

SunQM-2: Expanding QM from micro-world to macro-world: general Planck constant, H-C unit, H-quasi-constant, and the meaning of QM

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Abstract

A general form of Planck constant has been discovered as $h_{\text{general}} = h(m/m')$, where m is the mass of object we are studying (like electron, Earth, etc.), and m' is part of the scaling factor that scales h up or down. Although it is suitable for both micro- and macro- world's QM, as well as the EM- and G-forced QM, h_{general} is not constant at all. h_{general} is a variable unit (named H-C unit), which make us able to use the same QM formula for hydrogen atom to calculate the orbit movement for all kinds of attractive central-force system. The orbital energy can be written as $E_{n\text{-orbit}} = -Hmf_{n\text{-orbit}}$, where $H=h/m'$ is a quasi constant (for all objects on all different orbits with same r_1 , and for both G-force and EM-force simultaneously). A new concept of high-frequency multiplier quantum number n' has been proposed. A new concept of RF (rotation diffusion, or rotafusion) has been proposed. It is the complete randomization of unit vector $\mathbf{L}/|\mathbf{L}|$ at $n=1$ ground state. A new concept of interior $\{N,n\}$ QM analysis has been proposed.

Introduction

In memory of Max Planck (who proposed h constant in 1900), Albert Einstein (who proposed $E = hf$ in 1905), and de Broglie (who introduced the matter wave in 1924).

A series of papers^[1] are used to present my discovery of Solar system quantum mechanics $\{N,n\}$ structure: Paper-1, how to quantize the orbits of Solar system solely based on the quantum orbit relationship of $r_n = r_1 * n^2$, which was first obtained from Bohr model, and had also been proved to be correct in Schrodinger equation solution. Paper-2, how to quantize the orbit energy of Solar system using $E = hf$, and how it led to the discovery of a new constant, a generalized Planck constant (named H-C unit). Paper-3, how to seamlessly transform Schrodinger's equation/solution into Bohr-kind simple model for Solar system. Paper-5, how to use a new QM method (C-QM, based on interior $\{N,n\}$ QM, multiplier n' , $|R(n,l)|^2 |Y(l,m)|^2$ guided mass occupancy, and RF) to analyze our world in scales from string to universe. Paper-6, relativistic C-QM. Each of these main papers has its own supplementary papers to present my additional results/thoughts that related to the main paper. Papers are abbreviated as: Main paper: SunQM-1, SunQM-2, SunQM-3, SunQM-5, etc. Supplementary paper: SunQM-1s1, SunQM-1s2, SunQM-1s3, SunQM-2s1, SunQM-3s1, SunQM-3s2, SunQM-3s3, SunQM-3s6, SunQM-5s1, etc.

From QM text books we know that both Bohr model and Schrodinger equation solution give the same formula for hydrogen atom's orbital radius ($r_n = r_1 * n^2$), energy ($E_n = E_1 / n^2$), but a very different angular momentum (L_n) calculation. In the previous paper (SunQM-1), I quantized the Solar system's orbits (e.g., $\{0,2\}$ for Sun's surface, $\{1,5\}$ for Earth orbit) and established a Solar QM $\{N,n/6\}$ periodic table. In this paper (SunQM-2), I will present that how to quantize the orbit energy of Solar system using $E = hf$, and how it led to the discovery of a new quasi-constant, H-C unit. I will introduce a new concept of rotation diffusion (RF) to explain why Bohr's \mathbf{L} is different from Schrodinger's \mathbf{L} . Note: for $\{N,n\}$ QM nomenclature as well as the general notes for $\{N,n\}$ QM model, please see my paper SnQM-p1 section VII and VIII. Note: Microsoft Excel's number format is often used in this paper, for example: $x^2 = x^2$, $3.4E+12 = 3.4*10^{12}$, $5.6E-9 = 5.6*10^{-9}$.

I. The discovery of a generalized Planck constant H for planet orbits in Solar system.

After Max Planck quantized the energy of black-body radiation and Albert Einstein quantized photon, the famous formula $E=hf$ has become the icon of quantum mechanics. That formula was developed for the micro-world's QM. Since the Solar system's orbital structure has been successfully described with $\{N,n\}$ QM (in paper SunQM-1), its orbital energy have to be described by the same quantum energy formula. After many tries, I discovered a formula $E= -Hmf$ for planets' quantum orbit energy, which closely matched Planck/Einstein's quantum energy formula $E=hf$. There are many ways to deduce this planets' quantum orbit energy formula, here I only give the simplest way.

I-a. A planet's orbit energy in classical physics form $E = -(1/2) mv^2$ eq-1

A planet moving around Sun is a central G-force governed circular movement. It follows Neuton's second law in classical physics: $F=ma=mv^2/r$, and $F=G*Mm/r^2$, combining these two equations we obtain: $mv^2/r = G*Mm/r^2$, or $GM = r v^2$. Its classical orbit energy $E = K + U = (1/2)mv^2 - GMm/r$. Replacing GM by rv^2 , we obtain $E = K + U = (1/2)mv^2 - (r v^2)m/r$, or $E = -(1/2)mv^2$, where r , v , E are the orbital radius, orbital velocity, and orbital energy of this planet.

I-b. A planet's QM orbit energy in its particle form $E_n = -(1/2) mv_n^2$ eq-2

Here we need to replace r , v , and E in section I-a by r_n , v_n , and E_n , and rewrite the equations:

$$F=ma=m v_n^2/r_n, \text{ and } F=G*Mm/r_n^2,$$

$$mv_n^2/r_n = G*Mm/r_n^2, \text{ or } GM = r_n v_n^2,$$

$$E_n = K + U = (1/2)mv_n^2 - GMm/r_n = \text{constant},$$

$$E_n = K + U = (1/2)mv_n^2 - (r_n v_n^2)m/r_n, \text{ or } E_n = -(1/2)mv_n^2.$$

This formula shows only the particle property of a planet moving in a circular orbit.

I-c. A planet's QM orbit energy in its matter wave form $E_n = -Hmf_n$ eq-3

Now I try to introduce the matter wave property of a planet that moving in a circular orbit. Extended from de Broglie's theory, we know that any matter (including planet) has an associated matter wave with wavelength $\lambda=h/p$, where h is the Planck constant, and p is the momentum of this matter. For a circular orbit moving object (like an electron, here I also extended to a planet), its matter wave followed de Broglie's relationship $n\lambda=2\pi r$.

As we learned from the text book "*Modern Physics For Scientist & Engineering (3rd Stephen Thornton), page 180, Eq 5.34*", any wave (including de Broglie's matter wave $2\pi r n=n\lambda$) has a phase velocity (v_{phase} , or v_{ph}), which is different from its particle velocity. For wave-particle duality, particle velocity = group velocity (v_{group} , or v_{gr}). There is a relationship of $\lambda f = v_{\text{ph}} \neq v_{\text{gr}}$, where f is the matter wave (orbit) frequency. When a particle of mass ($=m$) moving non-relativistic with speed v , its phase velocity, if treated as a de Broglie wave, is $v_{\text{ph}} = \lambda f = v/2$, or $v = v_{\text{gr}} = 2v_{\text{ph}}$. It is deduced as:

$$v_{\text{ph}} = \lambda f = (h/p)(E/h) = E/p = (p^2/2m) / p = p / 2m = mv / 2m = v/2.$$

So for a particle-wave duality kinetic energy $K_n=(1/2)mv_n^2$, it can be written as $K_n = m v_n v_{\text{ph}-n}$, or $K_n = m v_n \lambda_n f_n$.

For a planet moving in a circular orbit n , if replacing its matter wave's $\lambda_n (=h/mv_n)$, into $\lambda_n * f_n = v_{\text{ph}-n}$, we obtain $(h/mv_n)*f_n = v_{\text{ph}-n} = v_n/2$, $v_n^2=2(h/m)f_n$, $v_n^2=2Hf_n$,

where $H=h/m$. So for its orbit energy $E_n = K_n + U_n = -(1/2) mv_n^2$, I am able to obtain

$$E_n = - Hmf_n$$

where $H= h/m$, named as "quasi-Planck constant H ". This formula shows that the QM wave property of a planet moving in a circular orbit!

This formula is closely related to (Einstein's photon energy formula) $E = hf$. We can even directly deduce $E = hf$ from $E_n = - Hmf_n$. To do that, we need to know that in $E = hf$, f is the difference of two orbital frequencies $f_{\Delta n} = f_{n+\Delta n} - f_n$. So $\Delta E = E_{n+\Delta n} - E_n = -Hmf_{n+\Delta n} - (-Hmf_n) = -Hm(f_{n+\Delta n} - f_n) = -(h/m)m f_{\Delta n} = -h f_{\Delta n}$.

The negative sign is due to $f_{\Delta n} < 0$. I will present how I used this formula to study the Solar QM $\{N,n\}$ structure in my paper SunQM-2s2. It had been known from the beginning of the QM era that if we applied $E=hf$ to macro-world object, the obtained quantum number n will be too large to be meaningful.

Here is the magic part: to make this formula truly useful for planets, after many tests, I discovered that for $E_n = -Hmf_n$, the m has to be the planet's mass, but for $H=h/m$, the m can be any value (even the electron's mass) but (usually) not planet's mass ! So we re-write planet's orbit energy $E_n = -Hmf_n$, where $H=h/m'$, m is planet's mass, f_n is orbit n 's matter wave frequency, h is Planck constant, and m' is a scaling factor (in unit of kg) that makes the calculated n meaningful in planet's QM.

