**I J K**) is Fig. 2 part c

IJ (*i* ***i*** *j* ***j*** *k* ***k***) (*i* ***j*** *j* ***k*** *k* ***i***) → (*i j k* –***i*** –***j*** –***k*** –***ik*** –***ji*** –***kj***)
JK (*i* ***j*** *j* ***k*** *k* ***i***) (*i* ***k*** *j* ***i*** *k* ***j***) → (*i j k* –***i*** –***j*** –***k*** –***ii*** –***jj*** –***kk***)
KI (*i* ***k*** *j* ***i*** *k* ***j***) (*i* ***i*** *j* ***j*** *k* ***k***) → (*i j k* –***i*** –***j*** –***k*** – ***ij*** –***jk*** –***ki***)

Reverse **IJ = – JI, JK = – KJ, KI = – IK. I, J, K** anticommutative, becomes the new *i j k* at the next level in the cycle. Because of nilpotency, *i j k* and ***i j k*** are dual and contain the same information. Also, **I, J, K** is made up only of these terms and contains the same information again so we can map **I, J, K** directly onto *i j k* and then pull out its dual by nilpotency and go round the cycle again continuously. The **I**, **J**, **K** are a **new** set of quaternions with the same structure as *i j k* but with only the same infomration as as contained in the usual *i j k* (real) and its dual ***i j k*** (vacuum).

We assume **I, J, K** are Dirac phase rotations of hypercube B. At phase K by stereographic projection (topological switching) a copy of A is boosted to a Riemann sphere "above" K. We assume the Riemann sphere by conformal scale-invariance inherent to the cosmology. This gives us + *i,j,k* and - *i,j,k* on opposite sides of the boosted Riemann sphere. The **I, J, K**s all rotate so they take turns cyclically being in position to be boosted.

5. Toward Completing the Hypercube: The 2nd Triplet

In addition to boosting into the higher space we wish to turn the nilpotent into an idempotent that transforms into the vacuum in a similar manner. By multiplying we also get three copies that are three sets of three in the same manner as in the nilpotent boost into the higher space above that are also spin-like. And that will likewise by stereographic projection boost new copies into Riemann spheres above K.

It is suggested that the orientation of the Cs in Fig. 5 is periodic and that all three take turns boosting orientation information to the hypercube in the higher space. Through more layers of boosting into the higher space, in conjunction with additional symmetric idempotent copies in the vacuum space we wish to end up with a structure that is like a bag of close-packed ping pong balls or rotating Riemann spheres where the original quaternion sphere always remains at the center, but through a rewrite system (where the nilpotents are annihilation and creation 'vectors') that continuously boost copies of the original orientation information from the center to the surface spheres and back again. It is possible to describe these cubes as component hyperspheres or dodecahedral polyhedrons.

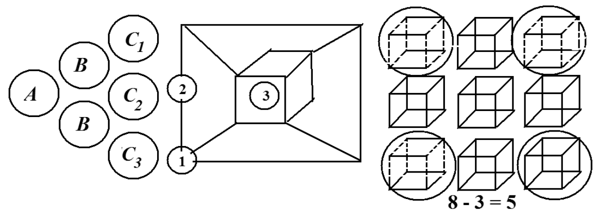

Figure 5. The first part cycles A, B, C (as in Figs.1&3) is the algebra progression as in Figs. 2 & 3. The center hypercube represents the orientation provided by the three phases of the cube from C1, C2 & C3. The 3rd part represents the 8 cubes exploded off a hypercube, suggesting that there are many orientations to explore with the new algebraic triplets.

5.1 *Higher Doublings - Planar To Riemann Sphere*

Quaternions

i j k anticommute SPACE (Real) ⇄
Complex Argand Plane *i*

i j k *anticommute* VACUUM (+C3) ⇄
Complex Argand Plane ***i***

-i -j-k *anticommute* VACUUM (- C3) ⇄
Stereographic Riemann Sphere ***I***

-i -j-k *anticommute* VACUUM ($\pm$ Virtual) ⇄
Stereographic Riemann Sphere **I**

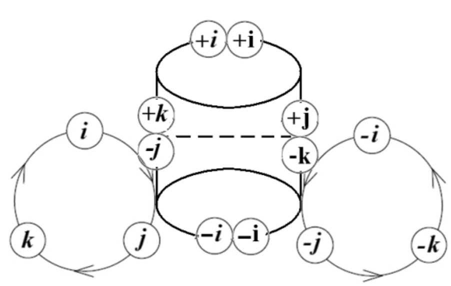

Figure. 6. Peter's intermediate skipped, central is straight to the spinor element (with additional duality to let the mechanism work cyclically), right is the symmetry reversing mechanism (when coupled at certain phase of the central spinor). To achieve causal separation it is postulated this structure is doubled again?

