Delphi

M. W. Roberts

Abstract

An optical communication system is described. The system provides a unique operational capability.

1. Introduction

A representation of the system is shown in Figure 1. The system is composed of a Source (Src), a Transmitter (Tx), and a Receiver (Rx).

The optical path length from the Source to the Transmitter is adjusted to be somewhat less than the optical path length from the Source to the Receiver. The Source, Transmitter, and Receiver are all assumed to be stationary.

To simplify the description of this system, the effects of optical filters, lenses, detector quantum efficiency and dark counts, and most other potential losses are not included in the following discussion.

2. Notation

In the following discussion and in the Appendix, both probability amplitude and probability will be calculated. As an example:

$$P[D4,D2;\Delta] = |pa[D4,D2;\Delta]|^2$$

In the above, $pa[D4,D2;\Delta]$ is the probability amplitude for the detection of a photon in detector D4 in the Transmitter followed by the detection of an associated photon in detector D2 in the Receiver with time difference between detections equal to Δ. $P[D4,D2;\Delta]$ is the probability for the same detection events.

Both intensity and amplitude variables are used in the following. As an example, for amplitude beam splitter ABS1:

$$R_1 = |r_1|^2 \text{ , } T_1 = |t_1|^2 \text{ and } R_1 + T_1 = 1$$

In the above, R_1 is the intensity reflectance, T_1 is the intensity transmittance, r_1 is the amplitude reflection coefficient, and t_1 is the amplitude transmission coefficient of ABS1.

3a. Source

The Source (Src) contains a single-mode, continuous wave (cw) pump laser (LSR1), a periodically-poled lithium niobate crystal (PPLN1), a dichroic mirror (DM1), a polarizing beam splitter (PBS1), and a beam stop (Stp).

Laser LSR1 has a stable output, and the coherence length of the pump photons from LSR1 is greater than 50 meters.

The PPLN1 crystal is temperature-controlled, and is set to allow collinear, non-degenerate, type II spontaneous parametric down-conversion (SPDC) in which a photon from pump laser LSR1 is annihilated and a signal and idler pair of photons is created [1]. The signal photon is horizontally (H) polarized, and the idler photon is vertically (V) polarized.

Short wavelength pump photons from laser LSR1 that are not down-converted in PPLN1 are reflected at long-pass dichroic mirror DM1 and are incident on beam stop Stp.

On average, one of every 10^6 of the pump photons is annihilated in a SPDC event within PPLN1 that creates a signal/idler pair of photons. The long wavelength signal and idler photons exit from PPLN1 and are transmitted through DM1 to PBS1. The H polarized signal photons are transmitted through PBS1 and travel to the Transmitter (Tx). The V polarized idler photons are reflected by PBS1 and travel to the Receiver (Rx).

3b. Receiver

The Receiver (Rx) contains two amplitude beam splitters (ABS1 and ABS2), three optical mirrors, and two detectors (D1 and D2). The detectors are capable of photon counting.

The amplitude beam splitters and three mirrors are arranged to form an unbalanced Mach-Zehnder interferometer (MZ). The unbalanced MZ provides a short path and a long path between ABS1 and ABS2 for the idler photons.

The short path through the MZ is transmission through ABS1 and then either transmission through ABS2 to detector D1 or reflection from ABS2 to detector D2. The long path through the MZ is reflection by ABS1, reflection by the three mirrors to ABS2, and then either transmission through ABS2 to detector D2 or reflection from ABS2 to detector D1.

One of the mirrors in the MZ is moveable. The moveable mirror allows the optical path length difference between the long and short paths through the MZ to be controlled. This mirror also allows for the adjustment of the phase of an idler photon at ABS2 that travels via the long path through the MZ.

The moveable mirror in the MZ is adjusted so that the time difference between the time an idler photon may be incident on detector D1 (or D2) via the short path, and the time the idler photon may be incident on detector D1 (or D2) via the long path through the MZ in the Receiver is equal to X.

The fixed time X should be of sufficient duration to allow the short path and the long path to be temporally distinct. Time X should be much longer than the coherence time of an idler photon but should also be much shorter than the coherence time of a pump photon from laser LSR1 in the Source.

The difference between the optical path length through the long path and the optical path length through the short path is equal to L.