I-d. Apply $E_n = -Hmf_n$ to Solar system (that is governed by G-force).

In table 1, I apply all three formulas (classical, QM particle, and QM wave) for planet orbit energy E_n calculation, and compare results. Notice that in Table1:

1) NASA's planets' data of r and v (in column 2 and column 4) is only used for the classical calculation of particle Energy. All QM calculated particle E_n or wave E_n (from column 13 to column 36) uses r_n (at column 10) & v_n (at column 11) that obtained purely from $\{N,n\}$ model .

2) The $\{N,n\}$ QM model only needs to know where we set the total $n=1$ (or what is r_1 value, it can be $\{0,1\}$'s r , or $\{2,1\}$'s r , ...). Once the r_1 is chosen, it will calculate out rest r_n values for all planets.

3) From $F=G*Mm/r_n^2$, $mv_n^2/r_n = G*Mm/r_n^2$, or $GM = r_n v_n^2$, or $v_n = \text{sqrt}(GM/r_n)$, we obtain $v_1 = \text{sqrt}(GM/r_1)$. eq-4

4) From $\lambda=h/mv$, or $h/m=\lambda_n v_n = 2\pi r_n /n *v_n = 2\pi v_1 r_1 = 2\pi r_1 * \text{sqrt}(GM/r_1)$, or $h/m = 2\pi * \text{sqrt}(GM r_1)$ eq-5
So H only related to r_1 . Once you chosen r_1 , you can calculate out H .

5) As explained in the paper SunQM-1, in the interior $\{N,n\}$, the orbit quantum number n of a planet is a quantized fraction number (e.g., $n/6$, $n/6^2$, ... etc.), and it means orbit is located at inner space of r_1 .

Table 1. Using $E_n = -Hmf_n$ to calculate the planet's orbit energy in Solar system's G-force field, and comparing result from classical, QM particle, and QM wave calculation.

NASA's data of planets		classical QM										particle										wave, set (0,1) as total n=1, calc $\lambda_n, f_n, H, m', E_n$										wave, set (1,1) as total n=1, calc $\lambda_n, f_n, H, m', E_n$										wave, set (2,1) as total n=1, calc $\lambda_n, f_n, H, m', E_n$										wave, set m=electron's mass, calc total n=?									
unit	mass	radius or planet orbit-r	planet orbit-v	orbit E (1/2)mv ²	(N,n) mod el, N, n	$v_n = \text{sqrt}(GM/r_n)$	$E_n = -(1/2)mv_n^2$	calc n	$\lambda_n = 2\pi r_n / n$	$f_n = v_n / \lambda_n$	$h/m' = 2\pi * \text{sqrt}(GM r_1)$	$E_n = -Hmf_n$	calc n	$\lambda_n = 2\pi r_n / n$	$f_n = v_n / \lambda_n$	$h/m' = 2\pi * \text{sqrt}(GM r_1)$	$E_n = -Hmf_n$	calc n	$\lambda_n = 2\pi r_n / n$	$f_n = v_n / \lambda_n$	$h/m' = 2\pi * \text{sqrt}(GM r_1)$	$E_n = -Hmf_n$	$m = m_e$	Hh/m'	$f_n = v_n / \lambda_n$	$E_n = -Hmf_n$	$\lambda_n = v_n / 2\pi f_n = 2\pi r_n / n$																																		
Sun core	1.74E+08				0	1	6	1	1.74E+08	873647		1						1/6																																											
SUN	1.99E+30	6.96E+08			0	2	6	2	6.96E+08	436204		2						2/6																																											
Mercury	3.30E+23	5.79E+10	47400	-3.71E+32	1	3	6	18	5.64E+10	48536	-3.89E+32	18	1.97E+10	1.23E-06	9.55E+14	6.94E-49	-3.89E+32	3	1.18E+11	2.06E-07	5.73E+15	1.16E-49	-3.89E+32	3/6	7.08E+11	3.43E-08	3.44E+16	1.93E-50	-3.89E+32	9.11E-31	7.27E-04	1.62E+12	1.50E-08	2.36E+19																											
Venus	4.87E+24	1.08E+11	35000	-2.98E+33	1	4	6	24	1.05E+11	36402	-3.23E+33	24	2.62E+10	6.94E-07	9.55E+14	6.94E-49	-3.23E+33	4	1.57E+11	1.10E-07	5.73E+15	1.16E-49	-3.23E+33	4/6	9.44E+11	1.93E-08	3.44E+16	1.93E-50	-3.23E+33	9.11E-31	7.27E-04	1.11E+11	2.00E-08	3.15E+19																											
Earth	5.97E+24	1.49E+11	29500	-2.65E+33	1	5	6	30	1.57E+11	29122	-2.53E+33	30	3.28E+10	4.44E-07	9.55E+14	6.94E-49	-2.53E+33	5	1.97E+11	7.60E-08	5.73E+15	1.16E-49	-2.53E+33	5/6	1.18E+12	1.93E-08	3.44E+16	1.93E-50	-2.53E+33	9.11E-31	7.27E-04	5.83E+11	2.50E-08	3.94E+19																											
Mars	6.42E+23	2.28E+11	24100	-1.86E+32	1	6	6	36	2.25E+11	24268	-1.89E+32	36	3.93E+10	3.08E-07	9.55E+14	6.94E-49	-1.89E+32	6	2.36E+11	5.14E-08	5.73E+15	1.16E-49	-1.89E+32	1	1.42E+12	8.57E-09	3.44E+16	1.93E-50	-1.89E+32	9.11E-31	7.27E-04	4.05E+11	3.00E-08	4.73E+19																											
Jupiter	1.90E+27	7.78E+11	13100	-1.63E+35	2	2	5.33	64.0	7.12E+11	13659	-1.77E+35	64.0	6.99E+10	9.77E-08	9.55E+14	6.94E-49	-1.77E+35	10.7	4.19E+11	1.63E-08	5.73E+15	1.16E-49	-1.77E+35	1.8	2.52E+12	2.71E-09	3.44E+16	1.93E-50	-1.77E+35	9.11E-31	7.27E-04	1.28E+11	5.33E-08	8.39E+19																											
Saturn	5.68E+26	1.43E+12	9700	-2.67E+34	2	3	5.33	95.9	1.65E+12	9100	-2.95E+34	95.9	1.05E+11	4.34E-08	9.55E+14	6.94E-49	-2.95E+34	16.0	6.29E+11	7.24E-09	5.73E+15	1.16E-49	-2.95E+34	2.7	3.77E+12	1.21E-09	3.44E+16	1.93E-50	-2.95E+34	9.11E-31	7.27E-04	5.70E+10	7.99E-08	1.26E+20																											
Uranus	8.68E+25	2.97E+12	6800	-2.01E+33	2	4	5.33	127.9	2.85E+12	6800	-2.03E+33	127.9	1.40E+11	2.46E-08	9.55E+14	6.94E-49	-2.03E+33	21.3	8.39E+11	4.07E-09	5.73E+15	1.16E-49	-2.03E+33	3.6	5.03E+12	6.79E-10	3.44E+16	1.93E-50	-2.03E+33	9.11E-31	7.27E-04	1.21E+10	1.07E-07	1.68E+20																											
Neptune	1.02E+26	4.51E+12	5400	-1.49E+33	2	5	5.33	159.9	4.45E+12	5460	-1.52E+33	159.9	1.75E+11	1.56E-08	9.55E+14	6.94E-49	-1.52E+33	26.7	1.05E+12	2.61E-09	5.73E+15	1.16E-49	-1.52E+33	4.4	6.29E+12	4.34E-10	3.44E+16	1.93E-50	-1.52E+33	9.11E-31	7.27E-04	2.05E+10	1.33E-07	2.10E+20																											
Pluto	1.46E+22	5.91E+12	4700	-1.61E+29	2	6	5.33	191.9	6.40E+12	4553	-1.51E+29	191.9	2.10E+11	1.09E-08	9.55E+14	6.94E-49	-1.51E+29	32.0	1.26E+12	1.81E-09	5.73E+15	1.16E-49	-1.51E+29	5.3	7.55E+12	3.02E-10	3.44E+16	1.93E-50	-1.51E+29	9.11E-31	7.27E-04	1.42E+10	1.60E-07	2.52E+20																											

Result and discussion (for Table 1):

1) For each planet, the QM wave formula $E_n = -Hmf_n$ generates the same orbit energy (in columns 18, 24, 30) as QM particle formula $E_n = -(1/2)mv_n^2$ (in column 12), and both $\{N,n\}$ QM modeled values are the same (or closely) as that of

classical physics result $E = -(1/2)mv^2$ (in column 5). This confirms that the $E_n = -Hmf_n$ is the correct QM orbit energy formula.

2) In columns 16, 22, 28, $H = h/m' = 2\pi \cdot \text{sqrt}(GMr_1)$, so H only relates to r_1 . Once you have chosen a $\{N,1\}$'s r as r_1 , H will be a constant for all planets. Therefore, H is a quasi-constant. However, each planet will have its own λ_n, f_n, E_n under this H.