These are not planar (as usual Q) but stereographic projections onto Riemann sphere. In the central portion closure (when it appears) subtracts the extra degrees of freedom in the extra duality. When not subtracted the

Riemann sphere may "flip" the symmetry to achieve the basis for the commutative mode node. Doubled again, perhaps something like that shown in Fig. 6 to make it hyperspherical and causally separated.

In the London Underground tunnel near Imperial College there is an entrance to The Victoria and Albert Museum. In front of the entrance is a display sign encased in a 3 or 4 meter long clear plastic tube about a meter in diameter. Inside the tube components of a large (V & A) rotate such that the A becomes the V and vice versa (with the help of the ampersand) [19,20]. These rotations are insufficient because they are planar and preserve the symmetry and closure of the original quaternion algebra [2,3]. With an additional copy on the Riemann sphere (mirror symmetries) 2 rotations occur simultaneously; but closure is not yet broken. This requires a 3rd doubling in order to acquire properties of Dirac spherical rotation. With sufficient degrees of freedom one preserves the usual anticommutative parameters of closure. The other by 'boosting' through a Necker-like topological switching reverses the symmetry and passes cyclically through a commutative mode. And then back down through a reverse Necker n-spinor switch to return to the origin or 2nd point in the line element reducing back to the usual quaternion form. This is also beautifully illustrated in the 'Walking of the Moai on Rapa Nui' [11,12].

5.2. *Nilpotnet Idempotent Vacuum Doublings*

The nilpotent when multiplied by $\underset{\sim}{k}$ becomes idempotent.

$$\left(i\underset{\sim}{k}E + if + \underset{\sim}{j}m\right)\left(i\underset{\sim}{k}E + if + \underset{\sim}{j}m\right)$$
$$\left(i\underset{\sim}{k}E + if + \underset{\sim}{j}m\right)\left(i\underset{\sim}{k}E + if + \underset{\sim}{j}m\right)$$
$$\left(i\underset{\sim}{k}E + if + \underset{\sim}{j}m\right)\left(i\underset{\sim}{k}E + if + \underset{\sim}{j}m\right)$$
$$+(\quad)(\quad) = 0$$

5.3. *As Generalized Equation*

The anticommutative

Anticommutative

$$(i_1 + j_1 + k_1)(i_2 + j_2 + k_2) =$$

Commutative Commutative Commutative

$$(i_1 i_2 + j_1 j_2 + k_1 k_2)(i_1 j_2 + j_1 k_2 + k_1 i_2)\ (i_1 k_2 + j_1 i_2 + k_1 j_2) =$$

I + J + K

a new iteration of the original i, j, k

5.4. *As Tensor Trasformation*

Transform (X Y Z)

$$\begin{pmatrix} i_1 j_1 & i_2 j_2 & i_3 j_3 \\ j_1 k_1 & j_2 k_2 & j_3 k_3 \\ k_1 i_1 & k_2 i_2 & k_3 i_3 \end{pmatrix} = \begin{pmatrix} \mathrm{I} \\ \mathrm{J} \\ \mathrm{K} \end{pmatrix}$$

6. Quaternion Mirrorhouses

Utilizing hypersets, to represent two parallel mirrors, a simple 2-mirror mirrorhouse may be represented as: X = {Y}, Y = {X} (ignoring the inversion effect of mirroring). In constructing a 3-mirror mirrorhouse interestingly the hyper-structure turns out to be precisely the structure of the quaternion imaginaries. Let i, j and k be hypersets representing three facing mirrors [13]. We then have

$$i = \{j,k\},\ j = \{k,i\} \text{ and } k = \{i,j\}$$

where the notation $i = \{j,k\}$ means, e.g. that mirror i reflects mirrors j and k in that order.

With three mirrors ordering is vital because mirroring inverts left/right-handedness. If we denote the mirror inversion operation by "-" we have i = {j,k} = -{k,j}, j = {k,i} = -{i,k} and k = {i,j} = -{j,i} which is the structure of the quaternion triple of imaginaries: i = j*k = -k*j, j = k*i = -i*k, k = i*j = -j*i.