Amplitude beam splitters ABS1 and ABS2 may be partially-silvered plate beam splitters. The characteristics of ABS1 and ABS2 are:

$$R_1 = |r_1|^2 = 0.50 \; ; \; T_1 = |t_1|^2 = 0.50$$
$$R_2 = |r_2|^2 = 0.50 \; ; \; T_2 = |t_2|^2 = 0.50$$

3c. Transmitter

The Transmitter (Tx) contains a Pockels cell (PC), a single-mode, continuous wave (cw) laser (LSR2), four dichroic mirrors (DM2-DM5), a periodically-poled lithium niobate crystal (PPLN2), a lithium niobate orthogonal-polarization compensating plate (LiN), a Soleil-Babinet compensator (SBC), a beam stop (Stp), two half-wave plates (HWP1 and HWP2), two polarizing beam splitters (PBS2 and PBS3), three mirrors, and three detectors (D3-D5). The detectors are capable of photon counting.

Laser LSR2 has a stable output, and the coherence length of the pump photons from LSR2 is greater than 50 meters.

Pockels cell PC may be used to rotate the polarization direction of a signal photon from the Source. If the PC is turned off, an H polarized signal photon will remain H polarized when it exits from the PC. If the PC is turned on, the signal photon will be V polarized when it exits from the PC.

Polarizing beam splitter PBS2 is set to transmit incident H polarized signal photons to detector D3, and to reflect incident V polarized signal photons to dichroic mirror DM2.

Short pass dichroic mirror DM2 transmits incident pump photons from LSR2 and reflects incident signal photons from PPLN1 in the

Source. The pump photons and signal photons are aligned so as to overlap at the output from DM2.

Long pass dichroic mirror DM3 transmits the combined beam of pump photons from LSR2 and signal photons from the Source.

The PPLN2 crystal is temperature-controlled, and is set to allow collinear, type I sum frequency generation (SFG) in which a photon from pump laser LSR2 and a signal photon from the Source are simultaneously annihilated, and a higher-frequency "SF" photon is created [1]. The SF photon is H polarized.

Most of the pump photons from laser LSR2 and signal photons from the Source pass through PPLN2 and are reflected by short pass dichroic mirror DM4. The pump photons then pass through short pass dichroic mirror DM5 and are incident on beam stop Stp. The longer-wavelength signal photons are reflected by DM5 and are incident on detector D5. The purpose of detector D5 is to monitor the number of signal photons that reach the Transmitter in order to verify proper system operation.

On average, one of every 10^2 of the signal photons from the Source is annihilated along with a pump photon from laser LSR2 in an SFG event within PPLN2 that creates an SF photon [1].

Short pass dichroic mirror DM4 transmits the short-wavelength SF photons which then travel to HWP1. Half-wave plate HWP1 is set with its "fast" axis at 22.5 degrees above horizontal. An H polarized SF photon that passes through HWP1 has its polarization direction rotated about the fast axis, and the SF photon exits from HWP1 +45 degree polarized.

Polarizing beam splitter PBS3 is set to transmit incident H polarized photons to detector D4, and to reflect incident V polarized photons to the moveable mirror (mm).

Components in the Transmitter are arranged as a "circulator" to provide two paths for an SF photon to reach detector D4. The circulator is adjusted so that the two paths precisely overlap at the output from PBS3 to detector D4 [2].

The first path in the circulator is from creation in PPLN2, through DM4, HWP1, and then transmission through PBS3 to detector D4. The second path is from creation in PPLN2, through DM4, HWP1, reflection from PBS3 and two mirrors, through HWP2, LiN, and the SBC, reflection from a mirror and DM3, then through PPLN2, DM4, and HWP1, and transmission through PBS3 to detector D4.

One of the mirrors in the circulator is moveable. The moveable mirror allows the optical path length difference between the first and second paths through the circulator to be controlled. This mirror also allows for the adjustment of the phase of an SF photon at PBS3 that travels via the second path through the circulator.

The moveable mirror in the Transmitter is adjusted so that the time difference between the time an SF photon may be incident on detector D4 via the first path, and the time the SF photon may be incident on detector D4 via the second path through the Transmitter is equal to X.

The fixed time X should be of sufficient duration to allow the first path and the second path to be temporally distinct. Time X should be much longer than the coherence time of an SF photon from PPLN2 but should also be much shorter than the coherence time of a pump photon from laser LSR2 in the Transmitter.

(Note that time X is also the time difference for an idler photon between the long path and the short path through Mach-Zehnder interferometer MZ in the Receiver.)