3) You can choose another $\{N,1\}$'s r as r_1 (e.g., choose $\{1,1\}$'s or $\{2,1\}$'s r as r_1), then H will have a different value, but it is still a quasi-constant for all planets. For each planet, this new H will generate different λ_n, f_n , but the E_n will keep unchanged for this planet for all different new H(s). So each planet on orbit does not have a unique λ, f . A planet's matter wavelength and frequency completely depends on which $\{N,1\}$'s r that you choose as r_1 ! For our Solar system, only $\{N,1\}$'s r can be chosen as r_1 (governed by its de Broglie's QM condition $n\lambda = 2\pi r$), so each planet has a set of limited number of λ_n and f_n . It is just like each planet has a set of quantum number n (s) that depending on which r_n you choose as r_1 . For example, Jupiter's matter wavelength can be simultaneously $6.99E+10, 4.19E+11, \text{ and } 2.52E+12$ meters, and its total n can be simultaneously = 64, 10.7, and 1.8, and its r_1 can be simultaneously = $\{0,1\}, \{1,1\}, \text{ and } \{2,1\}$'s r .

4) The $H = h/m'$ value is controlled by m' value. Although in theory m' can be any value, but conditioned by $n\lambda = 2\pi r$, only a set of limited m' is meaningful for our Solar system's QM. When we choose $\{0,1\}$, or $\{1,1\}$ or $\{2,1\}$'s r as r_1 , the m' is $6.94E-49$ kg, or $1.16E-49$ kg, or $1.93E-50$ kg. These m' values are much smaller than the smallest mass particle (electron, neutrino, $2.2eV/c^2 = 3.9E-30$ kg) in the Wikipedia "List of Particles". At this time, no physics meaning can be found to correlate to this extraordinary small m' calculated from $H = h/m'$ for planets in Solar system.

5) In column 31-35 of Table 1, I set m' to a very small value, but has a known physics meaning, e.g., an electron's mass m_e ($=9.109E-31$ kg). Then the calculated planets' n are around $E+19$, which are too large to see any QM effect.

Why m' matters? Because formula $E_n = -Hmf_n$ not only valid for G-central-forced orbit movement, but also valid for the EM-central-forced orbit movement! For example in the hydrogen atom, if we want to calculate electron's orbit energy, we only need to set m' as electron's mass m_e , so that $E_n = -Hmf_n = -(h/m_e)m_e f_n = hf_n$, it come back as the old micro-world QM formula!

I-e. Apply $E_n = -Hmf_n$ to hydrogen atom's proton-electron system governed by EM-force

So in Table 2, I present how to use the same $E_n = -Hmf_n$ formula to calculate the electron's orbit energy in hydrogen atom (for electromagnetic central-force, not the gravity central-force field). Notice that in Table 2:

$$1) \text{ Calculate } v_n: F = ma = mv_n^2/r_n, \quad F = 1/(4\pi\epsilon_0) * Ze^2/r_n^2, \quad mv_n^2/r_n = 1/(4\pi\epsilon_0) * Ze^2/r_n^2, \quad mr_n v_n^2 = 1/(4\pi\epsilon_0) * Ze^2, \text{ or} \\ v_n = e * \text{sqrt}[Z/(4\pi\epsilon_0 m r_n)] \quad \text{eq-6}$$

$$2) \quad r_n v_n^2 = 1/(4\pi\epsilon_0) * Ze^2/m, \quad E_n = K + U = (1/2) * m v_n^2 - 1/(4\pi\epsilon_0) * Ze^2/r_n = -(1/2) * 1/(4\pi\epsilon_0) * Ze^2/r_n, \text{ or} \\ E_n = -(1/2) * 1/(4\pi\epsilon_0) * Ze^2/r_n \quad \text{eq-7}$$

3) The observed or Ritz $\lambda_{n \rightarrow n-1}$ values (in column 12 of Table 2) are obtained from:

Kramida, A., Ralchenko, Yu., Reader, J., and NIST ASD Team (2015). NIST Atomic Spectra Database (ver. 5.3), [Online]. Available: <http://physics.nist.gov/asd> [2017, May 29]. National Institute of Standards and Technology, Gaithersburg, MD.

Table 2. Using $E_n = -Hmf_n$ formula to calculate the single electron's orbit energy in hydrogen atom's (or H-atom like) EM-force-field (Note: $m = m' = m_e$).

Sun distance to Milky Way Center=	28000 light yrs =	2.647E+20 meter
Sun orbit period =	225000000 yrs =	7.096E+15 sec
Sun orbit speed =		2.344E+05 m/s
mass of Milky Way galaxy =		6E+42 kg
mass of Milky Way galaxy center (black hole)=		8.57E+36 kg
effective center mass of Milky Way for Sun's orbit*=		2.18E+41 kg
* calculated from $F=ma=m v_n^2/r_n$, $F=G*Mm/r_n^2$, $m v^2/r_n = G*Mm/r_n^2$, $r_n v_n^2 = GM$, $M=r v^2 /G$		

- 2) For G-force and EM-force, we can use the same formula, only need to exchange GMm with $Ze^2/(4\pi\epsilon_0)$.
- 3) In column 11, from known r_1 (6.63E+19m) I calculate out $H (=2\pi * \text{sqrt}(GMr_1))=1.95E+26$ J.s/kg), and copy this H for all rest objects.
- 4) In column 13, I use this known H to calculate $r_1 (=H/2\pi)^2/(GM)$, see eq-5) for all sized objects.
- 5) In column 14, I use the known r_1 to calculate $n (= \text{sqrt}(r_n/r_1))$ for all sized objects (except Milky Way-Sun).
- 6) In column 15, nH is calculated for all sized objects. Notice that for each object, this value is constant no matter what r_1 you choose (see section I-g).
- 7) In columns 16-18 I calculate out λ_n , f_n , E_n for all sized objects.

Result and discussion of Table 3:

1) To show H is a constant in $E_n = -Hmf_n$, we need to show that for all sizes of objects, and for all forces (including G and EM), their orbital E can be calculated using a single H. From Table 1's result-2 we know that H only related to r_1 . So (in Table 3 column 10) I choose $r_1=6.63E+19$ meters (which is 1/4 of Sun's orbit-r to Milky way center, so that Sun's orbit-r to Milky way will be $n=2$ under this r_1), and then obtain the corresponding $H=1.95E+26$. Then I use this H to calculate all objects' orbital E_n (in column 18). Amazingly the results are the same as that of classical E (in column 9). So it confirms that H is a constant in formula $E_n = -Hmf_n$ for all sizes of objects, and for both G & EM forces.

Note-1: Under this r_1 , the calculated $n=2$ (in column 14) for Sun orbiting Milky way, but it does not mean this is a good QM number for Sun's orbit, because it does not consider the rest stars that orbiting the Milky way. A good (or true) n number for Sun's orbit must be able to simultaneously give good n numbers for most (if not all) stars orbiting Milky way under the same $\{N,n\}$ QM system.

Note-2: Under this r_1 , all other calculated $n \ll E-10$, completely meaningless for QM, but still meaningful for λ_n , f_n , E_n calculation.

2) To show H is a quasi-constant in orbital E_n , we need to show that H is not a unique value for $E_n = -Hmf_n$. So from column 19 to 27 of Table 3, I choose another $r_1=5.28E-11$ meters, now the corresponding $H=7.27E-4$, and we set it as the new constant for all sized objects. The calculated E_n (in column 27) still match the classical E (in column 9). Also from column 28 to 36, I picked a third $r_1 (=1.66E+8)$, try to make Earth's orbit-r has total $n=30$, now the new constant $H=9.31E+14$. Again, the calculated E_n for all objects (in column 36) still matches the classical E (in column 9). It shows that although H is a constant for all sized objects in $E_n = -Hmf_n$ calculation, H can be another constant value for the same all sized objects, so H is not unique. Therefore, H is a quasi-constant in formula $E_n = -Hmf_n$. Again, the calculated n , λ_n , f_n may not necessary be meaningful in QM. Only those $n(s)$ satisfy all following three conditions will be meaningful in QM: a) $n=$ (or \approx) integer number, b) $1 \leq n < 100$, and c) not only one object, but all objects (planets, or stars) in the same central forced system simultaneously have $n(s)$ satisfy conditions a) and b).

3) Modern physics has been looking for the unification of G-force with other three basic forces for long time. Here I used a single formula $E_n = -Hmf_n$ and a single quasi-constant H for orbital energy (governed by both G-force and EM force) calculation. This method strongly suggests that these two forces have the same origin. Does this formula and H quasi-constant unifies G-force and EM-force? I believe so, although I am not sure (since I am only a citizen scientist level QM researcher). Even if this is not a true unification of G-force with EM-force, I expect this result will have some impact on the future study of unification of G-force with other three forces.

4) Because $H=h/m'=2\pi \cdot \sqrt{GMr_1}$, it is interesting to see that the higher the central mass (or the larger the r_1 size), the higher the H value.

I-g. The quasi-constant character of H came from the $Hn = H'n' = H''n'' = \text{fixed value for a circular moving object with a fixed orbit } r_n$

From Table 3, $H=h/m'=2\pi \cdot \sqrt{GMr_1}$, $n=\sqrt{r_n/r_1}$, combining two we obtain $Hn=2\pi \cdot \sqrt{GMr_n}$ for the G-central-forced circular movement. Or, $H=h/m'=2\pi \cdot \sqrt{Ze^2/(4\pi\epsilon_0)r_1/m}$, $n=\sqrt{r_n/r_1}$, combining two we obtain $Hn=2\pi \cdot \sqrt{Ze^2/(4\pi\epsilon_0)r_n/m}$ for the EM-central-forced circular movement. Both deduced results demonstrate that for any circular moving object, if its orbit r_n is a fixed value, then its $H*n$ is also a fixed value. Which means, we can have a set of different numbers for n (either n, or n', or n'') for this object's orbit, and they satisfy $H*n=H'*n'=H''*n''$.