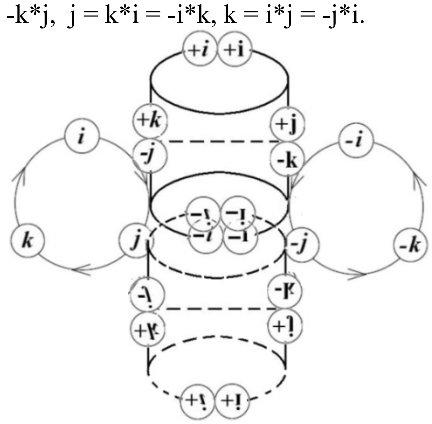

Figure 7. Possible new duality doublings in higher space.

The quaternion algebra therefore is the precise model of the holographic hyper-structure of three facing mirrors, where we see mirror inversion as the quaternionic anticommutation. The two versions of the quaternion multiplication table correspond to the two possible ways of arranging three mirrors into a triangular mirrorhouse.

Following Goertzel's [13] construction of a house of three mirrors; he demonstrates that the construction has the exact structure of the quaternion imaginaries. He lets i, j and k be hypersets representing the three facing mirrors such that i = {j,k}, j = {k,i} and k = {i,j} The notation i={j,k} means, e.g. that mirror i reflects mirrors j and k in that order. Goertzel points out that mirroring inverts left/right-handedness. Denoting mirror inversion by the minus sign "-" yields i = {j,k} = -{k,j}, j = {k,i} = -{i,k} and k = {i,j} = -{j,i} which is the exact structure of the quaternion imaginary triplets: i = j*k = -k*j, j = k*i = -i*k, k = i*j = -j*i. Goertzel then claims the quaternion algebra is therefore the precise model of the holographic hyper-structure of three facing mirrors, where mirror inversion is the quaternionic anticommutation.

The two versions of the quaternion multiplication table correspond to the two possible ways of arranging three mirrors into a triangular mirrorhouse.

i*j = k j*i = - k
j*k = i k*j = - i
k*i = j i*k = - j

Which with dual counterpropagating mirror reflections is like a standing-wave.

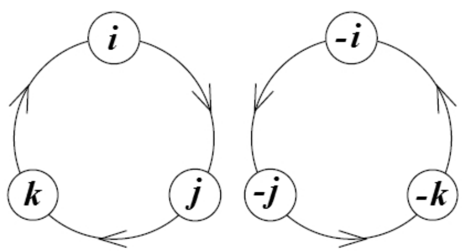

Figure 8. A) the usual quaternion series cycle, B) The dual counterpropagating standing-wave mirror image.

6.1. *Observation as Mirroring*

To map the elements inside the mirrorhouse/algebraic framework noted previously, it suffices to interpret the X = {Y}, Y = {X} as "X observes Y", "Y observes X" (e.g. we may have X = primary subset, Y = inner virtual other), and the above i = {j,k}, j = {k,i}, k = {i,j} as "i observes {j observing k}"
"j observes {k observing i}" and "k observes {i observing j}". Then we can define the - observation as an inverter of observer and observed, so that e.g. {j,k} = -{k,j}. We then obtain the quaternions

i = j*k = -k*j, j = k*i = -i*k, k = i*j = -j*i

where multiplication is observation and negation is reversal of the order of observation. Three inter-observers = quaternions.

This is where the standard mathematics of quataternion algebra has remained. In an actual mirrorhouse the reflections, like Bancos ghost extend into infinity. We wish to build an additional dual trivector mirrorhouse on top of the quaternion mirrorhouse that reduce back to the usual quaternion mirrorhouse. The idea is that in the additional mirrorhouses the cyclic reflections would reverse anticommutative effects to commutative effects.

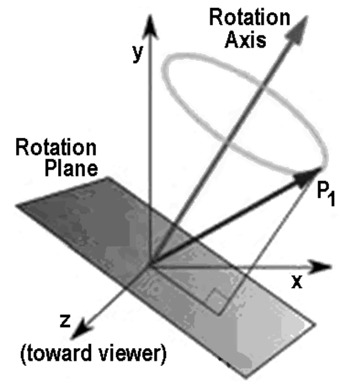

Figure 9. A quaternion can be used to transform one 3D vector into another.

The quaternions can be physically modeled by three mirrors facing each other in a triangle; and the octonions modeled by four mirrors facing each other in a tetrahedron (a cube is a tetrahedron), or more complex packing structures related to tetrahedra. Using these facts, we wish to explore the structure of spacetime as that of a quaternionic. Buckminster Fuller (1982) for example viewed the tetrahedron as an essential structure for internal and external reality. In addition in nature the structure of HD space is an interacting tiling of topologically switching adjacent quaternionic or octonionic mirrorhouses that break Hamilton's closure cyclicality with an inherent natural periodicity that oscillates from Non-Abelian to Abelian continuously.

