# Test of General Relativity Theory by Investigating the Conservation of Energy in a Relativistic Free Fall in the Uniform Gravitational Field 

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#### Abstract

This paper investigates the General Relativity Theory (GRT) by studying the accelerated motion simulating the relativistic free fall of a small test body in a uniform gravitational field. The paper compares the predictions of energy loss, perhaps by radiation, in an accelerated motion obtained from the GRT and from the Metric Theory of Gravity (MTG). It is found that the gravitational mass dependence on velocity in GRT is not correct, because it predicts a negative loss of energy while the MTG predicts correctly a positive loss. The energy loss for a true free fall in a uniform gravitational field of a curved space-time is also investigated. The energy conservation law is satisfied only for the MTG.


Introduction: The theories describing the accelerated motion and a free fall are well understood in both; the GRT and the MTG. In the GRT the inertial mass and the gravitational mass are assumed identical with identical dependencies on velocity. In the MTG, on the other hand, the gravitational mass depends on velocity differently than the inertial mass ${ }^{[1,2]}$. It is thus simple for both theories to derive equations describing the free fall velocity and from that the energy loss of a small test body that is accelerated in a simulated uniform gravitational field and in a true curved space-time gravitational field.

Theories: In the GRT the relation between the velocity $v$ and time is somewhat more complicated but can be easily derived as follows:

$$
\begin{equation*}
\frac{d}{d t}\left[\frac{m_{0} v}{\sqrt{1-v^{2} / c^{2}}}\right]=g \frac{m_{0}}{\sqrt{1-v^{2} / c^{2}}} \tag{1}
\end{equation*}
$$

where $m_{0}$ is the rest mass, $c$ the speed of light in a vacuum, and $g$ the constant of gravitational like acceleration. The left hand side of Eq. 1 is the relativistic formula for the inertial force and the right hand side is the formula for the gravitational like force that includes the gravitational mass dependence on velocity. The formula in Eq. 1 can be rearranged and simplified resulting in the following relation for the small body acceleration:

$$
\begin{equation*}
\frac{d v}{d t}=g\left(1-v^{2} / c^{2}\right) \tag{2}
\end{equation*}
$$

The energy loss will be calculated by comparing the potential energy that is exerted by lifting the small test body very slowly in the uniform gravitational field $g$ by a distance $z$ to the energy that is gained back when the body falls the same distance $z$. The test body potential energy is simply expressed as follows:

$$
\begin{equation*}
E=m_{0} g \cdot z \tag{3}
\end{equation*}
$$

This relation will be kept as reference energy. For the actual falling body energy the incremental energy gain by a fall can be expressed in terms of the velocity $v$ and the gravitational force $F$ as follows:

$$
\begin{equation*}
d E=F \cdot v \cdot \frac{d v}{(d v / d t)}=\frac{g \cdot m_{0}}{\sqrt{1-v^{2} / c^{2}}} \frac{v \cdot d v}{g\left(1-v^{2} / c^{2}\right)} \tag{4}
\end{equation*}
$$

[^0]After rearrangements and integration the expression for the energy as a function of velocity becomes:

$$
\begin{equation*}
E=\frac{m_{0} c^{2}}{\sqrt{1-v^{2} / c^{2}}} \tag{5}
\end{equation*}
$$

This result is expected and it is a nice confirmation of methodology used in Eq.1.
In the next step we will evaluate the energy difference $\Delta E$ given by Eq. 3 and Eq.5. However, to accomplish this task it is necessary to express the energy of the falling body in terms of the travelled distance $z$. This can be derived starting from Eq. 2 as follows:

$$
\begin{equation*}
\frac{d v}{\left(1-v^{2} / c^{2}\right)}=g \frac{d t}{d z} d z \tag{6}
\end{equation*}
$$

After multiplying both sides of Eq. 6 by velocity $v=d z / d t$, after integration, and after rearrangements the relation between the travelled distance $z$ during the fall and the velocity becomes equal to:

$$
\begin{equation*}
1-\frac{v^{2}}{c^{2}}=e^{\frac{-2 g \cdot z}{c^{2}}} \tag{7}
\end{equation*}
$$