The difference between the optical path length through the first path and the optical path length through the second path in the circulator is equal to L.

Half-wave plate HWP2 is set with its "fast" axis at 67.5 degrees above horizontal. A V polarized SF photon that passes through HWP2 has its polarization direction rotated about the fast axis, and the SF photon exits from HWP2 +45 degree polarized.

The SF photon then travels to lithium niobate plate LiN. The LiN is a polarization direction compensator for PPLN2. The LiN and SBC are adjusted so that an SF photon that enters the LiN +45 degree polarized will (subsequently) exit from PPLN2 also +45 degree polarized. The LiN provides coarse polarization adjustment, and the SBC provides fine polarization adjustment.

After passing through the SBC, the SF photon then reflects from a mirror and travels to DM3. Long-pass dichroic mirror DM3 reflects the short-wavelength SF photon which then travels to PPLN2.

The SF photon passes through PPLN2 and exits +45 degree polarized. [Some of the SF photons will be lost in PPLN2 due to difference frequency generation (DFG). However, the conversion efficiency of DFG should be roughly equal to that of SFG, therefore the loss should be approximately one of every 10^2 SF photons].

After exiting from PPLN2, the SF photon again passes through DM4 and travels to HWP1. A +45 degree polarized SF photon that passes through HWP1 has its polarization direction rotated about the fast axis, and the SF photon exits from HWP1 H polarized. The now H polarized SF photon then passes through PBS3 and is incident on detector D4.

In order for the system to function properly, the photon detected in detector D4 in the Transmitter must be created inside the circulator. Thus the need for the SF photon.

4. Frequencies and Wavelengths

The frequency and wavelength requirements for the SPDC process in the Source are:

$\omega_{P1} = \omega_S + \omega_I$
$(1/\lambda_{P1}) = (1/\lambda_S) + (1/\lambda_I)$, (free space)

In the above, ω_{P1} is the angular frequency and λ_{P1} is the wavelength of the pump photons from laser LSR1, ω_S is the angular frequency and λ_S is the wavelength of the signal photons, and ω_I is the angular frequency and λ_I is the wavelength of the idler photons.

The frequency and wavelength requirements for the SFG process in the Transmitter are:

$\omega_{SF} = \omega_S + \omega_{P2}$
$(1/\lambda_{SF}) = (1/\lambda_S) + (1/\lambda_{P2})$, (free space)

In the above, ω_{P2} is the angular frequency and λ_{P2} is the wavelength of the pump photons from laser LSR2, ω_S is the angular frequency and λ_S is the wavelength of the signal photons, and ω_{SF} is the angular frequency and λ_{SF} is the wavelength of the SF photons.

5a. Binary Zero

To send a binary zero from the Transmitter to the Receiver, Pockels cell PC in the Transmitter is turned off.

The V polarized idler photon of a down-converted pair travels from the Source to the Receiver. The idler photon then travels via either the short path or the long path through the MZ and is incident on either detector D1 or detector D2.

The H polarized signal photon of the pair travels from the Source to the Transmitter and passes through the PC unchanged. The H polarized signal photon then passes through PBS2 and is incident on detector D3.

The signal photon follows a direct path from the Source to the Transmitter. Therefore, the time of detection of a signal photon in detector D3 fixes the time of creation of that signal and idler photon pair in PPLN1 in the Source. The time of detection of the idler photon then identifies which path was followed by that idler photon as it passed through the MZ in the Receiver.

There is no ambiguity as to which path was followed by either the signal photon in the Transmitter or the idler photon in the

Receiver. Consequently, no two-photon interference occurs in the binary zero case [2].

Since there is no two-photon interference, the probabilities that an idler photon will be incident on detector D1 or detector D2 in the Receiver in the binary zero case are:

$$P^{(0)}[D1] = (T_1 \cdot T_2)+(R_1 \cdot R_2)=[(0.50)\cdot(0.50)]+[(0.50)\cdot(0.50)] = 0.50$$
$$P^{(0)}[D2] = (T_1 \cdot R_2)+(R_1 \cdot T_2)=[(0.50)\cdot(0.50)]+[(0.50)\cdot(0.50)] = 0.50$$

These probabilities apply for all signal and idler photon pairs. No SF photons are created in the Transmitter in the binary zero case.

5b. Binary One

To send a binary one from the Transmitter to the Receiver, Pockels cell PC in the Transmitter is turned on.