6.2. *Mirror Symmetry Experiment*

One of us (RLA) curious to experience firsthand the symmetries of a quaternion mirrorhouse got permission on 30 Jan 2012, 1:30 - 2 PM to use Sealing room 6, in the Oakland, CA LDS Temple for an experiment.

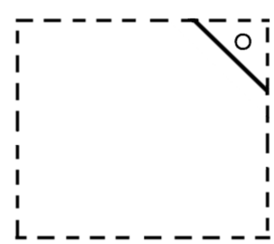

Figure 10. Quaternion mirror house experiment. The dashed line-sided square represents the four mirrored walls of a large room. The solid line in the upper right corner a 6 foot mirror supported on a small table, and the circle a chair inside the mirror house corner.

Getting inside the mirror house 4 images appeared. I initially thought it might be 3 images because I had made a triangle, but the 2 corners reflected off the mirror behind my head create 4 proximal images. Not counting the observer it's like viewing an infinite series of quaternion vertices doubling *ad infinitum* with each mirror having the 4-fold quaternion vertex.

If I closed my right eye, the face in the left vertex mirror closes its left eye (eye on the left). Then in the 2nd level image it is the right eye in right vertex image that closes. This is standard mirror symmetry imagery, but interesting to experience and verify 1st hand because it helps pondering how to reflect, rotate and invert more complex image components in order to create a new cyclical quaternion algebra that will bot commute and anticommute.

7. Calabi-Yau Manifolds - Brief Review

In superstring theory, the extra dimensions (XD) of spacetime are sometimes conjectured to take the form of a 6-dimensional Calabi-Yau manifold which led to the idea of mirror symmetry. The classical formulation of mirror symmetry relates 2 Calabi-Yau 3-folds M & W whose Hodge numbers three folds M $h^{1,1}$ and $h^{1,2}$ are swapped. Mirror symmetry is a special example of T-duality: the Calabi-Yau manifold may be written as a fiber bundle whose fiber is a three-dimensional torus. The simultaneous action of T-duality on all three dimensions of this torus is equivalent to mirror symmetry. Mirror symmetry allowed the physicists to calculate many quantities that seemed virtually incalculable before, by invoking the "mirror" description of a given physical situation, which can be often much easier.

There are two different, but closely related, string theory statements of mirror symmetry.

1. Type IIA string theory on a Calabi-Yau M is mirror dual to Type IIB on W.

2. Type IIB string theory on a Calabi-Yau M is mirror dual to Type IIA on W.
This follows from the fact that Calabi-Yau hodge numbers satisfy $h^{1,1} \geq 1$ but $h^{2,1} \geq 0$. If the Hodge numbers of M are such that $h^{2,1}$ = 0 then by definition its mirror dual W is not Calabi-Yau. As a result mirror symmetry allows for the definition of an extended space of compact spaces, which are defined by the W of the above two mirror symmetries.

Essentially, Calabi–Yau manifolds are shapes that satisfy the requirement of space for the six "unseen" spatial dimensions of string theory, which are currently considered to be smaller than our currently observable lengths as they have not yet been detected. An unpopular alternative that we embrace and have designed experiments to test for [14] is known as large extra dimensions (LSXD). This often occurs in braneworld models - that the Calabi–Yau brane topology is large but we are confined to a small subset on which it intersects a D-brane.

8. Search for a Commutative - Anticommutative Cyclical Algebra

We view the Hubble Universe, H_R as a nilpotent state of zero totality arising from a Higher Dimensional (HD) absolute space of infinite potentia. The world of apparent nonzero states that we observe, R must be associated with a zero creating conjugate, R^* such that $(R)(R) \Rightarrow (R)(R^*)$ in order to maintain the fundamental nilpotent condition of reality.

Since the quaternion set is isomorphic to the Pauli matrices we believe the Quaternion and Octonion sets are not merely a form of pure mathematics but indicative that the physical geometry of the universe is a fundamental generator of these algebras. Baez [1] stated that the properties of these algebras have: "so far…just begun to be exploited". Our purpose in this present work is to discover a new type of dualing in the cyclic dimensionality that alternates between the principle of closure and openness, i.e. anticommutivity and commutivity.

