Proof of Riemann Hypothesis

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***Abstract*—This article describes how to calculate the formula for calculating all primes.**

***Index Terms*—algorithm**

I. Riemann Hypothesis Definition

There is a pattern in the distribution of primes among the positive integers.

II. Riemann Hypothesis Proof Algorithm

A. Distinguishing the Sequence of Odd Numbers

Let $\mathbb{N}$ be natural numbers, including zero, and $\mathbb{N}^*$ be natural numbers without zero.

First two primes (by condition) are:

$$1, 2. \tag{1}$$

Prime number 2 is significant for dividing the sequence into two equal sequences of even (x) and odd (y) numbers:

$$x \in \{2M \mid M \in \mathbb{N}^*\}, \tag{2}$$

$$y \in \{2M + 1 \mid M \in \mathbb{N}^*\}. \tag{3}$$

Starting from $M = 2$ (2) describes the set of composite numbers x_{comp} by condition:

$$x_{comp} \in \{2M \mid M \in \mathbb{N}^*,\ M \geq 2\}. \tag{4}$$

Thus further we will consider the sequence of odd numbers $\{y\}$ (3) to determine the pattern in the distribution of primes (y_o).

The sequence of odd numbers $\{y\}$, except for y_o, also includes the set of composite odd numbers y_{comp}:

$$y_{comp} \in \{y_o y \mid y_o \geq 3,\ y \geq 3\}. \tag{5}$$

Expression (3) without limitations describes the distribution of first y_o in the sequence of odd numbers within the segment from 3 to the first $y_{comp} = 3^2 = 9$.

Let's represent (3) as the following expression:

$$y_o = 1^2 + 2 \cdot 1 \cdot M_1 + 2, \tag{6}$$

where $M_1 \in \mathbb{N}$.

Therefore, this segment can be represented in the following way:

$$1^2 < y < 3^2. \tag{7}$$

The following segment, where (6) for determination of y_o will be limited by exception of the set of composite numbers $\{3y \mid y > 3\}$, will end with the first y_{comp} to which $y_o = 3$ will bear no relation. By definition it is $y_{comp} = 5^2 = 25$. Thus we can conclude the following.

B. Conclusion 1

All segments compliant with the specific pattern of distribution of y_o are limited by $y_{comp} = y_{on}^2$ and $y_{comp} = y_{o(n+1)}^2$. Let's analyze the first such segment.

C. The first segment of odd numbers from 1 to 9

Distribution of y_o is described by (6).

Let's calculate first y_o after (1):

$$3, 5, 7. \tag{8}$$

D. The second segment of odd numbers from 9 to 25

In order to exclude the composite numbers y_{comp} from the set $\{3y \mid y > 3\}$, $y_o = 1$ in (6) shall be replaced by $y_o = 3$ and summand 2 shall be replaced by variable ± 2 to cover all y_o in this segment:

$$y_o = 3^2 + 2 \cdot 3 \cdot M_3 \pm 2 = 3^2 + 2(3M_3 \pm 1), \tag{9}$$

where $M_3 \in \mathbb{N}$.

Let's calculate next y_o in the sequence:

$$11, 13, 17, 19, 23. \tag{10}$$

E. The third segment of odd numbers from 25 to 49

For this segment y_o value shall be equal in two expressions - in (9) and in the following expression in order to exclude the composite numbers $\{5y \mid y > 5\}$:

$$y_o = 5^2 + 2 \cdot 5 \cdot M_5 \pm 2z_5 = 5^2 + 2(5M_5 \pm z_5), \tag{11}$$

where $M_5 \in \mathbb{N},\ 1 \leq z_5 \leq 2$.

Starting from the second segment, expression for y_o depends on the value of M_3. According to Conclusion 1 and (9) it is possible to calculate the lower and upper limits for M_3 in any segment of $y_{on}^2 < y < y_{o(n+1)}^2$:

$$\frac{y_{on}^2 - 9 \pm 2}{6} \leq M_3 \leq \frac{y_{o(n+1)}^2 - 9 \pm 2}{6}. \tag{12}$$

For this segment M_3 value in (9) will change:

$$3 \leq M_3 \leq 7. \tag{13}$$

Let's compare (9) and (11):

$$3^2 + 2 \cdot 3 \cdot M_3 \pm 2 = 5^2 + 2 \cdot 5 \cdot M_5 \pm 2z_5. \qquad (14)$$

Let's express M_5 from (14):

$$M_5 = \frac{3M_3 \pm 1 - 8 \mp z_5}{5}. \qquad (15)$$

Substitute (15) into (11):

$$y_o = 5^2 + 2\left(5 \cdot \frac{3M_3 \pm 1 - 8 \mp z_5}{5} \pm z_5\right), \qquad (16)$$

where $3 \leq M_3 < 7,\ 1 \leq z_5 \leq 2$.

