

Kinetic Energy and Conservation of Momentum

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In the history of physics, kinetic energy has been represented by two expressions. One from Issac Newton, the other from Special Relativity. Both expressions are expected to describe a physical system that demands conservation of momentum. By examining the expression of momentum in a projectile motion, the kinetic energy from Issac Newton is found to obey conservation of momentum while the kinetic energy from Special Relativity is found to violate conservation of momentum.

I. INTRODUCTION

In 17th century, Issac Newton proposed a definition of force, $F=m*a$. Based on this definition, both kinetic energy and momentum can be derived.

In 20th century, Special Relativity[1] proposed a new definition of kinetic energy. This results in new definitions of both momentum and force.

However, the physics law, conservation of momentum, remains intact. Any definition of kinetic energy is expected to generate a force that results in the conservation of momentum.

This paper examines both expressions of momentum in a projectile motion. The x-component of total momentum is calculated for both expressions of kinetic energy.

The concept of relativistic mass becomes less popular in modern physics. Relativistic force and relativistic momentum do not share the same relativistic mass. The momentum of a particle is represented by either $\gamma(v) * m(0) * v$ or $m(v) * v$. Both representations are equivalent to each other mathematically. In this paper, $\gamma(v) * m * v$ is chosen to emphasize Lorentz Factor, $\gamma(v)$, in Lorentz Transformation.

$$\frac{dm}{dv} = \frac{dm(0)}{dv} = 0 \quad (1)$$

II. PROOF

Consider two-dimensional motion.

A. Kinetic Energy and Momentum

In Newtonian Mechanics, force F is defined as multiplication of mass and acceleration.

$$F = m * a \quad (2)$$

In Special Relativity, kinetic energy K is defined as

$$K = (\gamma(v) - 1) * m * C^2 = \left(\frac{1}{\sqrt{1 - \frac{v^2}{C^2}}} - 1 \right) * m * C^2 \quad (3)$$

These two definitions generate different expressions for momentum. However, the derivation of momentum from

kinetic energy has not been changed. Kinetic Energy K is defined as integration of force over distance.

$$K = \int F dx \quad (4)$$

Momentum P is defined as integration of force over time.

$$P = \int F dt \quad (5)$$

$$\frac{dP}{dt} = F \quad (6)$$

In Newtonian Mechanics,

$$K = \frac{1}{2} * m * v^2 \quad (7)$$

$$P = m * v \quad (8)$$

In Special Relativity,

$$K = (\gamma(v) - 1) * m * C^2 \quad (9)$$

$$P = \gamma(v) * m * v \quad (10)$$

The difference in expression indicates that only one expression of momentum can be correct. By applying conservation of momentum to both expressions of momentum in a physical system such as projectile motion, the correct expression can be distinguished.

B. Projectile Motion

A particle moves along x axis under a force along y axis. The acceleration in x direction is zero. The acceleration in y direction is A. The single force on this particle demands that total momentum in x direction P_x should remain constant.

$$\frac{dP_x}{dt} = F_x = 0 \quad (11)$$

C. Conservation of Momentum

Let a_x be the acceleration on x direction. Let a_y be the acceleration on y direction.

$$a_x = 0 = \frac{dv_x}{dt} \quad (12)$$

$$a_y = A = \frac{dv_y}{dt} \quad (13)$$

In Newtonian Mechanics, change of momentum in x direction is zero.

$$\frac{dP_x}{dt} = \frac{d(m*v_x)}{dt} = m * a_x = 0 \quad (14)$$

In Special Relativity, change of momentum in x direction is not zero.

$$\frac{dP_x}{dt} = \frac{d(\gamma(v)*m*v_x)}{dt} = \frac{d\gamma(v)}{dt} * m * v_x \quad (15)$$

$$= \gamma(v)^3 * \frac{v}{C^2} * \frac{dv}{dt} * m * v_x \quad (16)$$

$$= \gamma(v)^3 * \frac{v}{C^2} * \frac{d\sqrt{v_x^2+v_y^2}}{dt} * m * v_x \quad (17)$$

$$= \gamma(v)^3 * \frac{v}{C^2} * \frac{v_y}{v} * A * m * v_x \quad (18)$$

Total momentum in x direction remains constant in Newtonian Mechanics but not in Special Relativity.

III. CONCLUSION

Special Relativity violates conservation of momentum in projectile motion.

Conservation of momentum fails to hold if momentum is defined as $\gamma(v) * m * v$. The failure of this physics law is due to the introduction of Lorentz factor, $\gamma(v)$, from Lorentz Transformation[8][11].

Lorentz Transformation was proposed on the assumption that the speed of light is independent of inertial reference frame.

As the result of this incorrect assumption[3], Lorentz Transformation violates Translation Symmetry[4] and Conservation of Momentum[10] in physics. Translation Symmetry requires conservation of simultaneity[5], conservation of distance[6], and conservation of time[7]. All three conservation properties are broken by Lorentz Transformation.

Therefore, Lorentz Transformation is an invalid transformation in physics. Consequently, any theory based on Lorentz Transformation is incorrect in physics. For example, Special Relativity.

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