For our purposes we wish to apply this to an extended version of Cramer's Transactional Interpretation (TI) of quantum theory where according to Cramer the present instant of a quantum state is like a standing-wave of the future-past [5]. Cramer never elaborates on what he means by "standing-wave". In our extension the standing-wave transactional present is a metaphorically more like a hyperspherical standing wave with HD Calabi-Yau mirror house symmetry because the richer topology of brane transformations is equipped to handle the duality of closure and openness essential to our derivation of a new algebra we hope to use as the empirical foundation of Unified Field Mechanics, U_F [14].

We symbolize closure and openness as a form of wave-particle duality, which in our model we elevate to a principle of cosmology. So that rather than having the usual fixed closure built in to the quaternion algebra, we manipulate the standing-wave mirror house cycles so that closure is periodic instead. To do this we might keep a background doubling as a synchronization backbone rather than the 'reduction by a factor of 2' which as Rowlands states:

> [4] …describing a set of operators such as *i, j, k* as 'cyclic' means reducing the amount of independent

information they contain by a factor of 2, because *k* for example arises purely from the product of *jk*. It could even be argued that the necessity of maintaining the equivalence of Q_4 and C_2 x C_2 x C_2 representations is the determining factor in making the quaternion operators cyclic.

Rowlands further discusses (pg. 15) the dualing of Q_4:

[4] by complexifying it to the multivatiate vector group, 1, -1, *i, -i,* ***i*, -*i*, *j*, -*j*, *k*, -*k*, *ii*, -*ii*, *ij*, -*ij*, *ik*, -*ik*** of order 16, which has a related C_2 x C_2 x C_2 x C_2 formalism and which may also be written, 1, -1, *i*, -*i*, *i***i**, -*i***i**, *i***j**, -*i***j**, *i***k**, -*i***k**, **i, -i, j, -j, k, -k,** where a complex quaternion such as, ***ii*** becomes the equivalent of the multivariate vector **i**.

There are two options

1) $(AB)(AB) \rightarrow (R^*)$ (Anticommutative)
2) $(AB)(AB) \rightarrow (R)$ (Commutative)

Option 1 was chosen by Hamilton when he discovered that the 3-dimensionality of the quaternion algebra required closure. The closed cycle of the anti-commuting option (zero creating) with distinguish-able terms; but the commutative option has a series of terms that are completely indistinguishable (ontological) because the anticommuting partner must be unique. There are a number of forms of these open and closed dualities.

Planes through the origin of this 3D vector space give subalgebras of *O* isomorphic to the quaternions, lines through the origin give subalgebras isomorphic to the complex numbers, and the origin itself gives a subalgebra isomorphic to the real numbers.

What we really have here is reminiscent of a description of octonions as a`twisted group algebra'. Given any group *G*, the group algebra $\mathbb{R}[G]$ consists of all finite formal linear combinations of elements of *G* with real coefficients. This is an associative algebra with the product coming from that of *G*. We can use any function

$$\alpha : G^2 \rightarrow \{\pm 1\}$$

to `twist' this product, defining a new product

$$*\mathbb{R}[G] \times \mathbb{R}[G] \rightarrow \mathbb{R}[G]$$

by: $g * h = \alpha(g,h)gh$ Where $g, h \in G \subset \mathbb{R}[G]$. One can figure out an equation involving α that guarantees this new product will be associative. In this case we call α a `2-cocycle'. If $\alpha$ satisfies a certain extra equation, the product will also be commutative, and we call $\alpha$ a `stable 2-cocycle'. For example, the group algebra $\mathbb{R}[\mathbb{Z}_2]$ is isomorphic to a product of 2 copies of $\mathbb{R}$, but we can twist it by a stable 2-cocyle to obtain the complex numbers. The group algebra $\mathbb{R}[\mathbb{Z}_2^2]$ is isomorphic to a product of 4 copies of $\mathbb{R}$, but we can twist it by a 2-cocycle to obtain the quaternions. Similarly, the group algebra $\mathbb{R}[\mathbb{Z}_2^3]$ is a product of 8 copies of $\mathbb{R}$, and what we have really done in this section is describe a function α that allows us to twist this group algebra to obtain the octonions. Since the octonions are nonassociative, this function is not a 2-cocycle. However, its coboundary is a `stable 3-cocycle', which allows one to define a new associator and braiding for the category of $\mathbb{Z}_2^3$-graded vector spaces, making it into a symmetric monoidal category [15]. In this symmetric monoidal category, the octonions are a commutative monoid object. In less technical terms: this category provides a context in which the octonions *are* commutative and associative [1].