Using this relation in the formula for the energy difference the result becomes as follows:

$$
\begin{equation*}
\Delta E=m_{0} g \cdot z+m_{0} c^{2}-m_{0} c^{2} e^{\frac{g \cdot z}{c^{2}}}=m_{0} g \cdot z+m_{0} c^{2}-m_{0} c^{2}\left(1+\frac{g \cdot z}{c^{2}}+\frac{1}{2}\left(\frac{g \cdot z}{c^{2}}\right)^{2}+\cdots\right) \tag{8}
\end{equation*}
$$

In this equation the rest mass equivalent energy was added to the reference energy to be consistent with the rest energy of the falling body. It is necessary that for the zero distance the energy difference and the velocity of falling body are both zero. By expanding the result into a power series as is shown in Eq. 8 and neglecting the higher order terms the energy loss will be as follows:

$$
\begin{equation*}
\Delta E \cong-\frac{m_{0} c^{2}}{8}\left(\frac{2 g \cdot z}{c^{2}}\right)^{2} \cong-\frac{m_{0} c^{2}}{8}\left(\frac{v^{2}}{c^{2}}\right)^{2} \tag{9}
\end{equation*}
$$

This is a very strange result. It seems that the falling body is gaining some additional energy from an unknown source on top of the energy that is predicted from the free fall by Eq.5. This is not reasonable and it is pointing to a problem that exists in the GRT for a long time. The gravitational mass cannot depend on velocity the same way as the inertial mass. This problem will become clear from the results presented next.

The similar expression introduced in Eq. 1 is used, but with the gravitational mass depending on velocity as follows ${ }^{[1,2]}$ :

$$
\begin{equation*}
m_{g}=m_{0} \sqrt{1-v^{2} / c^{2}} \tag{10}
\end{equation*}
$$

This leads to the following formula:

$$
\begin{equation*}
\frac{d}{d t}\left[\frac{m_{0} v}{\sqrt{1-v^{2} / c^{2}}}\right]=g \cdot m_{0} \sqrt{1-\frac{v^{2}}{c^{2}}} \tag{11}
\end{equation*}
$$

After rearrangements the formula is simplified with the result as follows:

$$
\begin{equation*}
\frac{d v}{d t}=g\left(1-v^{2} / c^{2}\right)^{2} \tag{12}
\end{equation*}
$$

Following the same procedure as above for the GRT case the differential of energy will be:

$$
\begin{equation*}
d E=g \cdot m_{0} \sqrt{1-\frac{v^{2}}{c^{2}}} \frac{v \cdot d v}{g\left(1-v^{2} / c^{2}\right)^{2}} \tag{13}
\end{equation*}
$$

This becomes, after integration, identical to formula in Eq.5. Both theories, the GRT and the MTG, thus give the same expression for the energy, which is expected and confirms once more that the calculating procedure is correct. For the calculation of energy loss the same procedure will also be followed as before. From Eq. 12 the relation between the distance and velocity is:

$$
\begin{equation*}
\frac{d v}{\left(1-v^{2} / c^{2}\right)^{2}}=g \frac{d t}{d z} d z \tag{14}
\end{equation*}
$$

which becomes after multiplying both sides of equation by velocity and after integration equal to:

$$
\begin{equation*}
\frac{1}{\left(1-v^{2} / c^{2}\right)}=1+\frac{2 g \cdot z}{c^{2}} \tag{15}
\end{equation*}
$$

The energy loss is then calculated also as before. By adding the rest mass energy to the reference results in the following:

$$
\begin{equation*}
\Delta E=m_{0} g \cdot z+m_{0} c^{2}-m_{0} c^{2} \sqrt{1+\frac{2 g \cdot z}{c^{2}}}=m_{0} g \cdot z+m_{0} c^{2}-m_{0} c^{2}\left(1+\frac{g \cdot z}{c^{2}}-\frac{1}{2}\left(\frac{g \cdot z}{c^{2}}\right)^{2}+\cdots\right) \tag{16}
\end{equation*}
$$

Again, by using the expansion of this result into a power series and neglecting the higher order terms the energy loss becomes equal to:

$$
\begin{equation*}
\Delta E \cong \frac{m_{0} c^{2}}{8}\left(\frac{2 g \cdot z}{c^{2}}\right)^{2} \cong \frac{m_{0} c^{2}}{8}\left(\frac{v^{2}}{c^{2}}\right)^{2} \tag{17}
\end{equation*}
$$

This is a similar result as in Eq. 9 except that the energy loss is now positive as it should be. This confirms the correctness of the gravitational mass dependence on velocity and therefore soundly disproves the validity of GRT.