From $H=h/m'=2\pi \cdot \sqrt{GMr_1}$, we know that the H value depends on where you choose r_1 . Once you have chosen r_1 , the object's orbit r will have a defined n. Table 4 shows one example: for Mercury, Venus, Earth, Mars, we can choose either {1,1}'r as r_1 , or {0,1}'s r as r_1 , each will give a different H, although Hn and $H'n'$ should be the same.

Table 4. To demonstrate that Hn equals to a fixed value for each planet, no matter what r_1 (or n) you choose.

NASA's data of planets				classical	using {0,1}'s r as r_1			using {1,1}'s r as r_1		
	mass	Sun's radius or planet orbit-r	planet orbit-v	$E=(1/2)mv^2$	$n=\sqrt{r_n/r_1}$	$H=h/m'=2\pi \cdot \sqrt{GMr_1}$	$Hn=$	$n=\sqrt{r_n/r_1}$	$H=h/m'=2\pi \cdot \sqrt{GMr_1}$	$Hn=$
unit	kg	m	m/s	J		J.s/kg	J.s/kg		J.s/kg	J.s/kg
Sun core {0,1}		1.74E+08			1	9.55E+14				
SUN {1,1}	1.989E+30	6.96E+08						1	5.73E+15	
Mercury	3.3E+23	5.79E+10	47400	-3.71E+32	18.2	9.55E+14	1.74E+16	3.0	5.73E+15	1.74E+16
Venus	4.87E+24	1.08E+11	35000	-2.98E+33	24.9	9.55E+14	2.38E+16	4.2	5.73E+15	2.38E+16
Earth	5.97E+24	1.49E+11	29800	-2.65E+33	29.3	9.55E+14	2.79E+16	4.9	5.73E+15	2.79E+16
Mars	6.42E+23	2.28E+11	24100	-1.86E+32	36.2	9.55E+14	3.46E+16	6.0	5.73E+15	3.46E+16

Column 15 and column 24 in Table 3 also shows that $H*n = \text{fixed value}$ for Solar system in Milky way, or for Earth in Solar system, or for electron in hydrogen atom, even theirs H or n can be very different values.

It is not every r_n can be chosen as r_1 . Only {N,1} in each N super-shell can be chosen as total $n=1$ (or as r_1). Therefore, the rule of $Hn = H'n'$ is only valid and useful between different N super-shells, but not within each N super-shell. Within each N super-shell, H is a constant for every n(s). For example, $Hn = H'n'$ is NOT suitable for hydrogen atom's electron orbit n between $n=1$, and $n=2$ (or any n), simply because that hydrogen atom electron orbits $n=1, 2, 3$, etc. belong to the same N super-shell, so they all have the same H (see column 9 in Table 2). Same thing for planets: all planets ($n=3, 4, 5, 6$) have the same H (see column 17 in Table 1). So this rule ($Hn = H'n'$) is only suitable for {N,n} QM system where there are multiple N super-shells, or equivalently, there are multiple r_1 (s) in this QM system!

I-h. The simultaneous multiple n for a single orbit in Solar QM {N,n} structure means a planet's real orbit n is composed of a base-frequency n that is modified by many multiplier n'(s).

That a single orbit (in Solar QM {N,n} structure) has multiple n simultaneously (see section I) has a real physics meaning. The lowest n (integer, and >0) number gives the major QM effect, the higher n numbers gives the minor QM effect: This is exactly like that in wave analysis, the base frequency gives the basic tone, the multiplier gives the modification of tone. Here I give one example to explain why. In the Solar {N,n} model, Earth has $n=5$, $5*6=30$, $5*6^2=180$, $5*6^3=1080$, ... etc. simultaneously. Its {1,5} modeled orbit should have orbit $r_n=r_1*n^2=1.74E+8 * 30^2=1.57E+11$ m. The real orbit r of Earth is =1.49E+11 m. The corresponding $n=\sqrt{1.49E+11/1.74E+8} \approx 29.26$, or $n \approx 4.88, 29.26, 175.58, 1053.47, \dots$ etc. simultaneously.

We know that the QM effect needs its quantum number $n = \text{integer}$. In column 3 of Table 4b below, I calculate the Earth's real n deviate from the integer number (using relative value $= [\text{Round}(n') - n'] / n'$). The result shows that the higher the n' , the lower the relative n' deviate from the integer number. For example, at $n' = 1053.47$, the relative deviation is only -0.0004 . So now the n' can be (comfortably) round as the integer 1053. Then we can explain Earth's QM as: it has a base QM quantum number $n = 5$ that make Earth in a $\{1, 5\}$ orbit. It also has the multiplier $n' = 1053$, which is -27 smaller than the strict quantum number $n' = 1080$ (of $n = 5$), so Earth's orbit r is modified from $1.57E+11$ m to $1.49E+11$ by this minor QM effect. This small modification of n' from 1080 to 1053 does not mess up its base QM effect as $n = 5$!

Table 4b. Calculate n' deviation from integer

Model {1,5}	Earth {1,5}	
	$n' = \sqrt{r/r_1}$	n' deviate from integer
$n =$		
5	4.88	0.0252
30	29.26	-0.0090
180	175.58	0.0024
1080	1053.47	-0.0004

Actually, a true orbit n of Earth is a combination of mixture of the base-frequency $n = 5$ (which is the major contributor) and many multiplier $n' = 29, 176, 1053, \dots$ (which are minor contributors). It is just like the piano's mid-C tone, the major frequency $= 261.6$ Hz contributes to the base tone of mid-C, and many minor multiplier frequencies makes the tone to be piano's, not other instrument's. So the real orbit n of all eight planets in the Solar system (or any celestial orbit n) are a base-frequency n that is modified by the multiplier n' . The mixture of base/multiplier n of a planet's QM must be caused by the interference of base/multiplier frequencies of its multiple matter waves. This means when a planet (or any object) moving in an orbit, its matter wave has a base wave and many multiplier waves simultaneously. So in paper SunQM-1, all I did was that I extracted out these base-frequency $n(s)$ and re-presented these base-frequency $n(s)$ in a $\{N, n/6\}$ QM model !

In paper SunQM-3s2, I will use this base/multiplier frequency theory to calculate the snapshot pictures of pre-Sun ball disk-lyzation process. In paper SunQM-3s4, I will use the same theory to study the Saturn's ring formation, planet's flattening, etc.

II. The new concept of H-C unit for both macro-world's QM and micro-world's QM.

In Table 3, by exchanging $G M m$ with $Z e^2 / (4 \pi \epsilon_0)$, we are able to use formula $E_n = -H m f_n$ (which was developed for planet's orbital energy calculation) to calculate the orbital energy for electron in hydrogen atom. Here I'd like to demonstrate that all formulas developed from hydrogen atom QM (like the Bohr radius a_0 , E_n , etc.) can be directly used for planet, simply by replacing Planck Constant h by a generalized quasi-constant $h_{\text{general}} = (h/m_e) m_{\text{planet}}$.

II-a. Bohr radius a_0 formula can be directly use to calculate the planet's QM orbit's r_1

The standard Bohr radius formula is $a_0 = (h/2\pi)^2 / m_e / (Z e^2 / 4 \pi \epsilon_0)$. In this formula, replacing as $a_0 \rightarrow r_1$, $h \rightarrow h_{\text{gen}} = (h/m_e) m_{\text{planet}}$, $m_e \rightarrow m_{\text{planet}}$, $(Z e^2 / 4 \pi \epsilon_0) \rightarrow G M m_{\text{planet}}$, it become $r_1 = (h/2\pi / m_e * m_{\text{planet}})^2 / m_{\text{planet}} / (G M m_{\text{planet}})$, or

$$r_1 = (h/2\pi / m_e)^2 / (GM) \quad \text{eq-8}$$

The result of this equation is shown in column 5 of Table 5, and they are almost same as result in column 35 of Table 1, where $H = h/m_e$.

II-b. Bohr Model's orbital energy formula can be directly used to calculate the planet's QM orbital energy E_n .

From Giancoli's book pp1005, eq-37-14a, the standard Bohr Model's orbit energy formula is $E_n = -(1/2) * [Ze^2 / (4\pi\epsilon_0 (h/2\pi)n)]^2 m_e$. In this formula, replacing as $h \rightarrow h_{gen} = (h/m_e)m_{planet}$, $m_e \rightarrow m_{planet}$, $(Ze^2 / 4\pi\epsilon_0) \rightarrow GMm_{planet}$, it become $E_n = -(1/2) * [GMm_{planet} / (h/2\pi/m_e) / m_{planet} / n]^2 m_{planet}$, or

$$E_n = -[GM / (h/2\pi/m_e) / n]^2 m_{planet} / 2 \tag{eq-9}$$

Remember here we use $h/m' = h/m_e$, so the planet's n(s) are around E19 ~ E20 (see column 6 of Table 5). The result of E_n is shown in column 7 of Table 5, and they are same as result in column 5 of Table 1.

Table 5. To demonstrate that all formulas in Bohr Model for hydrogen atom can be directly used for planets' QM calculation.