We do not believe an Octonion approach is correct for developing our new algebra and only provides a *gedanken* medium for exploring dualities and doublings. As in the Fano plane (Fig.11) an ordinary 3-cube has 8 possible quaternionic vertices. A hypercube as in Fig. 5 explodes into eight 3-cubes for a total of 64 quaternionic vertices (see Fig. 4). Without yet having finished formalizing the new algebra we believe 64 symmetry components are required for its invention. See [4] for a discussion pertaining to 64 quaternionic elements.

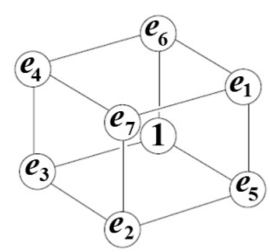

Figure 11. The Fano plane is the projective plane over the 2-element field $\mathbb{Z}_2$ consisting of lines through the origin in the vector space $\mathbb{Z}_2^3$. Since every such line contains a single nonzero element, we can also think of the Fano plane as consisting of the seven nonzero elements of $\mathbb{Z}_2^3$. If we think of the origin in $\mathbb{Z}_2^3$ as corresponding to $1 \in O$, we get this picture of the octonions.

The quaternions are a 4-dimensional algebra with basis 1, *i.j.k* . To describe the product we could give a multiplication table like Tbl 1, but it is easier to remember that:

- 1 is the multiplicative identity,
- *i,j* and *k* are square roots of -1,

- $ij = k$, $ji = -k$, and all identities obtained from these by cyclic permutations of (i,j,k).

We can summarize the quaternion rule in a diagram, Fig. 8. When we multiply two elements going clockwise around the circle we get the next element: for example, *i,j* = *k*. But when we multiply two going around counterclockwise, we get *minus* of the next element: for example, *j,i* = -*k* [1].

This cyclicality is key to our continuous-state commutative anticommutative leap-frogging of the metric. Einstein believed 'the photon provides its own medium and no ether is required'. This is true in general but the medium must be imbedded in a cosmology. Therefore is the Rowland's concept that the fermion vertex is fundamental similar, i.e. needing a cosmological embedding. Correspondence to HAM cosmology could provide the proper embedding. This is a primary postulate herein.

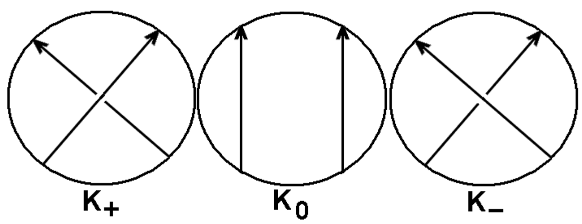

Figure 12. Crossing change triple. Imagine embedded in Fig. 6-8.

In Fig. x the the ordered set (K_+,K_-,K_0) is called a crossing-change-triple. Let (K_+,K_-,K_0) be a twist-move-triple See [16-18] for a variations and in depth discussion.

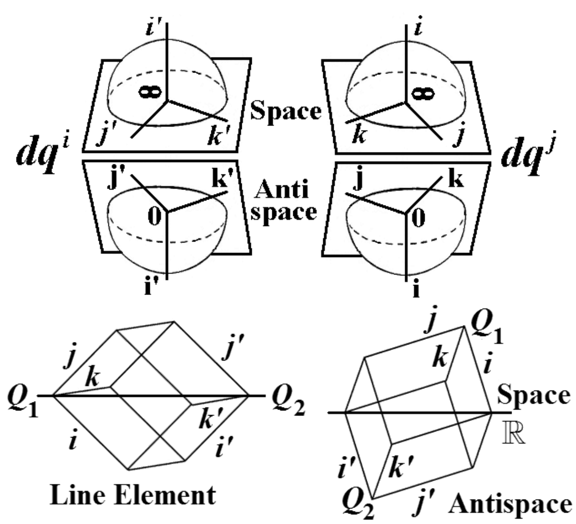

Figure 13. Riemann sphere quaternionic line elements.

Riemann sphere manifolds are key to algebraic topology. The Riemann sphere is usually represented as the unit sphere $x^2 + y^2 + z^2 = 1$ in real 3D space, R^3 (x,y,z) where two stereographic projections from the unit sphere are made onto the complex plane by $\zeta = x + iy$ and $\xi = x - iy$, written as

$$\varsigma = \frac{x+iy}{1-z} = \cot\left(\tfrac{1}{2}\phi\right)e^{i\theta}$$

and

$$\xi = \frac{x-iy}{1+z} = \tan\left(\tfrac{1}{2}\phi\right)e^{-i\theta}.$$

In order to cover the whole unit sphere two complex planes are required because each stereographic projection alone is missing one point, either the point at zero or infinity. The extended complex numbers consist of the complex numbers C together with ∞. The extended complex numbers are written as $\hat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$. Geometrically, the set of extended complex numbers is referred to as the Riemann sphere (or extended complex plane) seen as glued back-to-back at z = 0. Note that the two complex planes are identified differently with the plane $z = 0$. An orientation-reversal is necessary to maintain consistent orientation on the sphere, and in particular complex conjugation causes the transition maps to be holomorphic.