The V polarized idler photon of a down-converted pair travels from the Source to the Receiver. The photon then travels via either the short path or the long path through the MZ and is incident on either detector D1 or detector D2.

The time difference between the time an idler photon may be incident on detector D1 (or D2) via the short path, and the time the idler photon may be incident on detector D1 (or D2) via the long path through the MZ in the Receiver is set equal to X.

The probability amplitude that an idler photon will reach detector D1 or detector D2 via the short path is:

$$pa_{SHT}(D1) = t_1\, t_2 \exp(i\Phi_I) = \exp(i\Phi_I)/2$$
$$pa_{SHT}(D2) = i\ t_1\, r_2 \exp(i\Phi_I)= [i \exp(i\Phi_I)]/2$$

The probability amplitude that an idler photon will reach detector D1 or detector D2 via the long path is:

$$pa_{LNG}(D1)=(i^4)\ r_1\ r_2 \exp((i\Phi_I)+(i\omega_I L/c))= \exp((i\Phi_I)+(i\omega_I L/c))/2$$
$$pa_{LNG}(D2)=(i^3)\ r_1\ t_2 \exp((i\Phi_I)+(i\omega_I L/c))=[-i \exp((i\Phi_I)+(i\omega_I L/c))]/2$$

The phase angle Φ_I is random, because the location within PPLN1 at which an idler photon is created varies from one photon to the next.

The H polarized signal photon of the pair travels from the Source to the Transmitter and has its polarization direction rotated to V polarized as it passes through the PC. The now V polarized signal photon reflects from PBS2 and travels to PPLN2.

Most signal photons follow a direct path through PPLN2 and are incident on detector D5. On average, one in 10^2 of the signal photons is annihilated along with a pump photon from laser LSR2 in an SFG event within PPLN2 that creates a higher-frequency "SF" photon [1].

The short wavelength, H polarized SF photon exits from PPLN2 and passes through short pass dichroic mirror DM4. The SF photon has its polarization direction rotated to +45 degrees as it passes through HWP1. With 50% probability, the SF photon passes through PBS3 becoming H polarized. The photon is then incident on detector D4. This is the first path to detector D4.

The probability amplitude that an SF photon will reach detector D4 via the first path is:

$$pa_{I}(D4) = [\exp(i\Phi_{SF})]/\sqrt{(2)}$$

The phase angle Φ_{SF} is random, because the location within PPLN2 at which an SF photon is created varies from one photon to the next.

With 50% probability, the SF photon reflects from PBS3, becoming V polarized, and travels via two mirrors to HWP2. The V polarized SF photon has its polarization direction rotated to +45 degrees as it passes through HWP2.

The SF photon then passes through orthogonal-polarization compensating plate LiN and Soleil-Babinet compensator SBC and travels around through the circulator, returning to PPLN2. The purpose of LiN and the SBC is to ensure that when the SF photon exits from PPLN2, the photon is +45 degree polarized.

After exiting PPLN2, the +45 degree polarized SF photon again passes through DM4 and has its polarization direction rotated to H polarized as it passes through HWP1. The now H polarized SF photon passes through PBS3 and is incident on detector D4. This is the second path to detector D4.

The probability amplitude that an SF photon will reach detector D4 via the second path is:

$$pa_{II}(D4) = [\exp((i\Phi_{SF})+(i\omega_{SF}L/c))]/\sqrt{(2)}$$

The time difference between the time an SF photon may be incident on detector D4 via the first path, and the time the SF photon may be incident on detector D4 via the longer second path through the circulator in the Transmitter is set equal to X.

If an SF photon is detected in detector D4 after travelling through the first path in the Transmitter, and its associated idler photon is detected in detector D1 or D2 after travelling through the short path of the MZ in the Receiver, then the time difference between these detections is equal to τ.

If an SF photon is detected in detector D4 after travelling through the second path in the Transmitter, and its associated idler photon is detected in detector D1 or D2 after travelling through the long path of the MZ in the Receiver, then the time difference between these detections is also equal to τ.

When the time between the detection of the SF photon in the Transmitter and its associated idler photon in the Receiver is equal to τ, there is an ambiguity as to “which path” the two photons travelled. This causes nonlocal, two-photon interference between the SF photon in the Transmitter and the idler photon in the Receiver [2].