NASA's data of planets						
	mass	Sun's radius or planet orbit-r	planet orbit-v	$r_1 = (h/2\pi/m_e)^2 / (GM)$, m	$n = \text{sqrt}(r_p/r_1)$	$E_n = -[GM / (h/2\pi/m_e) / n]^2 m_{planet} / 2$
unit	kg	m	m/s			
Sun core		1.74E+08				
SUN	1.99E+30	6.96E+08				
Mercury	3.30E+23	5.79E+10	47400	1.01E-28	2.395E+19	-3.78E+32
Venus	4.87E+24	1.08E+11	35000	1.01E-28	3.271E+19	-2.99E+33
Earth	5.97E+24	1.49E+11	29800	1.01E-28	3.842E+19	-2.66E+33
Mars	6.42E+23	2.28E+11	24100	1.01E-28	4.752E+19	-1.87E+32
Jupiter	1.90E+27	7.78E+11	13100	1.01E-28	8.778E+19	-1.62E+35
Saturn	5.68E+26	1.43E+12	9700	1.01E-28	1.189E+20	-2.64E+34
Uranus	8.68E+25	2.97E+12	6800	1.01E-28	1.716E+20	-1.94E+33
Neptune	1.02E+26	4.51E+12	5400	1.01E-28	2.113E+20	-1.50E+33
Pluto	1.46E+22	5.91E+12	4700	1.01E-28	2.42E+20	-1.64E+29

Result (of Table 5):

- 1) Bohr radius formula can be directly used to calculate planet's r_1 (though it is not the good r_1 we like to see, because its n is too big to be meaningful for QM). The reason why n is too big is that in formula $h_{gen} = (h/m')m_{planet}$, I used $m' = m_e$. If I use a much smaller m' , e.g., $m' = 1.16E-49$ kg (see column 23 in Table 1), then Earth's $n=5$.
- 2) Bohr Model's orbital energy formula can be directly used to calculate the planet's QM orbital energy E_n .

II-c. H-C unit, a new concept for both macro-world's QM and micro-world's QM.

All above results suggest that the Planck constant (obtained for electron's QM) actually is a quasi-constant, it can be variable according to the body mass we are dealing with. The general form of h should be written as:

$$h_{general} = h_{planck} * m_{body} / m' \tag{eq-10}$$

where m_{body} is the mass of a body we are dealing with which moving circularly in a central force field, and m' is part of a scaling factor and it has a unit of kg. While the scaling factor m_{body}/m' scales $h_{general}$ value up (or down), the product of $nh = n_{Planck} * h_{Planck} = n_{general} * h_{general} = \text{constant}$ (or more accurately, proportional to a fixed value like r_n , or v_n , or E_n for this body)! For example, when dealing with planets in Solar system, while the $h_{general}$ scaled up by a factor = m_{planet}/m' , (or $h_{planet} = h_{Planck} m_{planet} / m'$), n is scaled down by the same factor m_{planet}/m' . QM calculated orbit r, or v, or E will keep the same value ($r_n = r_n$, $v_n = v_n$, and $E_n = E_n$). The only purpose to scale $h_{general}$ up or down is to keep n in a proper range (=1, 2, 3, ... <100, not E+20, or E-20) so that this n value is meaningful for QM.

In other words, h_{planck} is a unit (which happened to be obtained from the work dealing with hydrogen atom's electron), and n is the quantum number for that unit and n is obtained through QM calculation. If we dealing with planet (or galaxy, or proton, or quark, etc.), we need only to scale up (or down) the unit, and then we can obtain the meaningful n quantum number through the same QM calculation!

So here I introduced a new H-C unit ($h_{\text{general}} = H * m_{\text{body}}$). The new concept associated with this H-C unit is: in old way, we keep unit constant, and change formula or variable input value for calculation. In the new way: we keep QM formula (of hydrogen atom) unchanged, but only change H-C unit accordingly!

In other words, h_{general} is not a constant at all, it is a variable unit (named H-C unit), which make us able to use the same QM formula for hydrogen atom to calculate the orbit movement for all kinds of attractive-central-force system. Of course, in the system, there should be no electron-electron-type repelling interaction, and planet-planet (N-body) interaction should be (weak enough to be) ignored. A list of those attractive central-force systems is: G-force (solar system, galaxy), EM-force (single electron in hydrogen atom, He-ion, Li-ion, etc.). I believe that H-C unit is also valid for the strong force (and weak force) formed QM structure. This forms the basis of why QM orbital energy formula $E_n = -Hmf_n$ (or $E_n = -h_{\text{gen}}f_n$) is suitable for attractive central force of either G, or EM, or even strong force, weak force. It is also can be seen that for one kg of mass that doing orbit movement, $H=h_{\text{gen}}$. Therefore $H(=h/m')$ is the h_{gen} per one kg of mass that doing orbit movement, so it has unit of J.s/kg

II-d. Some useful formulas for both macro-world's QM and micro-world's QM.

If we know the value of $h_{\text{general}} (=h_{\text{gen}})$, then following formulas can help us to easily calculate out the rest (central-forced orbit) QM parameters:

$$n = 2\pi m v_{n\text{-orbit}} r_{n\text{-orbit}} / h_{\text{gen}}$$

$$r_1 = h_{\text{gen}}^2 / (4\pi^2 GMm^2) = [H/(2\pi)]^2 / (GM), \text{ for G-force}$$

$$a_0 = 4\pi\epsilon_0 (h_{\text{gen}}/2\pi)^2 / (Ze^2 m_e), \text{ for a single electron in hydrogen like atom, EM-force, (Z should be included in } a_0 \text{ to make } E_n = -hf_n \text{ as normal),}$$

$$r_{n\text{-orbit}} = n^2 r_{1\text{-orbit}}$$

$$r_{n\text{-orbit}} = n^2 a_0, \text{ (do NOT include Z here !)}$$

$$v_{n\text{-orbit}} = \sqrt{GM/r_{n\text{-orbit}}}, v_n = v_1 / n$$

$$v_{n\text{-orbit}} = e * \sqrt{Z/4\pi\epsilon_0 m r_{n\text{-orbit}}}, \text{ for EM-force.}$$

$$\lambda_{n\text{-orbit}} = 2\pi r_{n\text{-orbit}} / n, \lambda_n = n * \lambda_1$$

$$f_{n\text{-orbit}} = v_{n\text{-orbit}} / \lambda_{n\text{-orbit}} = v_{n\text{-orbit}} / (2\pi r_{n\text{-orbit}} / n), f_n = f_1 / n^2$$

$$E_{n\text{-orbit}} = -h_{\text{general}} f_{n\text{-orbit}}$$

Note: r_n, v_n, E_n change only upon n , NOT upon $H, h_{\text{gen}}, h', h'',$ or Z .

$$E_n = E_1 / n^2. E_n = -Hmf_n = -Hmf_1 / n^2$$

$H = \text{constant for all } n(s)$.

III. The meaning of Planck constant (and the H-C unit) is a quantum action of RF rotation.

In wiki "Planck constant", it says "*The Planck constant ... is the quantum of action*". One web site "<https://physics.stackexchange.com/questions/153807/what-is-the-point-of-the-reduced-plancks-constant-hbar-h-bar-why-don-t>" says "*Planck constant is a quantum of action. But what kind of action?*" has answer: "*a rotation*". So we understand why we have to put 2π , as it refers to a complete rotation". I like this explanation that Planck constant is a quantum action of rotation!

Here I moved forward one more step: H-C unit (or $h_{\text{gen}}=Hm$) is a unit of quantum action of RF rotation! It can be scaled up/down as required. Let me first explain what is RF. RF is a unit vector diffusion phenomenon in a way that only the vector's orientation is randomized, the vector length is keep unchanged. So it is a rotational diffusion, and I created a new

word "rotafusion", or abbreviated as "RF", to name the rotation diffusion. In hydrogen atom QM, the electron orbiting a nucleus generates a vector angular momentum \mathbf{L}_n at QM state n . When we comparing Schrodinger equation solution $L^2 = n(n-1)[h/(2\pi)]^2$ to Bohr's result $L = n h/(2\pi)$, it is obvious that at high n , they are almost same. The major difference is at low n , especially at $n=1$ (see Table 6 below).

One day (in early 2016) I suddenly realized that there is a simple way to solve the discrepancy: if I keep the \mathbf{L} vector length unchanged (=Bohr's \mathbf{L}), meanwhile randomize its orientation (like diffusion), then I can obtain Schrodinger's \mathbf{L} as the time averaged result of its vector's random RF. The reason of $L = n h/(2\pi)$ becoming $|L| = \text{sqrt}[n(n-1) h/(2\pi)]$ is that $\text{sqrt}[n(n-1)]$ reflect the time-averaged projection of RF movement of vector \mathbf{L} ! So Bohr's L does not include the RF effect, while Schrodinger's L does include the RF effect.

The physics meaning of RF is that at the QM $n=1$ ground state, the unit vector of the orbit's angular momentum $\mathbf{L}/|L|$ diffused to all directions in 3D space so that it is time-averaged value =0. The RF effect can be directly derived from Schrodinger equation solution. Here I give a (non-strict, or qualitative) derivation: in $n=1$'s wave function $|\text{nlm}\rangle = R(1,0)Y(0,0)$. The radial wave function $R(n,l)$ is always spherical symmetric. The spherical harmonic function $Y(0,0)=1/2/\text{sqrt}(\pi)$, suggesting the matter wave at $n=1$ is spherical symmetric, so that it has to be RF. I believe that the RF effect can also be directly derived from the known uncertainty principle $[L_x, L_y] = i \hbar L_z$ (I tried it, but not successful yet, see my paper SunQM-2s1).