The transition maps between ζ-coordinates and ξ-coordinates are obtained by composing one projection with the inverse of the other. They turn out to be $\zeta = 1/\xi$ and $\xi = 1/\zeta$, as described above. Thus the unit sphere is diffeomorphic to the Riemann sphere. Under this diffeomorphism, the unit circle in the ζ-chart, the unit circle in the ξ-chart, and the equator of the unit sphere are all identified. The unit disk $|\zeta| < 1$ is identified with the southern hemisphere $z < 0$, while the unit disk $|\xi| < 1$ is identified with the northern hemisphere $z > 0$. A Riemann surface does not have a unique Riemannian metric which means we may develop a relevant metric for the noetic transformation.

This implies a dual or mirror metricity where one is fixed in reality or 3-space and the 'mirror' confined to complex space where it then cyclically 'topologically switches' into real space and the real space metric takes its place in complex space. A component of the metric fixing-unfixing when applied to the 'Walking of the Moai on Rapa Nui' relates to recent work of Kauffman:

> When the oscillation (**HAM continuous-state**) is seen without beginning or end, it can be regarded in two distinct ways: as an oscillation from marked to unmarked, or as an oscillation from unmarked to marked. In this way the re-entering marked becomes the recursive process generated by the equation and J is an algebraic or imaginary fixed point for the recursion.

$$R(x) = \overline{\;x\;}|$$

The analog with the complex numbers is sufficiently strong that we now shift to the complex numbers themselves as a specific instantiation of this recursive process. In the body of the paper we will return to the more rarified discussion leading from the distinction itself. We begin with recursive processes that work with the real numbers **+1** and **-1**. Think of the oscillatory process generated by **R(x) = -1/x** [17].

The fixed point is **i** with $\mathbf{i^2 = -1}$, and the processes generated over the real numbers must be directly related to the idealized **i**. There are two i*terant views*: **[+1,-1]** and **[-1,+1]**.

These are seen as points of view of an alternating process that engender the square roots of negative unity.

... +1 -1 +1 -1 +1 -1 +1 -1 +1 -1 +1 -1 +1 -1 +1 -1 ...

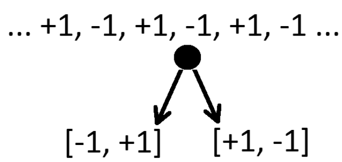

Figure 14. Quantizing the Iterant views.

To see this, we introduce a *temporal shift operator,* **h** such that **[a,b]h** = **h [b,a]** and **h h = 1** so that concatenated observations can include a time step of one-half period of the process **...abababab...** . “”[17].

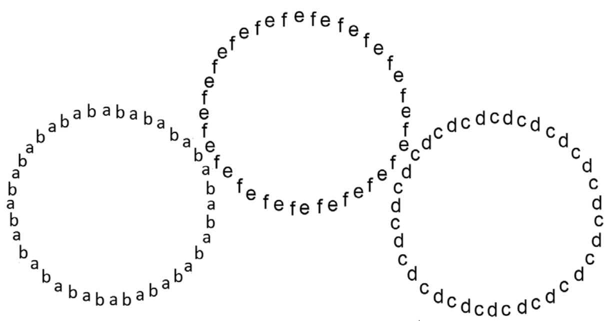

Figure 15. Triune temporal shift operator for 1st triad dualing.

Figure 15 reminds us of the ‘walking of the Moai on Rapa Nui’ [11,12] because if for example the left foot is the ababab iterator, when it is lifted the cdcdcd iterator represents the ‘fixed metric’ and the efefef acts as the shift operator enabling a ‘complex crossing change’ from commutivity to anticommutivity or vice versa. However as we already stated several more triune sets of additional mirror house doublings or dualings are required.

The 3rd complex metric is involved in making an evolution from dual quaternions to a 3rd quaternion we choose to name a trivector that acts as a baton passing mechanism between the space-antispace or dual quaternion vector space. The trivector facilitates a ‘leap-frogging’ between anti-commutative and commutative modes of HD space. This inaugurates a Mobius or transformation between the Riemann dual stereographic projection complex planes.