The positive energy loss is perhaps justifiable by a gravitational wave radiation and the force of its radiation reaction because the body is accelerated in a Minkowski flat space-time. However, it is difficult to justify the negative energy loss as is derived in GRT. Where does the energy come from?

From the above derivations it is clear that the root cause of this problem is the incorrect dependency of gravitational mass on velocity in GRT. The absolute identity of inertial and gravitation masses for all the velocities as postulated in an ad hoc fashion in that theory is not sustainable. Fortunately the MTG eliminates this problem and it seems that this is the correct theory of gravity.

For a better understanding of the magnitude of radiated energy the graph of the energy loss as a function of the travelled distance during the fall for a mass of $10,000 \mathrm{~kg}$ is shown in FIG.1.

MTG energy conservation test for a free fall in the uniform gravitational field:

$$
\mathrm{g}=9.80665 \frac{\mathrm{~m}}{\mathrm{~m}^{2}} \quad \mathrm{z}:=0 \cdot \mathrm{~m}, 0.2 \cdot \mathrm{~m} . .100 \cdot \mathrm{~m} \quad \mathrm{mo}:=10^{4} \cdot \mathrm{~kg} \quad \Delta \mathrm{E}(\mathrm{z}):=\frac{\mathrm{mo} \cdot \mathrm{c}^{2}}{8} \cdot\left(\frac{2 \mathrm{~g} \cdot \mathrm{z}}{\mathrm{c}^{2}}\right)^{2}
$$

FIG. 1 the dependency of energy loss due to radiation for a 10,000kg test body free falling in a uniform gravitational field equal to Earth's gravity on the travelled distance $z$ of the fall.

Finally, it is also necessary to investigate the possibility that the energy is conserved by considering that the uniform gravitational field distorts the space-time and this should be included in derivations.

The metric for such a uniform gravitational field space-time is not the GRT based metric. The detail description is available elsewhere ${ }^{[3]}$ with the differential metric line element derived as shown below:

$$
\begin{equation*}
d s^{2}=\frac{(c d t)^{2}}{\left(1+g \cdot z / c^{2}\right)^{2}}-d x^{2}-d y^{2}-\frac{d z^{2}}{\left(1+g \cdot z / c^{2}\right)^{2}} \tag{18}
\end{equation*}
$$

Using the Lagrange formalism, the Lagrangian describing the motion of a small test body in the spacetime described by this metric is as follows:

$$
\begin{equation*}
L=\frac{(c d t / d \tau)^{2}}{\left(1+g \cdot z / c^{2}\right)^{2}}-\left(\frac{d x}{d \tau}\right)^{2}-\left(\frac{d y}{d \tau}\right)^{2}-\frac{(d z / d \tau)^{2}}{\left(1+g \cdot z / c^{2}\right)^{2}} \tag{19}
\end{equation*}
$$

with the first integrals that are readily found to be:

$$
\begin{equation*}
\frac{d t}{d \tau}=\left(1+\frac{g \cdot z}{c^{2}}\right)^{2} \tag{20}
\end{equation*}
$$

$$
\begin{equation*}
\frac{d v}{d t}=\frac{g}{\left(1+g \cdot z / c^{2}\right)^{3}} \tag{21}
\end{equation*}
$$

and considering that the Lagrangian is also a first integral we have:

$$
\begin{equation*}
1-\frac{v^{2}}{c^{2}}=\frac{1}{\left(1+g \cdot z / c^{2}\right)^{2}} \tag{22}
\end{equation*}
$$

The generalized gravitational force equation valid in the curved space-time with the gravitational mass depending on velocity as was introduced in Eq. 10 is defined as follows:

$$
\begin{equation*}
F_{g}=-g^{z z} \frac{\partial \varphi}{\partial z} m_{0} \sqrt{g_{t t}} \sqrt{1-v^{2} / c^{2}} \tag{23}
\end{equation*}
$$

By substituting into Eq. 23 the values for the metric coefficients and the square root of velocity term from Eq. 22 , the result is a constant force independent of velocity. It is interesting that while the gravitational mass depends on velocity as shown in Eq. 10 the gravitational force does not depend on velocity for the MTG in a curved space-time.