The relative $|L|$ value change (in column 4 of Table 6, calculated as $\{n-\text{sqrt}[n(n-1)]\}/n$) can be used to reflect the level of RF: so at $n=1$ ground state it is 100% RF, $n=2$ decreased to 29% RF, $n=3$ further decreased to 18% RF, and at $n=36$ it is only 1% RF.

Table 6. Comparison of Schrodinger equation solution $L^2 = n(n-1)[h/(2\pi)]^2$ with Bohr's result $L = n h/(2\pi)$.

n	Bohr's $ L $ = n	Schrodinger's $ L $ = $\text{sqrt}[n(n-1)]$	% of RF =(B-S)/B
unit=	$h/(2\pi)$	$h/(2\pi)$	
1	1	0.0	100%
2	2	1.4	29%
3	3	2.4	18%
4	4	3.5	13%
5	5	4.5	11%
6	6	5.5	9%
36	36	35.5	1%

Actually the " $i\hbar$ " may be the true unit of quantum action of RF rotation. I believe that the complex number sign " i " of " $i\hbar$ " in equations (or operators) reflects the real-time RF movement. For example: uncertainty principle $[L_x, L_y] = i\hbar L_z$, $[x^\wedge, p^\wedge] = i\hbar$, $p^\wedge = -i\hbar \nabla$, $E^\wedge = i\hbar \partial/\partial t$. Meanwhile, all equations with \hbar (but without i) are the time-averaged RF projection. For example, $|L| = \text{sqrt}[n(n-1)] \hbar$ reflects the time-averaged RF of vector \mathbf{L} projected on $h/(2\pi)$ unit (see paper SunQM-2s1 for details).

So in Table 6, at $n=1$, Bohr's $L_{n=1} = h/(2\pi)$ tells us that $h/(2\pi)$ is a quantum action of rotation, and Schrodinger's $L_{n=1} = 0 \cdot h/(2\pi)$ tells us that $h/(2\pi)$ is a quantum action of RF rotation.

To make RF happen, it needs mass occupancy =100% (or close to 100%). In Schrodinger equation model, electron has 100% occupancy in n to $n+1$ orbit space shell, so its vector \mathbf{L} has RF. In Bohr model, electron take <1% of occupancy in n to $n+1$ orbit space shell (because it does circular movement only at r_n , in x - y plane), so its vector \mathbf{L} has no RF. Inside Sun of {0,1}RF ball, it is 100% mass occupancy, so mass (each molecule's vector \mathbf{L}) has RF. At {1,5}o orbit, Earth has <1% mass occupancy of orbit space {1,5}o, so the vector \mathbf{L} of Earth (orbiting around Sun) has no RF (see paper SunQM-2s1 for details).

It is easy to understand RF in hydrogen atom where a single electron orbiting proton at $r_1 = a_0$, after RF of its vector \mathbf{L} , the circular orbit forms a spherical surface. After add the elliptical orbit (like that of Sommerfeld's, but randomize both the length and the orientation of the semi-major axis), then the electron's orbit trace covers all spherical shell space (mainly from r_n to r_{n+1} in a simplified model, see paper SunQM-3s2). We can even view it in a slow motion movie (in classic mechanics)

like Mercury (pretend to be electron) orbiting Sun (pretend to be proton) in an apsidal precession motion, except that Mercury's precession orbit is in 2D (x-y plan), the electron's precession orbit around proton needs to be in 3D (r, θ , ϕ), so the semi-major axis' length also become a variable.

In 2016, after I gradually understood the electron's RF effect, I also figured out how to apply the RF effect to the hydrogen atoms inside the Sun. All atoms inside Sun are pretty much localized even they are doing thermal motion. The limited mean free path makes atoms unable to do the (free) precession movement for long time. **So inside the Sun it is the collection of matter waves (of all atoms) that doing the RF.** For a specific atom, at the end of its free path it collides with a 2nd atom, the 1st atom's original motion is stopped, but its matter wave is transferred to the 2nd atom, and the 2nd atom carries this matter wave (now it is a virtual matter wave) on its free path until it collides with the 3rd atom and transfer this virtual matter wave to the 3rd atom. The Sun has countless of atoms, and each atom carries (and transfers) several matter wave mode simultaneously (to different direction), so the collection of matter waves form RF effect. This RF theory, together with Solar QM {N,n} structure, can easily explain why Sun has temperature of 1E+6 degree at both inside Sun and corona, but only 5800K at Sun surface (see paper SunQM-2s1 for details).

For EM-central-force caused {N,n} QM, $h/2\pi$ is a unit of quantum action of RF rotation that has a value =1.055E-34 (J.s) for an electron orbiting around proton at $r_1=5.29E-11$ m. For G-central-force caused {N,n} QM, the equivalent $h_{gen}/2\pi$ is a unit of quantum action of RF rotation that has a value around 1.52E+14 (J.s) for a 1 kg object orbit around Sun with $r_1=r$ of {0,1} (=1.74E+8 m). It is calculated as: $h_{gen}/2\pi = H*m/2\pi$, (or = $h/2\pi *(m/m') = m * \sqrt{GM/r_1}$), for each 1 kg object orbiting a Sun-massed center, they are $h_{gen}=1.52E+14$ (J.s) for orbits {0,n=1..5}o, $h_{gen}=9.12E+14$ (J.s) for orbits {1,n=1..5}o, $h_{gen}=5.47E+15$ (J.s) for orbits {2,n=1..5}o (see Table 1 columns 16, 22, 28 for $H=h/m'$, and using $h_{gen}=H*1kg/(2\pi)$).

IV. The meaning of QM, the (1D and 3D) matter wave resonance, and the interior {N,n} QM analysis.

There are many ways to explain the QM meaning. Here is my version of QM meaning explanation.

IV-a. QM effect is the matter wave resonance effect.

Basically, QM effect is the matter wave resonance effect. Following are some examples:

- 1) According to text books, the reason of Bohr model's electron being able to do stable circular orbit movement is that its matter wave λ matches de Broglie condition $n\lambda=2\pi r$. In my explanation, the circular orbit is a one-dimensional matter wave resonance chamber, it amplifies the matter wave at $\lambda=1\times, 1/2\times, 1/3\times, \dots$ of circumference $2\pi r$ through wave resonance effect.
- 2) Guided by this matter wave theory, and combined with {N,n} QM model, the Earth's {1,5} circular orbit is a (one dimension) matter wave resonance chamber. It amplifies those matter waves with λ satisfying $n=5, 30, 180, \dots$, for $n\lambda=2\pi r$ through wave resonance in this chamber.
- 3) A Solar system is a 3D spherical matter wave resonance chamber. Let us set the {4,6}={5,1} as the spherical chamber's wall. Inside of this chamber, a mass of 1.99E+30 kg at the center generates a spherical matter wave interference pattern, so that it amplifies one set of matter waves at {N,n//6} shells start from {0,3} to {4,6} as planetary system (see the strict math deduction result in my paper SunQM-3, and SunQM-5). It also amplifies another set of matter waves at {N,n//6} shells start from center to {0,2} as Sun body.

IV-b. Furthermore, QM effect is the matter wave resonance effect combined with RF effect.

In wave physics we know that the short λ wave propagates in a more defined direction than that of the long λ wave. The same thing happened for the circular moving matter wave: in formula $n\lambda_n=2\pi r_n$, the higher the n , the shorter the λ_n , and the more accurate defined direction the wave propagates. This is exactly why in each $\{N,1\}=\{N-1,6\}$ shell in Solar system, the belt replaces the planet (e.g., Kuiper belt at $\{2,6\}=\{3,1\}$, and Asteroid belt at $\{1,8\}\approx\{2,1\}$). Because $\{3,1\}$ shows the $n=1$ property, so its orbit should be in 100% RF, but meanwhile it is also $n=6$, so it should be almost no RF (or 9% RF see table 6). So as the averaged result, the objects in $\{2,6\}=\{3,1\}$ shell have significantly higher RF intensity than its neighbors in $\{2,5\}$ and $\{3,2\}$ shells, which means, fragments in this shell intended to have higher orbit inclination and eccentricity, so they are unable to be accreted into a single planet. That is why Kuiper belt will remain as a belt forever due to its $\{N,1\}$ shell property, while its neighbor $\{2,5\}$ shell's mass formed Neptune, and the mass in another side neighbor $\{3,2\}$ most likely also formed a single planet which has not been discovered yet. The Asteroid belt also formed because of it has both $n=8$, and $n\approx 1$ QM effect, and will stay there forever due to it is at nearly $\{N,1\}$ shell.