For detection of the SF photon in detector D4 and the idler photon in detector D1 with time difference equal to τ [3]:

$$pa^{(1)}[D4,D1;\Delta=\tau] = \{[\exp(i\Phi_{SF})]/\surd(2)\}\{t_1\ t_2\ \exp(i\Phi_I)\} +$$

$$\{[\exp((i\Phi_{SF})+(i\omega_{SF}L/c))]/\surd(2)\}\{i^4\ r_1\ r_2\ \exp((i\Phi_I)+(i\omega_I L/c))\}$$

$$= [\exp(i(\Phi_{SF}+\Phi_I))]\{[1/(2\surd(2))][1 + \exp((iL/c)(\omega_{SF} + \omega_I))]\}$$

Using $(\omega_{SF} + \omega_I) = (\omega_S + \omega_{P2}) + (\omega_{P1} - \omega_S) = (\omega_{P2} + \omega_{P1})$:

$$(L/c)(\omega_{SF} + \omega_I) = (L/c)(\omega_{P2} + \omega_{P1}) = 2\pi L[(\lambda_{P2} + \lambda_{P1})/(\lambda_{P2} \cdot \lambda_{P1})]$$

With $L = m\ [(\lambda_{P2} \cdot \lambda_{P1})/(\lambda_{P2} + \lambda_{P1})]$, (m is a positive integer):

$$\exp((iL/c)(\omega_{SF} + \omega_I)) = \exp(i2\pi m) = 1$$

$$pa^{(1)}[D4,D1;\Delta=\tau] = [\exp(i(\Phi_{SF}+\Phi_I))]/(\surd(2))$$

$$P^{(1)}[D4,D1;\Delta=\tau] = \left|pa^{(1)}[D4,D1;\Delta=\tau]\right|^2 = (1/2)$$

For detection of the SF photon in detector D4 and the idler photon in detector D2 with time difference equal to τ [3]:

$$pa^{(1)}[D4,D2;\Delta=\tau] = \{[\exp(i\Phi_{SF})]/\surd(2)\}\{i\ t_1\ r_2\ \exp(i\Phi_I)\} +$$

$$\{[\exp((i\Phi_{SF})+(i\omega_{SF}L/c))]/\surd(2)\}\{i^3\ r_1\ t_2\ \exp((i\Phi_I)+(i\omega_I L/c))\}$$

$$= [\exp(i(\Phi_{SF}+\Phi_I))]\{[i/(2\surd(2))][1 - \exp((iL/c)(\omega_{SF} + \omega_I))]\} = 0$$

$$P^{(1)}[D4,D2;\Delta=\tau] = \left|pa^{(1)}[D4,D2;\Delta=\tau]\right|^2 = 0$$

When the time difference between the detection of the SF photon in the Transmitter and its associated idler photon in the Receiver is not equal to τ, there is no ambiguity as to which path the two photons travelled, and, therefore, no two-photon interference occurs:

$$P^{(1)}[D4,D1;\Delta=(\tau-X)] = \left|[p_{II}(D4)][p_{SHT}(D1)]\right|^2 = 1/8$$

$$P^{(1)}[D4,D1;\Delta=(\tau+X)] = \left|[p_{I}(D4)][p_{LNG}(D1)]\right|^2 = 1/8$$

$$P^{(1)}[D4,D2;\Delta=(\tau-X)] = \left|[p_{II}(D4)][p_{SHT}(D2)]\right|^2 = 1/8$$

$$P^{(1)}[D4,D2;\Delta=(\tau+X)] = \left|[p_{I}(D4)][p_{LNG}(D2)]\right|^2 = 1/8$$

Summing the above, the probabilities that an idler photon will be incident on detector D1 or detector D2 in the Receiver in the binary one case are:

$P^{(1)}_{SF}[D1] = (1/2) + (1/8) + (1/8) = 0.75$
$P^{(1)}_{SF}[D2] = 0 + (1/8) + (1/8) = 0.25$

Note that these probabilities are different from those in the binary zero case. However, also note that these probabilities only apply in those instances when an SF photon is created in the Transmitter. When an SF photon is not created in the Transmitter, then the probabilities for the detections of idler photons in the Receiver are the same as in the binary zero case:

$P^{(1)}_{SIG}[D1] = P^{(0)}[D1] = 0.50$
$P^{(1)}_{SIG}[D2] = P^{(0)}[D2] = 0.50$

5c. Integration Time

On average, only one in 10^2 of the signal photons produces an SF photon in the Transmitter. Therefore, in the binary one case, of every 100 idler photons detected in the Receiver:

Number of detections in D1 = $(99 \cdot 0.5) + (1 \cdot 0.75) = 50.25$
Number of detections in D2 = $(99 \cdot 0.5) + (1 \cdot 0.25) = 49.75$
$N(D2) - N(D1) = 0.50$ (for $N_{Total} = 10^2$)

For 5 sigma confidence:
For binary one: N(D1) = 251,250 , N(D2) = 248,750
N(D1) – N(D2) = 2,500 (for $N_{Total} = 5.0 \times 10^5$)
1 sigma = $\sqrt{(2.5 \times 10^5)} = 500$; 5 sigma = 2,500
[For binary zero: $\langle N(D1)\rangle = 2.5 \times 10^5$, $\langle N(D2)\rangle = 2.5 \times 10^5$]

To achieve 5 sigma confidence, the integration time per bit for both binary one and binary zero must be of sufficient duration to allow for the detection of five hundred thousand photon pairs - the idler photon of the pair in the Receiver and its associated signal or SF photon in the Transmitter.

6. Conclusion

The optical path length from the Source to the Receiver is slightly greater than the optical path length from the Source to the Transmitter. The difference in optical path lengths is adjusted to be just sufficient to ensure that an idler photon will always be detected in the Receiver after its associated signal or SF photon is detected in the Transmitter (in the proper frame of the Source, Receiver, and Transmitter).

This communication system is unique because, once signal photons reach the Transmitter and idler photons reach the Receiver, the transfer of information from the Transmitter to the Receiver is virtually instantaneous, independent of the distance between the Transmitter and the Receiver. This is true, even if the distance is so large that the detection of an idler photon in the Receiver and the detection of its associated signal or SF photon in the Transmitter are space-like separated events.

The alternate setup, in which the optical path length from the Source to the Receiver is less than the optical path length from the Source to the Transmitter, will not be discussed – other than to refer to the title of this paper.

Appendix

The loss due to difference frequency generation (DFG) during the passage of the SF photon through PPLN2 on the second path through the circulator in the Transmitter may be reduced by increasing the polarization direction of the SF photon to an angle greater than 45 degrees.

The range of polarization direction angle Θ is:

$$45 \text{ degrees} < \Theta < 90 \text{ degrees}$$

The “fast” axis of half-wave plate HWP1 must be set at an angle $(\Theta/2)$ above horizontal, and the fast axis of half-wave plate HWP2 must be set at an angle $\{[(\pi/2)+\Theta]/2)\}$ above horizontal.

The LiN and the SBC must be adjusted to ensure that when the SF photon exits from PPLN2, the photon has a polarization direction of Θ degrees above horizontal.

As the angle Θ is increased, fewer SF photons will be lost to DFG, because fewer SF photons will meet the phase match requirement for DFG in PPLN2.

In this case, the characteristics of ABS1 and ABS2 should be:

$R_1 = |r_1|^2 = \cos^2\Theta$; $T_1 = |t_1|^2 = \sin^2\Theta$
$R_2 = |r_2|^2 = 0.50$; $T_2 = |t_2|^2 = 0.50$

The probabilities at the Receiver do not change in the binary zero case:

$P^{(0)}[D1]=(T_1 \cdot T_2)+(R_1 \cdot R_2)=[(\sin^2\Theta)\cdot(0.50)]+[(\cos^2\Theta)\cdot(0.50)] = 0.50$
$P^{(0)}[D2]=(T_1 \cdot R_2)+(R_1 \cdot T_2)=[(\sin^2\Theta)\cdot(0.50)]+[(\cos^2\Theta)\cdot(0.50)] = 0.50$

The probabilities change in the binary one case in those instances when an SF photon is created in the Transmitter:

$pa^{(1)}[D4,D1;\Delta=\tau] = \{[\cos\Theta][\exp(i\Phi_{SF})]\}\{[\sin\Theta/\sqrt{(2)}][\exp(i\Phi_I)]\} +$
$\{[\sin\Theta][\exp((i\Phi_{SF})+(i\omega_{SF}L/c))]\}\{i^4[\cos\Theta/\sqrt{(2)}][\exp((i\Phi_I)+(i\omega_I L/c))]\}$
$= [\sqrt{(2)}][\sin\Theta][\cos\Theta]$
$P^{(1)}[D4,D1;\Delta=\tau] = |pa^{(1)}[D4,D1;\Delta=\tau]|^2 = 2[\sin^2\Theta][\cos^2\Theta]$