Following the similar procedure as described above for the Minkowski flat space-time, but now for the curved space-time the force balance equation is as follows:

$$
\begin{equation*}
\frac{d}{d t}\left[\frac{m_{0} v}{\sqrt{1-v^{2} / c^{2}}}\right]=g \cdot m_{0} \tag{24}
\end{equation*}
$$

This results in the following equation for the acceleration:

$$
\begin{equation*}
\frac{d v}{d t}=g\left(1-v^{2} / c^{2}\right)^{3 / 2} \tag{25}
\end{equation*}
$$

Similarly as in Eq. 14 this result can be rearranged as follows:

$$
\begin{equation*}
\frac{d v}{\left(1-v^{2} / c^{2}\right)^{3 / 2}}=g \frac{d t}{d z} d z \tag{26}
\end{equation*}
$$

which again by multiplication of both sides by velocity and after integration changes into the following:

$$
\begin{equation*}
\frac{1}{\sqrt{1-v^{2} / c^{2}}}=1+\frac{g \cdot z}{c^{2}} \tag{27}
\end{equation*}
$$

By multiplying both sides of this equation by $m_{0} c^{2}$ results in the following expression:

$$
\begin{equation*}
\frac{m_{0} c^{2}}{\sqrt{1-v^{2} / c^{2}}}=m_{0} c^{2}+m_{0} g \cdot z \tag{28}
\end{equation*}
$$

This equation clearly states that the energy is conserved and no loss occurs during the fall. Again, the gravitational mass is not equal to the inertial mass as is assumed in the GRT; however, the gravitational force stays constant.

To complete the comparison with the GRT gravitational mass dependence on velocity in the curved space-time the GRT gravitational mass dependence on velocity is inserted into Eq. 23 resulting in:

$$
\begin{equation*}
\frac{d}{d t}\left[\frac{m_{0} v}{\sqrt{1-v^{2} / c^{2}}}\right]=\frac{g \cdot m_{0}}{\left(1-v^{2} / c^{2}\right)} \tag{29}
\end{equation*}
$$

and further in the relation for the acceleration:

$$
\begin{equation*}
\frac{d v}{d t}=g \sqrt{1-v^{2} / c^{2}} \tag{30}
\end{equation*}
$$

and finally after integration in the formula:

$$
\begin{equation*}
\sqrt{1-v^{2} / c^{2}}=1-g \cdot z / c^{2} \tag{31}
\end{equation*}
$$

The energy loss is calculated from the energy difference as before, which becomes as follows:

$$
\begin{equation*}
\Delta E=m_{0} g \cdot z+m_{0} c^{2}-\frac{m_{0} c^{2}}{\left(1-g \cdot z / c^{2}\right)}=m_{0} g \cdot z+m_{0} c^{2}-m_{0} c^{2}\left(1+\frac{g \cdot z}{c^{2}}+\left(\frac{g \cdot z}{c^{2}}\right)^{2}+\cdots\right) \tag{32}
\end{equation*}
$$

This finally leads to the result:

$$
\begin{equation*}
\Delta E \cong-\frac{m_{0} c^{2}}{4}\left(\frac{2 g \cdot z}{c^{2}}\right)^{2} \cong-\frac{m_{0} c^{2}}{4}\left(\frac{v^{2}}{c^{2}}\right)^{2} \tag{33}
\end{equation*}
$$

This proves again that the gravitational mass as postulated in the GRT cannot depend on velocity the same way as the inertial mass.

Conclusions: The paper derived simple expressions for the energy loss during the small test body accelerated motion in a simulated free fall of a uniform gravitational field in a flat Minkowski spacetime. It was shown that the loss derived according to the GRT is negative. This is unacceptable and this fact thus proves the invalidity of GRT. This problem has its root cause in the identical dependency of inertial mass and gravitational mass on velocity.

When a different dependency of gravitational mass on velocity is used, as is derived in the MTG, the positive energy loss is calculated.

For the uniform gravitational field when the space-time curvature is included in derivations the gravitational force stays constant for the MTG mass dependence on velocity even though the gravitational mass changes with velocity. The energy conservation law is satisfied in the MTG. However, for the GRT there is again a negative energy loss, which is not acceptable and proves again that the GRT is not a valid theory of gravity.

The presented results have fatal consequences for the GRT, because unquestionably prove its incorrectness. These findings thus have a significant impact on all the theories based on the GRT such as the Big Bang and similar ridiculous models of the Universe.

The author hopes that the main stream relativists finally recognize this problem and abandon the GRT with all its ridiculous claims of existence of Black Holes, Event Horizons, and the Big Bang Universe with its accelerating expansion to infinity from nothing.

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