Here let us define one way to characterize RF (at least semi-quantitatively) in our $\{N,n\}$ QM model:

- 1) In a Solar QM $\{N,n\}$ system dealing with a celestial body circular movement, I still use a classical circular orbit (rather than the probability density) to describe the motion.
- 2) Let us define that "one wave" means one complete (cycle of) wave, and "three waves" means three consecutive complete waves. So at $n=1$ circular orbit, it only needs one wave to finish one round ($\lambda=2\pi r$) of movement, and at $n=3$ circular orbit, it needs three consecutive waves to finish one round ($3\lambda=2\pi r$) of movement.
- 3) In a simplified model, let us define that the orbital angular momentum vector \mathbf{L} has 6 independent directions: $\mathbf{L} = \pm L_x, \pm L_y, \pm L_z$. A RF (= complete RF, or 100% RF) means that its vector \mathbf{L} has equal components on all 6 directions (so that the averaged vector $\mathbf{L} = 0$).
- 4) (inspired by $h/(2\pi)$ means a quantum action of rotation, see section III), let us simplify the vector \mathbf{L} 's RF model by assuming that each one complete circle of movement has a defined vector \mathbf{L} (meaning both the orientation and length of this \mathbf{L} vector is fixed), so it can only change its orientation (not the length) right after finishing one complete circle of movement, and just before start the next circle. In other words, the circular movement is quantized to every complete circle, the \mathbf{L} orientation can only change between each quantum, not within a quantum.
- 5) For a complete RF, it needs minimum of 6 round movement (each in one direction of $\pm L_x, \pm L_y, \pm L_z$) to get averaged vector $\mathbf{L} = 0$. For $n=1$ orbit, it means that it needs 6 waves to get a complete RF.

With this model, it is much easier to explain the interior $\{N,n\}$ QM.

IV-c. The interior $\{N,n\}$ QM structure analysis is a revolutionized new concept for QM (and even for the whole physics).

Since QM effect is a matter wave resonance in a chamber, it is nature to assign the chamber wall as $\{0,1\}$ and then to study the resonance (or QM) effect inside it. This is the interior $\{N,n\}$ QM analysis (see my paper SunQM-1). For an interior $\{N,n/6\}$ QM structure, it is more intuitive to write $\{-1,1\}$ as $\{0,1/6\}$, or $\{-2,3\}$ as $\{0,3/6^2\}$. But due to many times I need to write something like $\{-1,n=2..6\}$, and there is no simple way to express a collection of fraction numbers, so I have to still use $\{-1,1\}$, or $\{-2,3\}$. Only occasionally I will use $\{0,1/6\}$ for explanation.

From previous analysis we know that for $n=1$ (or $\{0,1\}$) QM ground state, a complete RF can be explained as a matter wave takes 6 waves to move circularly 6 rounds to cover all 6 directions for its vector \mathbf{L} . For $\{-1,1\}$ QM state, its $n=1/6$, what is its RF? $n\lambda=(1/6)\lambda=2\pi r, \lambda=6\times(2\pi r)$. Since its λ is $6\times$ of $2\pi r$, it takes only one wave to move circularly 6 rounds to cover all directions for vector \mathbf{L} . Here I name it as "level one RF", or symbolized as RF^1 . Then what about $\{-2,1\}=\{0,1/6^2\}=\{0,1/36\}$? Since its λ is $36\times$ of $2\pi r$, it takes only $1/6$ wave to move circularly 6 rounds to cover all 6 directions for vector \mathbf{L} . Here it is better to think 2 levels of vector \mathbf{L} randomization (RF): for the 1st level, all 6 directions will get RF, then for each one direction in the 1st level, we have another 6 rounds for completely RF it. So it becomes 2 levels of RF, I name it as RF^2 (meaning a second level randomization on top of the first level randomization)! Then for an interior QM $\{-15,1\}$, it must be fifteen levels of RF, or RF^{15} . Does this mean the higher level randomness (= more negative the N is), the more fundamental (or more stable?) the QM structure is?

Now let us look one example. Let us take Sun core as $\{0,1\}$, its matter wave $\lambda=2\pi r=2*3.14*1.74E+8=1.09E+9$ meters. Any proton inside Sun core has the interior QM size $\{-15,1\}$, so one wave of Sun core's λ makes 6^{15} round circular movement in proton, and makes 15 levels of RF! We know that (in traditional hydrogen atom QM where $n \geq 1$) $n=1$ ground state (or a complete RF state) is the most stable state (or $\{N,n\}$ structure) among all $n(s)$. This makes proton to be a super, super, ... (15 of it!) stable RF structure inside Sun core.

With above new concept, let us re-analyze the Solar $\{N,n\}$ structure using the interior $\{N,n\}$ QM analysis method. Let us first define "i $\{N,n\}$ " means it is a $\{N,n\}$ structure under the interior QM analysis (so the prefix "i" mean interior). As said in section IV-a, a Solar system can be treated as a spherical matter wave resonance chamber with its wall size at $\{4,6\}=\{5,1\}$. Now let us set this chamber's wall as $i\{0,1\}$. Inside it, a mass of $1.99E+30$ kg at the center generated a 3D spherical matter wave interference pattern which forms $i\{N,n\}$ QM structure. Following I will describe the Solar interior $\{N,n\}$ QM in either particle view or wave view. Notice that since we set $\{5,1\}$ as $i\{0,1\}$, the Sun core $\{0,1\}$ is automatically scaled down to $i\{-5,1\}$, the proton $\{-15,1\}$ is now $i\{-20,1\}$.

1) The particle view of Solar $i\{N,n\}$ QM.

Sun's gravitational interior QM generated a stable QM structure at size of $i\{-20,1\}$ (due to it has a very high level of RF, or RF^{20}), we call it proton (also can be neutron). There are total $1.19E+57$ of them (calculated as Sun's mass / proton's mass = $1.99E+30$ kg / $1.67E-27$ kg = $1.19E+57$). The space distribution of these $i\{-20,1\}$ QM structures follows the gravitational interior QM of $i\{0,1\}$, so that 99.9% of these $i\{-20,1\}$ QM structures stay in the ground state. Actually all of these $i\{-20,1\}$ QM structures stay in the ground state want to take the lowest energy state position which is at exactly the center of $i\{0,1\}$ structure. Due to $i\{N,n\}$ position exclusion principle (similar as Pauli exclusion principle), they cannot take the same space position, so most of them filled the $i\{N,n\}$ shell space from center to outer up to interior $i\{-5,1\}$, with 100% mass (or $i\{-20,1\}$ QM structure) occupancy and we call it Sun core. And rest of them filled a shell space up to $i\{-5,2\}$, which is also nearly 100% mass (or $i\{-20,1\}$ QM structures) occupancy (we call it Sun surface). All this forms a interior $i\{-5,2\}$ (equals to the exterior $\{0,2\}$) super large QM structure and we call it Sun.

Besides 99.9% in ground state, there are $< 0.1\%$ of $i\{-20,1\}$ QM structures in the excited states like $i\{-4,n=3..6\}$, $i\{-3,n=2..6\}$, ... etc. Among it, there are $3.57E+51$ of them (calculated as Earth's mass / proton's mass = $5.97E+24$ kg / $1.67E-27$ kg = $3.57E+51$) forms a stable QM structure in shell of $i\{-4,5\}$, we call it Earth. Among it, there are $4.19E+28$ of them (calculated as my body's mass / proton's mass = 70 kg / $1.67E-27$ kg = $4.19E+28$) forms a stable QM structure also in shell of $i\{-4,5\}$, we call it a human body. In this $i\{N,n\}$ model, we only have the total mass, and the space limitation for this mass, then the gravitational interior QM will determine the elementary particles, the mass distribution, ... etc.

2) The matter wave view of Solar $i\{N,n\}$ QM.

Note: MWRC meaning "matter wave resonance chamber", MWP meaning "matter wave packet". The whole Solar system is a huge combination of resonance matter waves which is confined by the centric G-force of its mass. We can treat it as a (3D spherical) matter wave resonance chamber (MWRC), it is equivalent to a $i\{0,1\}$ QM structure. There are all kinds of matter waves in this $i\{0,1\}$ chamber, but MWRC amplifies only some of them (by resonance), so they become stable matter wave. Most of matter waves in the chamber are suppressed. One of the amplified matter waves is the $i\{-20,1\}$ matter wave, and it becomes super stable MWP because it has high level of RF (RF^{20}). It carries mass of $1.67E-27$ kg, and we call it proton (or neutron). There are total $1.19E+57$ of this $i\{-20,1\}$ MWP (Sun's mass / proton's mass = $1.99E+30$ kg / $1.67E-27$ kg = $1.19E+57$), and in our current Solar system's MWRC, 99.9% of these $i\{-20,1\}$ MWP stay in the ground state. Actually all of these $i\{-20,1\}$ MWP stay in the ground state want to take the lowest energy state position which is at exactly the center of $i\{0,1\}$ structure. Due to $i\{N,n\}$ position exclusion principle (similar as Pauli exclusion principle), they cannot take the same space position, so most of them filled the $i\{N,n\}$ shell space from center to outer up to interior $i\{-5,1\}$, with 100% mass (or $i\{-20,1\}$ matter waves) occupancy and we call it Sun core. The rest of them filled a shell space up to $i\{-5,2\}$, which is also nearly 100% mass (or $i\{-20,1\}$ MWP) occupancy (we call it Sun surface). All this forms a interior $i\{-5,2\}$ (equals to the exterior $\{0,2\}$) super large matter wave packet structure and we call it Sun.

Besides 99.9% in ground state, there are $< 0.1\%$ of $i\{-20,1\}$ MWP in the excited states like $i\{-4,n=3..6\}$, $i\{-3,n=2..6\}$, ... etc. Among it, there are $3.57E+51$ of $i\{-20,1\}$ MWP in the shell space $i\{-4,5\}$ (= exterior $\{1,5\}$). They superimposed (through disk-lyzation and accretion) to form a big wave-packet, we call it Earth's matter wave packet. This

wave packet's movement is confined in a 1D-MWRC at $i\{-4,5\}$ we call it Earth's orbit, In this $n=5$ 1D-chamber, all matter waves are confined to resonance at $2\pi r = 5$ of λ , or $5*6=30$ of λ' , or $5*6^2=180$ of λ'' , etc., no matter it is a single $i\{-20,1\}$ matter wave packet, or a large wave-packet made of $3.57E+51$ of $i\{-20,1\}$ matter waves.