$pa^{(1)}[D4,D2;\Delta=\tau] = \{[\cos\Theta][\exp(i\Phi_{SF})]\}\{i[\sin\Theta/\sqrt{(2)}][\exp(i\Phi_I)]\} +$
$\{[\sin\Theta][\exp((i\Phi_{SF})+(i\omega_{SF}L/c))]\}\{i^3[\cos\Theta/\sqrt{(2)}][\exp((i\Phi_I)+(i\omega_I L/c))]\}$
$= 0$
$P^{(1)}[D4,D2;\Delta=\tau] = |pa^{(1)}[D4,D2;\Delta=\tau]|^2 = 0$

$P^{(1)}[D4,D1;\Delta=(\tau-X)] = |[p'_{II}(D4)][p'_{SHT}(D1)]|^2 = (\sin^4\Theta)/2$
$P^{(1)}[D4,D1;\Delta=(\tau+X)] = |[p'_{I}(D4)][p'_{LNG}(D1)]|^2 = (\cos^4\Theta)/2$
$P^{(1)}[D4,D2;\Delta=(\tau-X)] = |[p'_{II}(D4)][p'_{SHT}(D2)]|^2 = (\sin^4\Theta)/2$

$P^{(1)}[D4,D2;\Delta=(\tau+X)] = \left|[p'_{I}(D4)][p'_{LNG}(D2)]\right|^{2} = (\cos^{4}\Theta)/2$

Summing the above, in those instances when an SF photon is created in the Transmitter, the probabilities that an idler photon will be incident on detector D1 or detector D2 in the Receiver in the binary one case are:

$P^{(1)}_{SF}[D1] = 2[\sin^{2}\Theta][\cos^{2}\Theta] + [(\sin^{4}\Theta)+(\cos^{4}\Theta)]/2$
$P^{(1)}_{SF}[D2] = [(\sin^{4}\Theta)+(\cos^{4}\Theta)]/2$

References

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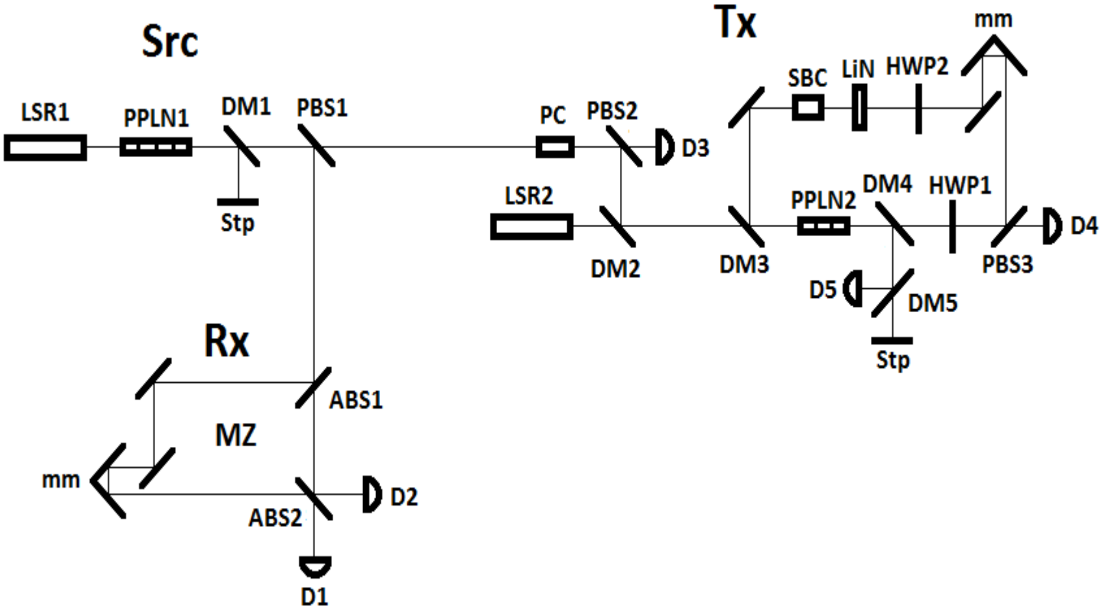

Figure 1: System Design