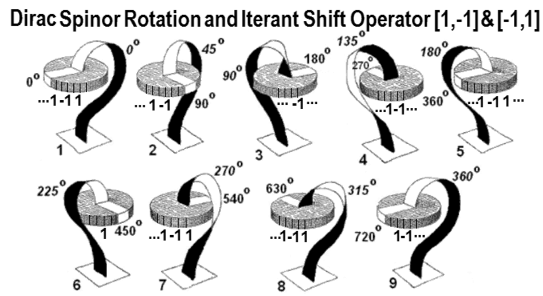

Figure 16. Temporal *i* clock shift operator with addition of Dirac spherical rotation in preparation for Mobius transformation crossing changes. See Fig. 12.

In the midst of the HAM continuous-state dimensional reduction compactification process (a U_F process) there is a central (focused around the singularity or LCU) coordinate ‘fixing-unfixing’. In prior work we [5] have related this to the integration of G-em coordinates, arbitrarily fixing one metric. That work proceeded discovery of the continuous-state process wherein now an inherent cyclicality occurs in first fixing one metric and then fixing the other. This is an essential aspect of the gating mechanism for the U_F dynamics to surmount quantum uncertainty.

Geometrically, a standard Möbius transformation can be obtained by first performing stereographic projection from the plane to the unit 2-sphere, rotating and moving the sphere to a new location and orientation in space, and then performing stereographic projection (from the new position of the sphere) to the plane. These transformations preserve angles, map every straight line to a line or circle, and map every circle to a line or circle. Möbius transformations are defined on the extended complex plane (i.e. the complex plane augmented by the point at infinity): $\hat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$.

This extended complex plane can be thought of as a sphere, the Riemann sphere, or as the complex projective line. Every Möbius transformation is a bijective conformal map of the Riemann sphere to itself. Every such map is by necessity a Möbius transformation. Geometrically this map is the Riemann stereographic projection of a rotation by 90° around $\pm i$ with period 4, which takes the continuous cycle $0 \to 1 \to \infty \to -1 \to 0$.

An LSXD point particle representation of a fermionic singularity [10]. The 8 + 8 or 16 ($\mathbb{C}^{\pm 4}$ complex space) 2-spheres with future-past retarded-advanced contours are representations of HD components of a Cramer 'standing-wave' transaction. This can be considered in terms of Figs. X,Y,Z as Calabi-Yau dual mirror symmetries. To produce the quaternion trivector representation (Fig. x) a 3rd singularity-contour map is required which is then also dualed, i.e. resulting in 6 singularity/contour maps. This many are required to oscillate from anticommutivity to commutivity to provide the cyclic opportunity to violate 4D quantum uncertainty [6]!

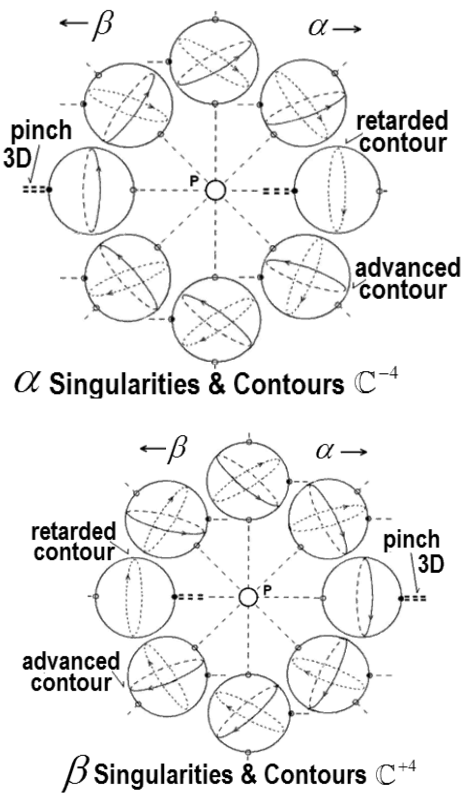

Figure 17. Two of six HD Nilpotent trivector r-qubit symmetries.

Figure 17 begins to illustrate the mirror symmetries involved in the more complex dualities. If the central points P represent mirrored 3-cubes each of which already has 8 quaternionic vertices, then each sphere in Fig. 17 represents one of eight 3-cubes exploded off a hypercube which is each mirrored in the α and β singularities and contours. We believe one more mirror house doubling is required beyond that scenario to complete a commutative-anticommutative iterative shift operator!

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