Calculate next y_o in the third segment:

$$29, 31, 37, 41, 43, 47. \qquad (17)$$

F. Conclusion 2

Based on the results of analysis of first, second and third segments of odd numbers we can conclude the following:
Each successive segment compliant with the pattern of distribution of y_o depends on the pattern of distribution of y_o in all previous segment starting from the second segment.
Let's analyze the following segment for final determination of the pattern of distribution of y_o in segments $y_{on}^2 < y < y_{o(n+1)}^2$.

G. The fourth segment of odd numbers from 49 to 121

For this segment y_o value shall be equal in two expressions - in (16) with different values of variables:

$$7 \leq M_3 < 19, \qquad (18)$$

$$1 \leq z_5 \leq 2, \qquad (19)$$

$$M_5 = \frac{3M_3 \pm 1 - 8 \mp z_5}{5} \in \mathbb{N}^*, \qquad (20)$$

and in the following expression to exclude the composite numbers y_{comp} from the set $\{7y \mid y > 7\}$:

$$y_o = 7^2 + 2 \cdot 7 \cdot M_7 \pm 2z_7 = 7^2 + 2(7M_7 \pm z_7), \qquad (21)$$

For this segment from (21) it follows that:

$$M_7 \in \mathbb{N}, \qquad (22)$$

$$1 \leq z_7 \leq 3 \qquad (23)$$

Let's compare (16) and (21):

$$5^2 + 2\left(5 \cdot \frac{3M_3 \pm 1 - 8 \mp z_5}{5} \pm z_5\right) = \\ = 7^2 + 2 \cdot 7 \cdot M_7 \pm 2z_7. \qquad (24)$$

Express M_7 from (24):

$$M_7 = \frac{5 \cdot \frac{3M_3 \pm 1 - 8 \mp z_5}{5} \pm z_5 - 12 \mp z_7}{7}. \qquad (25)$$

Substitute M_7 from (25) into (21):

$$y_o = 7^2 + 2(7 \cdot \frac{5 \cdot \frac{3M_3 \pm 1 - 8 \mp z_5}{5} \pm z_5 - 12 \mp z_7}{7} \pm z_7), \qquad (26)$$

where (18), (19), (20), (22), (23) and (25) are true.

Let's calculate the successive values of y_o in in the fourth segment of odd numbers:

$$53, 59, 61, 67, 71, 73, 79, 89, \\ 97, 101, 103, 107, 109, 113. \qquad (27)$$

H. General Expression of Distribution of Primes

Thus we can determine the specific patterns, comparing (16) and (26).
Let's present the general expression of distribution of y_o in n-th segments $y_{on}^2 < y < y_{o(n+1)}^2$ taking these patterns into consideration:

$$y_o = y_{on}^2 + 2(y_{on} M_{y_{on}} \pm z_{y_{on}}) \qquad (28)$$

Variables (28) are calculated using the following formulas:

$$1 \leq z_{y_{oc}} \leq \frac{y_{oc} - 1}{2}, \qquad (29)$$

where $2 \leq c \leq n$ is the number of the number in the sequence of odd primes;

$$M_{y_{on}} = \\ = \frac{y_{o(n-1)} M_{y_{o(n-1)}} \pm z_{y_{o(n-1)}} - \frac{y_{on}^2 - y_{o(n-1)}^2}{2} \mp z_{y_{on}}}{y_{on}}, \qquad (30)$$

where $M_{y_{on}} \in \mathbb{N}$;

$$M_3 < M_{y_{ob}} < M_{y_{on}}, \qquad (31)$$

where M_3 is calculated according to (12);

$$M_{y_{ob}} = \\ = \frac{y_{o(b-1)} M_{y_{o(b-1)}} \pm z_{y_{o(b-1)}} - \frac{y_{ob}^2 - y_{o(b-1)}^2}{2} \mp z_{y_{ob}}}{y_{ob}}, \qquad (32)$$

where $M_{y_{ob}} \in \mathbb{N}^*,\ 3 < y_{o(b-1)} < y_{ob} < y_{on}$.

In order to form the full sequence of y_o the n-th segments shall be analyzed in sequence. But calculation of y_o from segments to segment becomes more difficult. Thus the third segment of odd numbers in (16) has 5 variables, the fourth segment of odd numbers in (26) has 8 variables. But nevertheless, (26) unequivocally describes the distribution of y_o in sequence of numbers. If it is necessary to calculate y_o in n-th segment, avoiding the previous segments, all $y_o \leq y_{o(n+1)}$ from previous calculations shall be known. The required range will be set by summand y_{on}^2 and values of M_3 (12). While solving the problem all (31) for this n-th segment shall be calculated in sequence.

I. Final Conclusion

Riemann Hypothesis is true. Distribution of primes among the positive integers has its own pattern. But for odd numbers of y the sections compliant with the specific pattern of distribution of primes y_o are limited by composite numbers y_{on}^2 and $y_{o(n+1)}^2$. Distribution of y_o in such n-th sections, starting from the third segment of odd numbers, is calculated according to the (28). The full sequence of y_o is achieved by consequent analysis of n-th sections, starting from the first segment of odd numbers.

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References