3) Noble gas elements are super stable, not only because they have the n shell completely filled, but also due to that a completely filled n -shell (or even l sub-shell) will greatly increase the RF, which provide extra stability! This is not only correct for EM-forced nucleus-electron system, it is also correct to G-forced Sun-planet system.

4) It should be pointed out that the $i\{-20,1\}$ MWRC not only amplifies Solar system's matter wave mode with RF^{20} , it also amplifies our galaxy's matter wave with RF^{23} , Virgo super cluster's matter wave with RF^{25} , even our universe's matter wave with RF^x (where x is still unknown). Besides, it also amplifies matter waves of quark, string, etc. So each different sized $i\{N,1\}$ MWRC amplifies a whole set (or a whole spectrum) of matter waves (from string to universe) with its own characteristic weight (here weight means the statistic weighting on all candidates), meanwhile, each matter wave is amplified in a whole set different sized $i\{N,1\}$ MWRC with different weight. So the relation between matter wave and MWRC is like the relation between frequency domain and time domain in Fourier transformation. Therefore, Fourier transformation can be a useful method to study the $i\{N,n\}$ QM.

IV-d. The best way to study our universe is to use the interior $\{N,n\}$ QM analysis.

We can expand this analysis to the galaxy, Virgo super cluster, or even to our universe. Now let us apply this interior $\{N,n\}$ QM analysis to our universe, and set the whole universe as the most top QM structure $i\{0,1\}$. Because we do not know how big the universe is, let us assume its size is $\{X,1\}$ under $Sun\{0,1\}$. According to my paper SunQM-1s2 (Table 1), our observable universe is at size of $\{11,5\}$, so X is a integer number ≥ 12 . Inside this $i\{0,1\}$ QM structure, there are many $i\{-X+10,1\}$ QM structures, one of them we call it the Virgo super cluster. It is formed by our universe's matter wave propagate (or resonance) in a $\{-X+10,1\}$ space, with $RF^{(X-10)}$. Also inside this $i\{0,1\}$ QM structure, there are even more of $i\{-X+8,1\}$ QM structures (we call them galaxies). It is formed by our universe's matter wave propagate (or resonance) in a $\{-X+8,1\}$ space, with $RF^{(X-8)}$. Again inside this $i\{0,1\}$ QM structure, there are many many $i\{-X,1\}$ QM structures, One of them we call it Sun core. It is formed by our universe's matter wave propagate (or resonance) in a $\{-X,1\}$ space, with RF^X . Again inside this $i\{0,1\}$ QM structure, there are many, many, many $i\{-X-15,1\}$ QM structures (we call them proton or neutron). It is formed by our universe's matter wave propagate (or resonance) in a $\{-X-15,1\}$ space, with $RF^{(X+15)}$.

Again, just like the short λ wave goes straight line and long λ wave goes curved line, a proton's matter wave goes straight inside Sun core, and a Sun core's matter wave goes curved line in proton (by RF^{15}). Or, a Sun core's matter wave goes straight inside universe, and a universe's matter wave goes curved line in Sun core by RF^X . Here the important concept is, our universe can be treated as the most top QM structure (with fixed total mass and fixed space, the relative slow universe expanding can be ignored here), its gravitational interior QM will determine the elementary particles, the mass distribution, ...etc.

Now if somebody says the proton is not element enough as the elementary particles, and want to use quark ($\{-17,1\}$ under $Sun\{0,1\}$) as elementary particle, then in above analysis we only need to replace X by $X-2$, that is all. But we know at the size of quark, the strong force play the key role, G-force is no longer playing any role. So let us define a degenerated force of four forces (G-, EM-, Strong-, and Weak-) as F4-force, a degenerated force of three forces (EM-, Strong-, and Weak-) as F3-force, and a degenerated force of two forces (EM- and Weak-) as F2-force. At scale of celestial, F4-force differentiated (or removed degeneracy) into G-force. At scale of $1E+6$ to $1E-11$ meters, F4-force differentiated (or removed degeneracy) mainly into EM-force. At scale below $1E-11$ meters, F4-force differentiated (or removed degeneracy) into Strong-force, Weak-force, and EM-force. So our previous (G-force driven) interior QM should be re-write as F4-force driven interior QM. When using this (F4-force driven) interior QM to analyze our universe, we can get a top-down view of the whole picture of our universe. Again, in this $i\{N,n\}$ model, we only have the total mass, and the space limitation for this mass, then the F4-force interior QM will determine the elementary particles, the mass distribution, the matter wave interference (or resonance) pattern, ...etc.

The traditional (exterior) QM provides us a bottom-up view of universe, a lot of details, but more difficult to get the whole picture of our universe. In contrast, the interior QM presents us a top-down view of the universe, which may have a profound impact on the current theory of our universe. So the key new concept here is our universe is a $\{0,1\}$ resonance chamber, and it is our universe's interior $\{0,1\}$ QM generated many different lower levels of $\{-N,1\}$ QM structures like galaxies, stars, protons, etc. To me, it is better than the concept that many protons build up our universe.

A new QM, named as C-QM, is created based on my studies in papers of SunQM-1, SunQM-2, SunQM-3, and their supplementary papers. C-QM is the combination of the interior $\{N,n\}$ QM, multiplier n' , $|R(n,l)|^2 |Y(l,m)|^2$ guided mass occupancy, and RF. C-QM is suitable for both macro-world and micro-world. In my paper SunQM-5, I will present how I have used the C-QM to study our real world (from string to universe).

Conclusion:

- 1) A general form of Planck constant has been discovered as $h_{\text{gen}} = h(m/m')$, where m is the mass of object we are studying (like electron, Earth, etc.), and m' is part of the scaling factor that scales h up or down. This h_{gen} is suitable for both micro- and macro- world's QM, as well as the EM- and G-forced QM. So for an object moving in a circular orbit, its QM orbital energy can be written as $E_n = -h_{\text{gen}} f_{n\text{-orbit}}$ (just like the traditional QM formula $E=hv$). Or we can write it as $E_n = -Hmf_{n\text{-orbit}}$, where $H=h/m'$ is a quasi constant (for all objects on all different orbits with same r_1 , and for both G-force and EM-force simultaneously).
- 2) A new concept of H-C unit is proposed: h_{general} is not a constant at all, it is a variable unit (named H-C unit), which makes us able to use the same QM formula for hydrogen atom to calculate the orbit movement for all kinds of attractive central-force system.
- 3) The simultaneous multiple $n(s)$ for a single orbit in Solar QM $\{N,n\}$ structure means a planet's real orbit n is composed of a base-frequency n and also modified by many multiplier $n'(s)$. The mixture of base/multiplier n of a planet's QM is caused by the interference of base/multiplier frequencies of its multiple matter waves.
- 4) A new concept of RF (rotafusion) has been proposed. It is the complete randomization of unit vector $\mathbf{L}/|L|$ at $n=1$ ground state.
- 5) A new concept of interior $\{N,n\}$ QM analysis has been proposed. It is based on the $\{N,n\}$ QM structure and RF theory. All structures in our universe, from string to universe itself, can be studied using the interior $\{N,n\}$ QM analysis.

References

- [1] A series of my papers that to be published (together with current paper):
- SunQM-1: Quantum mechanics of the Solar system in a $\{N,n/6\}$ QM structure.
 - SunQM-1s1: The dynamics of the quantum collapse (and quantum expansion) of Solar QM $\{N,n\}$ structure.
 - SunQM-1s2: Comparing to other star-planet systems, our Solar system has a nearly perfect $\{N,n/6\}$ QM structure.
 - SunQM-1s3: Applying $\{N,n\}$ QM structure analysis to planets using exterior and interior $\{N,n\}$ QM.
 - SunQM-2: Expanding QM from micro-world to macro-world: general Planck constant, H-C unit, H-quasi-constant, and the meaning of QM.
 - SunQM-2s1: RF theory (unfinished).
 - SunQM-3: Solving Schrodinger equation for Solar quantum mechanics $\{N,n\}$ structure.
 - SunQM-3s1: Using 1st order spin-perturbation to solve Schrodinger equation for nLL effect and pre-Sun ball's disk-lyzation.
 - SunQM-3s2: Using $\{N,n\}$ QM model to calculate out the snapshot pictures of a gradually disk-lyzing pre-Sun ball.
 - SunQM-3s3: Using QM calculation to explain the atmosphere band pattern on Jupiter (and Earth, Saturn, Sun)'s surface.

SunQM-3s6: Predict radial mass density distribution for Earth, planets, and Sun based on $\{N,n\}$ QM probability distribution.

SunQM-5: C-QM (a new version of QM based on interior $\{N,n\}$, multiplier n' , $|R(n,l)|^2 |Y(l,m)|^2$ guided mass occupancy, and RF) and its application from string to universe.

SunQM-5s1: White dwarf, neutron star, and black hole re-analyzed by using C-QM.

[2] The citation of wiki "Solar core" means it is obtained from the Wikipedia online searching for "Solar core". Its website address is: https://en.wikipedia.org/wiki/Solar_core. This website address can be generalized for all other searching items.

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